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Sharing scenarios facilitate division performance in preschoolers

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ABSTRACT

Understanding division is critical for later mathematical achievement. Yet division concepts are difficult to grasp and are often not explicitly taught until middle childhood. Given the structural similarity between sharing and division, we investigated whether contextualizing division problems as sharing scenarios improved preschool-aged children's abilities to solve them, as compared with other arithmetic problems which do not share structural similarities with sharing. Preschoolers ($N = 113$) completed an addition, subtraction, and division problem in either a *sharing context* that presented arithmetic via contextualized sharing scenarios, or a comparable, linguistically-matched *non-social context* (randomly assigned). Children were assessed on their formal, verbal responses and their informal, non-verbal, action-based responses (abilities to solve the problems using manipulatives) to these arithmetic problems. Most critically, context predicted children's performance on the division, but not the addition or subtraction trial, supporting a structural link between sharing and division. Results also revealed that children's action-based responses to the arithmetic problems were much more accurate than their verbal ones. Results are discussed in terms of the conceptual link between division and sharing.

1. Introduction

Division is key for understanding critical numerical concepts such as fractions, algebraic operations, and other rational numbers (e.g., Bailey, Siegler, & Geary, 2014; Booth & Newton, 2012; DeWolf, Bassok, & Holyoak, 2015; Siegler et al., 2012; Siegler & Pyke, 2013; Stafylidou & Vosniadou, 2004). In spite of its importance, division concepts are difficult to grasp and are the last arithmetic operation taught according to current US curricula (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Yet, work in the domain of prosocial development has found children as young as preschool age are capable of engaging in structurally similar activities such as dividing resources equally amongst two parties in the interest of promoting the norm of fairness (e.g., Chernyak, Sandham, Harris, & Cordes, 2016; Chernyak, Harris, & Cordes, 2019; Sarnecka & Wright, 2013). In the current study, we explored whether contextualizing math problems as social sharing scenarios would lead to better performance on division problems in particular.

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1.1. Learning formal division

The Common Core suggests that division be taught in third grade, making it the last arithmetic operation introduced in the classroom (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Although understanding that division is the inverse of multiplication is a Common Core standard for third graders, research shows that very few 6th–8th grade students and adults use an inversion strategy when performing division problems. Instead, both children and adults rely on using less sophisticated calculation strategies, suggesting a relatively weak understanding of division (Dubé & Robinson, 2018; Robinson & Dubé, 2009; Robinson & Ninowski, 2003; Robinson, Ninowski, & Gray, 2006). Given children and adults' limited understanding of division, it is critical to find ways to facilitate early division understanding. Importantly, division forms the foundation upon which children acquire more advanced mathematics (e.g., fractions, algebra; DeWolf et al., 2015). Thus, supporting children's early understanding of division will better prepare them to complete more advanced math later on.

1.2. Sharing

While children do not grasp formal division until middle childhood, even preschool-aged children understand a theoretically-related concept: sharing. Prior work demonstrates that by preschool, children understand the principles of equal sharing both conceptually (Sarnecka & Wright, 2013; Schmidt & Sommerville, 2011; Smith, Blake, & Harris, 2013) and behaviorally (Chernyak et al., 2016, 2019; Muldoon, Lewis, & Berridge, 2007; Olson & Spelke, 2008). The discrepancy between children's success on sharing tasks and their difficulty with division is particularly surprising given that sharing and division are structurally similar – both involve attending to a dividend (resources to be shared) as well as a divisor (number of entities receiving the resources; Correa, Nunes, & Bryant, 1998; Squire & Bryant, 2002a; 2002b; see also Muldoon, Lewis, & Freeman, 2009). Despite their structural similarity and the fact that sharing is a social instance of division, the literature has consistently considered these two concepts as distinct. That is, work in numerical cognition and arithmetic has proceeded separately from work on sharing behavior (see Squire & Bryant, 2002a; 2002b for exceptions). Considering the structural similarities between sharing and division, in this work, we reasoned that presenting division problems as sharing scenarios may be a particularly fruitful way to scaffold children's understanding of division.

1.3. Division and sharing

While many educators use sharing contexts to introduce division concepts, no prior work has systematically investigated whether sharing scenarios *promote* division performance above and beyond non-social ones. A series of studies conducted by Squire and Bryant (2002a), (2002b) demonstrate that children can perform division in the context of a sharing scenario. In their work, children were instructed to distribute different numbers of candies equally to a given number of dolls (i.e., divide 12 candies between 3 dolls, Squire & Bryant, 2002a, 2002b, see also Correa et al., 1998; Frydman & Bryant, 1988). Children's success on this task led the researchers to conclude that sharing scenarios *may* support children's division understanding, yet they had no control group, making it impossible to determine whether children's understanding of division was better in the sharing scenario than in a non-sharing scenario. That is, children's division understanding was only assessed in a sharing context so performance in the sharing context was never compared to children's performance in a non-social context, leaving it unclear whether sharing scenarios promote division performance over non-social ones.

1.4. Social contexts for learning

Prior work suggests contextualizing problems may facilitate children's cognitive development. Substantial data shows that children are more competent problem-solvers when working in certain contexts, especially social ones. For example, research reveals storytelling contexts facilitate learning more generally (Bruner, 1987; Schank & Abelson, 1995, 1995) and also within the domain of mathematics (Casey, Erkut, Ceder, & Young, 2008; Casey et al., 2008; Cordova & Lepper, 1996; Jennings, Jennings, Richey, & Dixon-Krauss, 1992). Moreover, previous work has shown the benefit of presenting math problems as ecologically valid, real-life situations. For example, child vendors selling groceries on the streets in Brazil are unable to perform symbolic math problems, yet have no difficulty making change for their customers (Carraher, Carraher, & Schliemann, 1985; Saxe, 1988) and adults are better at arithmetic problem solving while shopping for groceries compared to taking a math test (Lave, Murtaugh, & De La Rocha, 1984). Contextualized problems thus have the advantage of mimicking the types of problems children may encounter in their daily lives, and making abstract math problems more concrete.

Although prior work has shown children can perform division in the context of a sharing scenario (Squire & Bryant, 2002a, 2002b), it is unknown whether the sharing scenario specifically, or the social nature of the sharing story facilitates performance. Because of the unique structural similarities between sharing and division, one may assume that early demonstrations of fair sharing may inherently give way to deeper understandings of formal division if children can connect the two. Alternatively, the contextualized nature of sharing as a whole may afford a similar benefit for *all* arithmetic problems by way of increasing motivation. In the present study, we aim to disentangle these possibilities by comparing children's division, addition, and subtraction performance in a sharing context to that of a non-social context. We investigate a) whether sharing provides a benefit above and beyond a non-social scenario and b) whether the performance benefits of sharing scenarios are unique to division problems, or apply to most arithmetic operations (addition and subtraction problems as well).

1.5. Response types

In addition to comparing arithmetic performance in sharing and non-social contexts, we also included two measures of children's arithmetic performance: an explicit, *verbal response*, in which we measured children's abilities to explicitly state the numerical answer to the problem, and a non-verbal, *action-based response* in which we measured children's ability to solve the problem implicitly (i.e., actively add, subtract, or divide resources in the manner requested by the experimenter). Several studies reveal that action-based responses are often more fruitful for tapping into young children's implicit conceptual knowledge (Alibali & Goldin-Meadow, 1993; Goldin-Meadow, Alibali, & Church, 1993; Ginsburg, Klein, & Starkey, 1998; Kibbe & Feigenson, 2015; Perry, Church, & Goldin-Meadow, 1988). Action-based responses, in contrast to verbal ones, may be particularly helpful for some children. For example, children from low income homes show substantially better performance on non-verbal compared to verbal calculation tasks, whereas middle-income children show similar performance on both response types (Jordan, Huttenlocher, & Levine, 1994). Action-based responses may improve performance either through the process of making abstract math equations more concrete (Martin & Schwartz, 2005; Manches, O'Malley, & Benford, 2010; Montessori, 1964; see also Carbonneau, Marley, & Selig, 2013 for a meta-analysis and Pouw, Van Gog, & Paas, 2014 for a review), through the use of manipulatives or other external representations, or through tapping into implicit conceptual knowledge, rather than formal, symbolic knowledge. As such, it may be the case that action-based responses may be privileged in situations in which task demands are challenging for the child.

1.6. The current study

In this study, we presented three types of arithmetic – addition, subtraction, and division – as either sharing scenarios or as non-social scenarios to preschool-aged children. We predicted that presenting division problems as sharing scenarios may facilitate preschoolers' abilities to solve them because there are special structural links between the cognitive process involved in sharing and division (e.g., Muldoon et al., 2009). In particular, we hypothesized that presenting division problems as sharing scenarios would draw children's attention to the importance of creating equivalent sets. Because addition and subtraction are not structurally similar to sharing, we did not predict sharing scenarios to also facilitate performance on these arithmetic problems. Thus, if there is a unique link between sharing and division, then context should predict children's performance on the division trial only. However, if sharing scenarios simply boost performance by presenting math problems in a more engaging and relevant context, then we would predict that presenting *any* arithmetic problem (not simply division) as a sharing scenario would enhance performance. Thus, including addition and subtraction trials was critical for determining whether sharing and division have a privileged relationship in arithmetic learning.

In order to determine whether our results generalized across differing levels of problem difficulty, half of the children completed problems with a small set size (4) and half with a larger, and thus more difficult set size (8). Since working with larger sets is more difficult for children (e.g., Geary, 1996; Mulligan, 1992; Posid & Cordes, 2015a, b; for a review see Ashcraft, 1992; Ashcraft & Guillaume, 2009), we wanted to ensure that any effects of the sharing context were consistent when working with more complex problems. Importantly, we chose these set sizes as they have been used in prior research on sharing (Chernyak et al., 2016, 2019).

Children solved problems using words only (verbal responses) and also with physical manipulatives (action-based responses) in order to explore how response modes may impact accuracy on our arithmetic problems. Given that previous research has shown children are able to produce action-based responses prior to verbal ones (e.g., Kibbe & Feigenson, 2015), we predicted that these responses would be more accurate than their verbal ones, regardless of context.

2. Method

2.1. Participants

One-hundred and thirteen children (62 females, 51 males) ranging from 37 to 72 months of age ($M = 57.94$ months, $SD = 8.30$) were tested through the laboratory ($n = 39$), or off-site in local museums, parks, or preschools ($n = 74$). Sample size was based on sample sizes used in previous work investigating children's understanding of sharing (Chernyak et al., 2016, 2019). Additionally, a formal power analysis using G*Power assuming a small to medium effect size with a moderate correlation between measures indicated that a sample size of 48 participants would be sufficient. We chose to have a minimum of $n = 20$ per condition (Simmons, Nelson, & Simonsohn, 2011), but continued until at least $n = 24$ per condition in order to achieve full counterbalancing. Additional children were tested to correct counterbalancing errors. An additional 16 children participated in this study but were excluded for not speaking English ($n = 1$), being outside of the selected age range ($n = 1$), parental interference ($n = 2$), no video recording (due to parental consent or equipment failure; $n = 8$), or experimenter error ($n = 4$).

We opted to focus on the preschool age, when children show the greatest age-related changes in their abilities to divide and compare equal sets (Chernyak et al., 2016, 2019) and when they are still unlikely to have encountered any formal arithmetic education, thus allowing us to explore how contexts may influence their understanding of these concepts before formal instruction. Demographic information was collected for a subset of children who participated in the laboratory (~35 % of children); however, some parents opted out of providing demographic information. Thus, we had demographic information for ~25 % of our sample. Of the participants who completed demographic information, 72.4 % identified as White/Caucasian, 10.3 % Asian, and 17.2 % as bi-racial. Maternal education was high amongst those who reported demographic information: 53.6 % of mothers reported having a Master's degree. Children participating in museums, parks, and preschools were recruited from the same areas and thus were expected to be of similar demographics. For example, approximately 69 % of visitors to a similar museum exhibit where the study was conducted

Table 1
Demographic Information divided by the four conditions. Children were randomly assigned to condition.

Condition	Mean Age	Male	Female	Testing Location: Lab	Testing Location: Off-site
Sharing, Small Set Size	57.58 months (SD = 8.64)	16	17	17	16
Sharing, Large Set Size	55.88 months (SD = 5.27)	11	13	5	19
Non-Social, Small Set Size	59.19 months (SD = 10.68)	15	16	12	19
Non-Social, Large Set-Size	58.88 months (SD = 6.67)	9	16	5	20

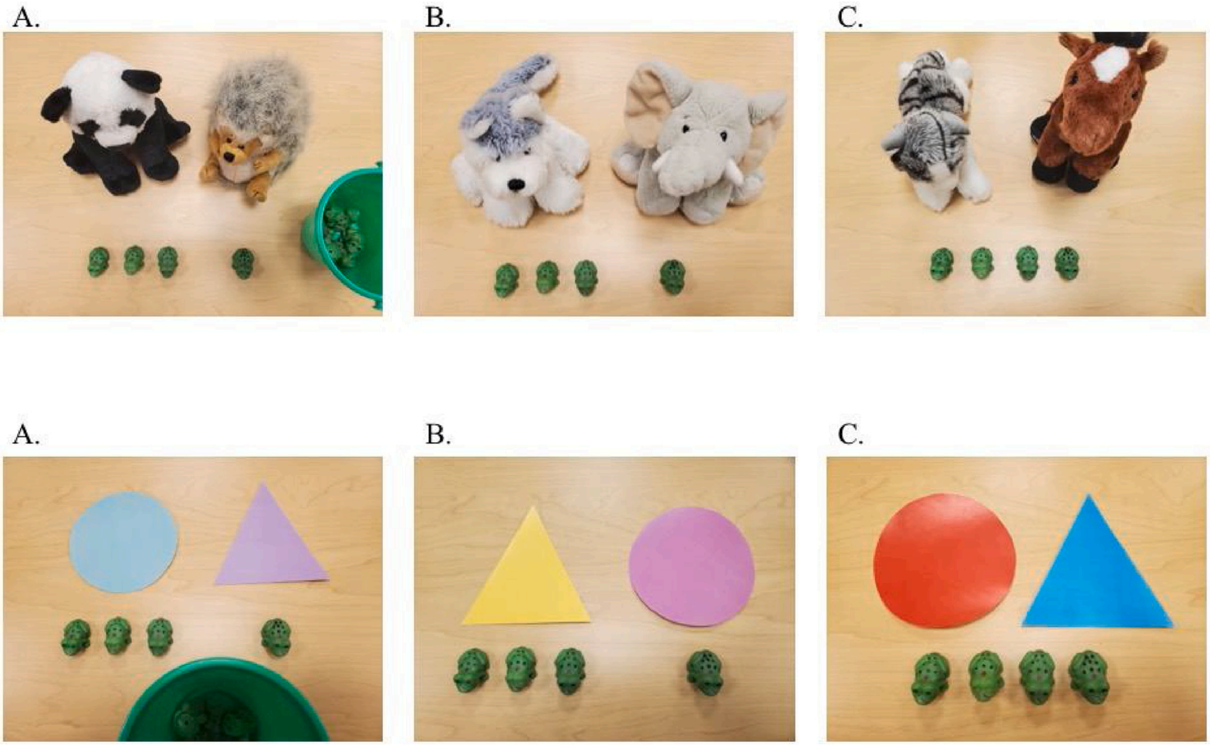

Fig. 1. Addition (A), Subtraction (B), and Division (C) Trials in the Sharing Condition (Top) and the Non-Social Sharing Condition (Bottom).

identify as White, and 50 % of the visitors to the exhibit had a graduate degree. Demographic information (average age, gender distribution, and testing location: in lab or off-site) for the participants in each of the four conditions are listed in Table 1.

2.2. Procedure

All procedures were approved by the Boston College Institutional Review Board. Children completed an arithmetic task in one of two contexts (sharing or non-social context) involving one of two set sizes (4 or 8). Although the arithmetic problems were different from those typically found in formal math assessments, the problems mimicked previous work used in studies of resource distribution (e.g., Chernyak et al., 2016, 2019) and assessments of children's informal math knowledge (Kibbe & Feigenson, 2015).

Children also completed two additional tasks: a counting assessment prior to the arithmetic task (Give-N, modeled after Le Corre & Carey, 2007) in order to test for children's counting proficiency[1]; and a nonsymbolic division task following the arithmetic task. Given the unique relation between sharing and division, we intended to investigate whether sharing contexts have an immediate benefit as well as any transfer effects on a subsequent non-social division task. Thus, the nonsymbolic division task was included so as to assess whether performance on the arithmetic task may transfer to a distinct division task. However, likely because our nonsymbolic division task was conceptually and perceptually distinct from our arithmetic tasks, we found no relation in performance between the two tasks. Thus, we have chosen not to discuss this task in the body of the paper. Information regarding the nonsymbolic division task can be found in the Appendix.

[1] Our original intent was to examine how counting performance might relate to sharing behavior. However, there was very little variability in our sample, with the majority of children ($n = 88$) being classified as Cardinal Principle knowers (i.e., proficient counters). Thus, we excluded Give-N from our primary analyses. All of the main behavioral analyses remain consistent when including Give-N performance.

2.2.1. Arithmetic task

We employed a 2 (Context Type: Sharing vs. Non-Social) x 2 (Set Size: 4 vs. 8) between-subject design, such that each child was assigned to one of four conditions. Half of the children were assigned to complete trials within a Sharing Context and half within a Non-Social Context. Within each context type, half of the children were assigned to the Small Set Size in which they completed trials using 4 small frog toys, and half were assigned to the Large Set Size in which they completed trials with 8 toys. Each condition consisted of three trials (order counterbalanced via a Latin Square design) (1) an *addition* trial, (2) a *subtraction* trial, and (3) a *division* trial, in which each child was tasked with equally distributing resources.

2.2.1.1. Sharing context. In the Sharing Context, on each trial, children were introduced to two plush animals (e.g., "Panda" and "Hedgie") and asked to divide resources equally between the animals in one of three different ways (depending on trial type).

In the *addition* trial, participants were presented with a panda that had 3 (or 6 in the Large Set) toys and a hedgehog that had only 1 (or 2) toy(s) (Fig. 1, Panel 1, A). Children were then shown a bucket full of toys and were told they could *add* additional toys from a bucket and give them to hedgehog so that both animals had the same number of toys ("You can give some of these toys (from the bucket) to [Hedgie] so that they both have the same."). In the *subtraction* trial, children were presented with a dog that had 3 (or 6) toys and an elephant that had 1 (or 2) toy(s) (Fig. 1, Panel 1, B). Children were told they could *take away* some of the dog's toys and give to the elephant so that they had same number of toys ("You can give some of [Doggie's] toys to [Ellie] so that they have the same."). Finally, in the *division* trial, children were presented with a cat and a pony (Fig. 1, Panel 1, C) and 4 (or 8) toys that were placed in the middle between the two animals. Children were told to *divide* the toys between the cat and pony so that they had the same number of toys ("You can give some of these toys to [Kitty] and some of these toys to [Pony] so that they both have the same.").

On each trial, children were asked to first provide a Verbal Response ("How many should you give to [Hedgie] so that they both have the same?"[2]), and then an Action-Based Response such that children were asked to evenly distribute the toys ("You can give some to [Hedgie] so they have the same."). In order to emphasize the social nature of the sharing context, children in the Sharing Context were asked to "give" toys to the characters. The three trials (addition, subtraction, division) and location of the animals/shapes (to the child's left or right) were counterbalanced.

2.2.1.2. Non-social context. The Non-Social Context was matched exactly except that children were introduced to two shapes (e.g., "red circle" and "blue square") and children were asked to distribute toys between the shapes.

In the *addition* trial, participants were presented with a circle shape that had 3 (or 6 in the Large Set) toys and a triangle that had only 1 (or 2) toy(s) (Fig. 1, Panel 2, A). Children were then shown a bucket full of toys and were told they could *add* additional toys from a bucket and put them on the triangle so that both shapes had the same number of toys ("You can put some of these toys (from the bucket) on the [triangle] so that they both have the same."). In the *subtraction* trial, children were presented with a triangle that had 3 (or 6) toys and a circle that had 1 (or 2) toy(s) (Fig. 1, Panel 2, B). Children were told they could take away some of the triangle's toys and put them on the circle so that they had same number of toys ("You can put some of the [triangle's] toys on the [circle] so that they have the same."). Finally, in the *division* trial, children were presented with a circle and a triangle (Fig. 1, Panel 2, C) and 4 (or 8) toys that were placed in the middle between the two shapes. Children were told to *divide* the toys between the circle and triangle so that they had the same number of toys ("You can put some of these toys on the [circle] and some of these toys on the [triangle] so that they both have the same.").

As in the Sharing Context, on each trial, children were asked to first provide a Verbal Response followed by an Action-Based, Behavioral Response. To emphasize the non-social nature of this context, children were asked to "put" the same number of toys on two unique shapes (see Fig. 1, Panel 2) rather than "give" them to the shapes. At the end of the study, parents were debriefed and children received a small toy for participation.

2.3. Data coding

All children were videotaped for later coding with the exception of 8 children whose parents either did not provide parental consent to video record or who experienced equipment failure during their participation. As mentioned previously, children who were not videotaped were excluded from the study. All videos were coded by two independent coders ($\kappa = .97$). Disagreements were settled by a third coder. We coded the following data for each child:

2.3.1. Arithmetic task: action-based performance

For each trial, we coded whether or not the child correctly split the toys equally between the two puppets (either a 1 or 0). Six children ($n_{\text{Subtraction}} = 2$, $n_{\text{Division}} = 4$) made equal splits using only a subset of the available toys and refused to use the rest of the toys when prompted. These trials were excluded from the analyses. Trials in which there was experimenter error ($n = 6$) or a poor camera angle which prevented the experimenter from determining whether the child made an equal split ($n = 6$) were also excluded. Given these criteria, 16 participants had missing data on 1 ($n = 13$) or 2 trials ($n = 3$) for the behavioral responses.

[2] In the division trial, children were asked two separate questions: "How many should you give to [Kitty] so they both have the same? And how many should you give to [Pony] so they both have the same?"

Table 2a
Spearman's Correlations of Correct Behavioral Responses on the Arithmetic Task Across Condition and Set Size.

	Addition	Subtraction	Division
Addition			
Subtraction	.484**		
Division	.340**	.415**	

Note:
** $p < .01$.

Table 2b
Spearman's Correlations of Correct Verbal Responses on the Arithmetic Task Across Condition and Set Size.

	Addition	Subtraction	Division
Addition			
Subtraction	.309*		
Division	.491**	.261	

Note:
* p < .05.
** $p < .01$.

Table 3
Proportion of Correct Behavioral Responses on the Arithmetic Task Across Condition, Set Size, and Trial Type.

Small Sets			
	Addition	Subtraction	Division
Sharing	.81 (.07)	.77 (.08)	.94 (.04)
Non-Social	.83 (.07)	.71 (.08)	.77 (.08)
Large Sets			
	Addition	Subtraction	Division
Sharing	.44 (.12)	.55 (.11)	.82 (.08)
Non-Social	.46 (.10)	.58 (.10)	.65 (.10)

Note. Standard error in parentheses.

2.3.2. Arithmetic task: verbal responses

We coded whether children's verbal responses were correct (coded as 1) or incorrect (coded as 0). Because children were asked two separate questions on the division trial, they could have provided two different verbal responses. In this case, the average of the two responses was considered. If a trial had been excluded for experimenter error (described above), then their verbal response was also coded as missing data. Counting was considered an incorrect response, however, if children counted and then also gave the final number (e.g., "1, 2, 3, that's 3"), the final number was coded as either correct or incorrect. Non-numerical responses (e.g., "this many") were coded as incorrect. Non-responses and uninterpretable data were coded as missing data. Although non-responses may have indicated that children did not know the answer, it is also possible that children failed to respond because they were too timid. As such, we coded non-responses as missing data.[3] Given these criteria, children could have had 1 ($n = 25$), 2 ($n = 24$) or 3 ($n = 41$) coded verbal responses. Twenty-three children had no coded verbal data. Children provided verbal responses on ~58 % of the trials.

3. Results

Preliminary analyses showed no main effects or interactions of gender or the order in which the arithmetic problems were completed (all p's > .05), so data were collapsed across these variables. Correlation matrices showing correlations between each trial type (addition, subtraction, and division) can be found in Table 2a, 2b.

3.1. Does the sharing context facilitate children's action-based responses on an arithmetic task?

Our first question concerned the predictors of success on the arithmetic task. Because children's responses were binary (coded as correct or incorrect), we used a binomial mixed model with compound symmetry as the covariance structure. The covariance structure was chosen as we expected each of the three arithmetic trials would be equally correlated with one another and the variance of each

[3] The number of verbal responses children provided (number out of 3) was not correlated with Age ($r = .042$, $p = .699$) or behavioral performance (number correct out of 3), $r = .161$, $p = .128$).

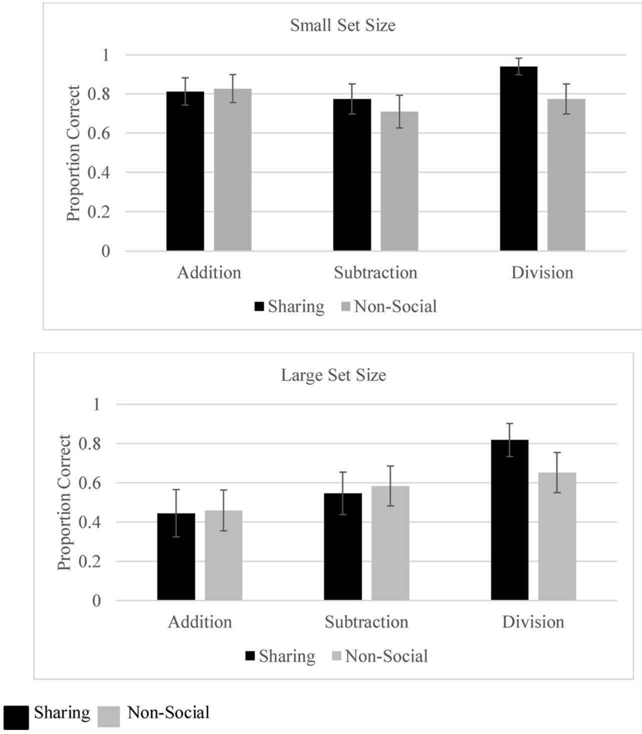

Fig. 2. Proportion of Correct Behavioral Responses on the Arithmetic Task Across Condition, Set Size, and Trial Type.

trial would be comparable. We first performed a regression using Trial Type (addition, subtraction, division; entered as a within-subjects repeated measure), Context (sharing vs. non-social context; dummy coded as 1 for sharing), Set Size (small vs. large; dummy coded as 1 for large), and Age (in months) as predictors and likelihood of providing a correct action-based response (equal sharing: 0 or 1) as the response variable. We also included all interactions of the manipulated variables (Trial Type, Set Size, and Context[4]). There was a significant effect of Age, $F(1, 306) = 22.77$, $p < .001$, revealing that older children were more likely to provide a correct response. There was also a main effect of Set Size, $F(1, 306) = 9.50$, $p = .002$, such that, as hypothesized, those working with the Small Set Size ($M = .81$, $SD = .40$) outperformed those working with the Large Set Size ($M = .59$, $SD = .50$). Additionally, there was a main effect of Trial Type, $F(2, 306) = 5.21$, $p = .006$. Follow-up McNemar's tests revealed that children were most accurate in the division trial ($M = .81$, $SD = .40$) relative to the addition ($M = .67$, $SD = .47$, $p = .019$) and subtraction ($M = .67$, $SD = .47$, $p = .015$) trials. Performance did not differ between the addition and subtraction trials, $p = 1.0$. No other main effects or interactions reached significance (p's $> .1$). See Table 3 and Fig. 2. The full regression model can be found in Table 4.

To further test our hypothesis that context may be uniquely important for the division trial, we first compared performance on each trial within each Context separately. McNemar's tests revealed that in the non-social condition, children performed equally on all three trials (p's $> .4$). However, those in the sharing condition performed significantly better on the division trial compared to the addition ($p = .002$) and subtraction ($p = .012$) trials. Performance in the sharing condition did not differ on the addition and subtraction trials ($p > .7$).

Next, we performed additional analyses exploring the effect of Context on each Trial Type separately. To do so, we conducted a logistic regression with Set Size, Context, and Age as predictors of performance on each trial type separately.

For the addition trial, the logistic regression model was significant $\chi^2(3) = 35.96$, $p < .001$, explained 41.5 % (Nagelkerke R^2) of the variance, and correctly classified 78.4 % of the cases. The Hosmer and Lemeshow Test indicated that the model fit the data well ($\chi^2(8) = 9.09$, $p = .335$)[5]. Set Size (β = −2.14, Exp(B) = .118, $p < .001$) and Age (β = .144, Exp(B) = 1.16, $p < .001$) significantly predicted accuracy, but Context (β = .045, Exp(B) = 1.05, $p = .930$) did not. Children working with large sets were 11 times less likely to provide a correct response than those working with small sets. Moreover, for every month increase in age, children were 16 % more likely to provide a correct response on the addition trial.

For the subtraction trial, the logistic regression model was significant $\chi^2(3) = 21.21$, $p < .001$ accounted for 25.1 % (Nagelkerke R^2) of the variance, and correctly classified 75.7 % of the cases. Again, the Hosmer and Lemeshow Test indicated that the model fit the data

[4] We confirmed that the three-way interactions were not adding any additional variance to the model and then removed them to increase our power. Thus, the models reported here do not include three-way interactions. Preliminary analyses revealed that Age did not interact with Condition or Set Size, and so those interactions were also not included in the models.

[5] If the model fits the data well, the chi-square value for the Hosmer and Lemeshow Test should be non-significant.

Table 4
Full Regression Model for the Behavioral Responses. Response: Likelihood of a Correct Behavioral Response.

	B	SE(*B*)	95 % CI of Beta
Context (Non-Social)	0.156	0.586	−0.997 – 1.309
Context (Sharing)	–	–	–
Trial Type (Addition)	0.687	0.511	−0.319 – 1.693
Trial Type (Division)	−1.384*	0.560	−2.487 to −.281
Trial Type (Subtraction)	–	–	–
Set Size (Small)	−0.897	0.597	−2.072 – 0.278
Set Size (Large)	–	–	–
Context (Non-Social) * Trial Type (Addition)	−.272	.588	−1.429 – 0.885
Context (Non-Social) * Trial Type (Division)	1.040	0.641	−0.222 – 2.302
Context(Non-Social) * Trial Type (Subtraction)	–	–	–
Context (Sharing) * Trial Type (Addition)	–	–	–
Context (Sharing) * Trial Type (Division)	–	–	–
Context (Sharing) * Trial Type (Subtraction)	–	–	–
Context (Non-Social) * Set Size (Small)	.293	0.733	−1.149 – 1.735
Context (Non-Social) * Set Size (Large)	–	–	–
Context (Sharing) * Set Size (Small)	–	–	–
Context (Sharing) * Set Size (Large)	–	–	–
Age	−0.112**	0.023	−0.158 to −0.066
Set Size (Small) * Trial Type (Addition)	−1.075	0.591	−2.239 to −0.089
Set Size (Large) * Trial Type (Addition)	–	–	–
Set Size (Small) * Trial Type (Division)	−0.151	0.633	−1.397 – 1.095
Set Size (Large) * Trial Type (Division)	–	–	–
Set Size (Small) * Trial Type (Subtraction)	–	–	–
Set Size (Large) * Trial Type (Subtraction)	–	–	–
Intercept	5.906**	1.360	3.230 – 8.582

Notes. Dashed lines indicate reference groups.
* $p < .05$.
** $p < .01$.

well ($\chi^2(8) = 5.60$, $p = .693$). There was a significant effect of Age (β = .117, Exp(B) = 1.12, $p < .001$), and a marginal effect of Set Size (β = −.782, Exp(B) = .457, $p = .090$), but again no effect of Context (β = .277, Exp(B) = 1.320, $p = .548$). For every month increase in age, children were approximately 12 % more likely to provide a correct action-based response on the subtraction trial.

Finally, we explored predictors of correct responding on the division trial. The logistic regression model was significant, $\chi^2(3) = 11.841$, $p = .008$, explained 16.8 % (Nagelkerke R^2) of the variance, and correctly classified 80.6 % of the cases. The Hosmer and Lemeshow Test indicated that the model fit the data well ($\chi^2(8) = 3.580$, $p = .893$). Importantly, Context (β = 1.253, Exp (B) = 3.502, $p = .024$) was a significant predictor of division performance, as was Age (β = .072, Exp(B) = 1.075, $p = .023$). Set Size (β = -.763, Exp(B) = .466, $p = .151$) did not predict performance. Thus, children in the sharing context were 3 times more likely to provide a correct behavioral response on the division trial than those in the non-social context. For every month increase in age, children were approximately 7 % more likely to provide an accurate division response.

Thus, as indicated in these analyses, the effect of social framing was specific to the division trial. Although the effect was small, children in the Sharing condition were significantly more likely to provide a correct response on the division, but not the addition or subtraction trial. In contrast, there was a larger effect of age which affected all trials, consistent with the fact that children in this age range are in the process of learning about number and arithmetic manipulations (e.g., Chernyak et al., 2016, 2019). Set size, which had a small effect, appeared to matter on the addition and subtraction trial only.

3.2. Does the sharing context facilitate children's verbal responses on an arithmetic task?

Next, we analyzed children's verbal responses. Providing a verbal response proved more difficult than providing a behavioral response, and many children failed to provide any verbal response on a majority of the trials (similar to Kibbe & Feigenson, 2015). These trials, along with uninterpretable data, were treated as missing data and excluded from analyses ($n_{addition} = 52$, $n_{subtraction} = 48$, $n_{division} = 43$), leaving 61 addition, 65 subtraction, and 70 division responses to be analyzed. We again used a binomial mixed model assuming a compound symmetry repeated covariance to perform a binary regression using Trial Type (addition, subtraction, division; entered as a within-subjects predictor), Context (sharing context vs. non-social context), Set Size (large vs. small), and Age as predictors and providing a correct versus incorrect verbal response[6]. Two-way interactions of our manipulated variables (Trial Type, Set Size, and Context) were also included. The model showed a main effect of Age, $F(1, 183) = 18.621$, $p < .001$, such that children gave more accurate responses as they got older. There was a significant effect of Set Size, $F(1, 183) = 18.413$, $p < .001$, such that those working with the Small Set Size ($M = .64$, $SD = .48$) provided correct verbal responses more frequently than those assigned to work with the Large Set Size ($M = .23$, $SD = .42$). There was also a main effect of Trial Type, $F(2, 183) = 9.173$, $p < .001$. Follow-up

[6] Three way interactions did not contribute significant variance to the model, and are not included in the models described here.

Table 5
Proportion of Correct Verbal Responses on the Arithmetic Task Across Condition, Set Size, and Trial Type.

Small Sets			
	Addition	Subtraction	Division
Sharing	.61 (.12)	.42 (.12)	.88 (.07)
Non-Social	.56 (.12)	.50 (.11)	.80 (.09)
Large Sets			
	Addition	Subtraction	Division
Sharing	.29 (.13)	.00 (.00)	.50 (.14)
Non-Social	.09 (.09)	.10 (.10)	.36 (.15)

Note. Standard error in parentheses.

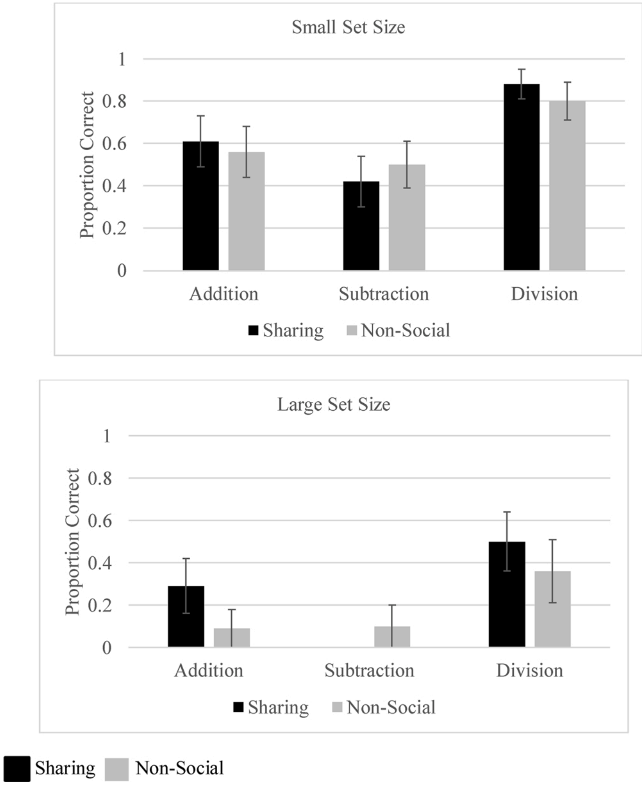

Fig. 3. Proportion of Correct Verbal Responses on the Arithmetic Task Across Condition, Set Size, and Trial Type.

McNemar's tests revealed that that children were significantly more likely to provide a correct verbal response on the division trial ($M = .70$, $SD = .46$) relative to the addition ($M = .43$, $SD = .50$, $p = .002$) and the subtraction trial ($M = .31$, $SD = .47$, $p < .001$). There was no significant difference in children's accuracy on the addition and subtraction trials, $p = .302$. No other main effects or interactions reached significance, p's $> .2$. See Table 5 and Fig. 3. The full regression model can be found in Table 6.

We again conducted binary logistic regressions for each trial type separately to explore whether any one trial drove the condition effects. We again tested a single model including Context, Set Size, and Age as predictors of verbal responses for each trial type separately.

For the addition trial, the logistic regression model was significant $\chi^2(3) = 23.524$, $p < .001$, explained 43.5 % (Nagelkerke R^2) of the variance, and correctly classified 78.3 % of the cases. The Hosmer and Lemeshow Test indicated that fit of the model fit the data well ($\chi^2(8) = 5.715$, $p = .679$). Set Size ($\beta = -1.571$, Exp(B) = .208, $p = .021$) and Age ($\beta = .162$, Exp(B) = 1.175, $p = .002$) significantly predicted accuracy of verbal responses, but Context ($\beta = .844$, Exp(B) = 2.327, $p = .217$) did not. Thus, children working with the large set size were 20 times less likely to provide a correct verbal response on the addition trial than those working with the small set size. Moreover, for each month increase in age, children were 17 % more likely to provide a correct response.

For the subtraction trial, the logistic regression model was significant $\chi^2(3) = 20.407$, $p < .001$, explained 38.4 % (Nagelkerke R^2) of the variance, and correctly classified 76.6 % of the cases. The Hosmer and Lemeshow Test indicated that fit of the model fit the data well ($\chi^2(8) = 6.772$, $p = .561$). Set Size ($\beta = -2.751$, Exp(B) = .064, $p = .011$) and Age ($\beta = .082$, Exp(B) = 1.086, $p = .039$)

Table 6
Full Regression Model for the Verbal Responses. Response: Likelihood of a Correct Verbal Response.

	B	SE(*B*)	95 % CI of Beta
Context (Non-Social)	−0.507	1.075	−2.628 – 1.614
Context (Sharing)	–	–	–
Trial Type (Addition)	−2.368	1.292	−4.917 – .180
Trial Type (Division)	−3.513**	1.270	−6.018 to −1.009
Trial Type (Subtraction)	–	–	–
Set Size (Small)	−2.802*	1.252	−5.272 to −0.331
Set Size (Large)	–	–	–
Context (Non-Social) * Trial Type (Addition)	1.499	0.937	−0.350– 3.348
Context (Non-Social) * Trial Type (Division)	1.391	1.004	−0.590 – 3.372
Context (Non-Social) * Trial Type (Subtraction)	–	–	–
Context (Sharing) * Trial Type (Addition)	–	–	–
Context (Sharing) * Trial Type (Division)	–	–	–
Context (Sharing) * Trial Type (Subtraction)	–	–	–
Context (Non-Social) * Set Size (Small)	−0.140	0.935	−1.984 – 1.704
Context (Non-Social) * Set Size (Large)	–	–	–
Context (Sharing) * Set Size (Small)	–	–	–
Context (Sharing) * Set Size (Large)	–	–	–
Age	−0.113**	0.026	−0.165 to −0.062
Set Size (Small) * Trial Type (Addition)	1.296	1.313	−1.295– 3.886
Set Size (Large) * Trial Type (Addition)	–	–	–
Set Size (Small) * Trial Type (Division)	0.639	1.305	−1.935 – 3.213
Set Size (Large) * Trial Type (Division)	–	–	–
Set Size (Small) * Trial Type (Subtraction)	–	–	–
Set Size (Large) * Trial Type (Subtraction)	–	–	–
Intercept	9.955**	1.995	6.019 – 13.891

Notes. Dashed lines indicate reference groups.
* $p < .05$.
** $p < .01$.

significantly predicted accuracy of verbal responses on the subtraction trial, but Context (β = −.492, Exp(B) = .612, $p = .441$) did not. Thus, children who were working with the larger set size were ~6 times less likely to provide a correct verbal response on the subtraction trial. For every month increase in age, children were ~9% more likely to provide a correct verbal response.

For the division trial, the model reached significance, $\chi^2(3) = 20.566$, $p < .001$, explained 36.8 % (Nagelkerke R^2) of the variance, and correctly classified 79.7 % of the cases. Again, the Hosmer and Lemeshow Test indicated that fit of the model fit the data well ($\chi^2(8) = 4.557$, $p = .804$). Set Size (β = −2.176, Exp(B) = .114, $p = .001$) and Age (β = .105, Exp(B) = 1.111, $p = .005$) significantly predicted verbal response accuracy, but Context (β = .760, Exp(B) = 2.138, $p = .238$) did not. Children working with the large set size were more 11 % less likely to provide a correct response on the division trial than those working with the small set size. Moreover, for every increase in month of age, children were 11 % more likely to provide a correct verbal response.

As with the behavioral responses, Set Size and Age had a small effect on the likelihood of providing an accurate verbal response. Unlike the behavioral responses, there were no Context effects on verbal responses, though there was a medium effect of Trial Type on verbal responses, with participants giving more correct verbal responses for the small set size problems.

3.3. Are action-based responses more accurate than verbal ones?

Lastly, we compared action-based and verbal responses on the arithmetic task. The number of correct action-based and verbal responses was correlated when controlling for age ($r = .315$, $p = .003$), indicating that children who provided a greater number of accurate action-based responses also provided a greater number of accurate verbal responses. Despite this, a paired samples *t*-test on number of correct action-based and verbal accuracy scores (out of 3) revealed that children provided correct action-based responses ($M = 2.07$, $SD = 1.07$) significantly more often than correct verbal responses ($M = 1.06$, $SD = .998$), $t(89) = -8.749$, $p < .001$. Follow-up exploratory McNemar's tests revealed that action-based responses were significantly more accurate than verbal responses on the addition ($p < .001$), subtraction ($p < .001$), and division trials ($p = .022$). Importantly, these statistics are already considering that on average children provided significantly more behavioral responses ($M = 2.81$, $SD = .47$) compared to verbal responses ($M = 2.18$, $SD = .84$, $t(89) = -7.036$, $p < .001$).

4. Discussion

4.1. Division and sharing

We began this work with the prediction that sharing contexts may facilitate division performance in three, non-mutually exclusive ways: (1) due to *shared structural similarities* between the two concepts, (2) due to *increased contextualization* by being embedded within a story format or ecologically valid context (Carpenter & Moser, 1982; Casey et al., 2008; Gilmore, McCarthy, & Spelke, 2007; Hughes, 1986; Riley, Greeno, & Heller, 1983; Siegler & Ramani, 2009; Vergnaud, 1982), and/or (3) due to the benefit of using manipulatives and *providing action based responses* (e.g., Kibbe & Feigenson, 2015; Pouw et al., 2014). Given that problems in both our sharing and non-social contexts were presented with manipulatives and with an opportunity to give action-based responses, performance on all arithmetic operations across both contexts should have been comparable if manipulatives or providing action-based responses alone facilitated performance. If contextualization in a sharing scenario drove our pattern of results, then we would expect improved performance on all three arithmetic trials in the sharing condition relative to the non-social condition. However, only division performance in the sharing context should be affected if performance was enhanced by the unique structural link between sharing and division. Results of our study support the latter possibility: context predicted children's performance on the division, but not addition or subtraction trial. The effect of social contextualization on the division trial held even when controlling for age, which is a strong predictor of early arithmetic performance, and also appeared across both set sizes tested. Children, however, did not benefit from any and all contextualization: specifically, context predicted children's *division* performance, but not their addition or subtraction performance. It is important to note that we observed condition differences on the division trial despite the fact that the non-social context was almost exactly matched in language, structure, and interaction with the experimenter.

In addition to enhanced performance on the division trial in the sharing context, our data also suggest that age and set size played a role in children's overall arithmetic performance. As one might expect, both action-based and verbal responses became more accurate as children became older, likely due to an increased understanding of number. Importantly though, age did not interact with any of our variables of interest. That is, children – regardless of age – equally benefitted from the sharing context. This suggests that a wide range of children may benefit from introducing division in a contextualized, sharing scenario, further emphasizing this manipulation's utility in the classroom. Given that larger set sizes are more difficult to manipulate than small set sizes (e.g., Ashcraft, 1992; Geary, 1996; Mulligan, 1992; Posid & Cordes, 2015a, b), it was not surprising that set size also impacted children's performance. While the number of items children were asked to manipulate impacted the addition and subtraction trials, it did not affect performance on the division trial. Exactly why set size did not have an impact on the division trial is unclear and future work will be important for determining why set size affected certain trial types, but not others.

Importantly, our results cannot be accounted for by a difference in difficulty across problem types. That is, it is possible that the addition and subtraction trials may have been structurally more difficult and thus taxed young children's abilities too much to benefit from any condition manipulation (i.e., to solve a subtraction problem, children were tasked with comparing the two sets, calculating a difference between them, and then understanding how to manipulate that difference to equalize the sets). While addition, subtraction, and division involve inherently different computations, and are thus not possible to equate, we nonetheless believe there are several reasons that indicate that structural similarity to sharing, rather than relative ease of the division trial, drove our results. If division was simply an easier calculation, one would expect performance on the division trial to be better – even in the non-social condition. This, however, was not the case as performance on all three trials was comparable in the non-social condition (p's $> .4$), yet division performance was significantly enhanced in the sharing condition.

Moreover, any perceptual differences across our sharing and non-social condition also cannot account for our pattern of results. For example, one could argue that our sharing condition involved 3D plush animals may have been more perceptually rich than the non-social condition which included 2D colorful shapes, differences which may have led to children to pay more attention to the task, resulting in improved performance overall[7] . However, if these superficial differences across contexts were to improve performance on the arithmetic task, then context should have predicted children's performance on all three arithmetic trials, as all three trials in the sharing context were equal in terms of perceptual features. Instead, the effect of sharing context was localized to the division trial, suggesting that perceptual richness did not account for our effects.

4.2. Response types

Our results also replicate previous literature indicating that children's verbal responses lagged behind their action-based ones in both our sharing and non-social contexts, particularly on the addition and subtraction trials (e.g., Alibali & Goldin-Meadow, 1993; Goldin-Meadow et al., 1993). Children could rarely solve problems verbally, despite being able to solve them behaviorally on a large proportion of the trials, suggesting that children's embodied problem-solving in math contexts may develop earlier than their abilities to engage in symbolic, verbal-based arithmetic. Performance on verbal responses remained poor even during our key experimental condition, which introduced problems as sharing scenarios. This finding is consistent with work showing that children's gestures while solving math equivalence problems often display a more mature understanding compared to their verbal responses (Perry et al., 1988) and that despite being unable to provide an accurate verbal response, preschoolers are capable of performing nonsymbolic algebra (Kibbe & Feigenson, 2015). In our study, children could solve problems behaviorally through their action-based responses before they were able to talk about them explicitly, suggesting that children recognize mathematical principles implicitly earlier on. In line with this finding is work suggesting physically interacting with manipulatives, which was only possible when giving an action-based response, facilitates children's performance (see Pouw et al., 2014 for a review). It is possible that verbal responses require a more formal symbolic understanding of mathematics, whereas action-based responses may only require an implicit or informal understanding of such principles. Our results, along with many others, suggest that informal, action-based responses may be more accessible

[7] It is acknowledged that while some work finds perceptually rich items to *hinder* performance (Carbonneau et al., 2013; McNeil & Jarvin, 2007; McNeil, Uttal, Jarvin, & Sternberg, 2009), other research suggests that increasing the perceptual features of objects children are unfamiliar with *facilitates* performance (e.g., Petersen & McNeil, 2013).

than verbal responses, at least early in development (e.g., Alibali & Goldin-Meadow, 1993; Goldin-Meadow et al., 1993; Kibbe & Feigenson, 2015).

While the preschoolers in our study benefitted from providing action-based responses, it is unclear whether older children would show a similar advantage. If action-based responses are useful because they provide children with the opportunity to reveal implicit knowledge in the absence of knowledge, one might expect children of all ages to benefit from providing action-based responses. However, it is also possible that providing action-based responses may only be helpful for children who are in the process of learning difficult concepts, such as arithmetic. Given the focus on providing verbal, symbolic responses in the classroom, it is unclear whether older children may be conferred a similar advantage. Future work will be critical for determining whether action-based responses are equally beneficial across development or are more useful during the learning process.

4.3. Social contexts for learning

The current work provides important implications for mathematics education. Our research further emphasizes the benefits of presenting division problems as sharing scenarios to young children (Squire & Bryant, 2002a, b). Moreover, our results suggest that children may be able to solve math problems non-verbally before being able to produce the answer verbally, thus demonstrating the benefit of introducing arithmetic concepts as behavioral problems in the classroom. Fortunately, presenting division as a sharing scenario and allowing for action-based responses are easy to implement and does not require many additional resources within the classroom. While our work suggests that sharing scenarios may promote division understanding, there are many possible directions for future investigations. For instance, we designed the present sharing scenario (involving resource distribution between two stuffed animals) in order to compare the effects to a comparable, non-social scenario (dividing resources between two shapes). This design, however, only represents one type of sharing situation with which children may have experience. In fact, there are many instances in which sharing is presented to children (i.e., the equal sharing of cookies during snack time; sharing toys fairly between siblings, etc.) that would be similar, although not identical, to the current sharing scenario. Given the current findings, we would predict other sharing scenarios (i.e., emphasizing the equal sharing of cookies during snack time) to also promote division performance. However, future work is needed to confirm whether all sharing scenarios equally promote division.

4.4. Learning formal division

Future research will also be important for investigating whether behavioral sharing problems scaffold later formal, symbolic division understanding. Although our current study suggests that sharing scenarios facilitate children's performance on an informal arithmetic task, it is unclear whether 1) sharing situations would confer similar advantages when learning formal division and 2) learning division within a sharing context may transfer to other non-social contexts. For instance, does learning division in a sharing context facilitate performance on paper and pencil division tasks? If presenting division problems in a sharing context promotes math thinking more broadly, then one would expect a similar advantage to learning formal division in a sharing scenario. Many informal situations such as conversations about resource distribution (Chernyak, 2018), as well as, playing board games (Siegler & Ramani, 2008, 2009) and computer games (Berkowitz et al., 2015), have been shown to promote math talk and thus math thinking. Moreover, previous work has suggested that fraction knowledge is enhanced when children also have an understanding of fair sharing (Empson, 1999, 2001). If sharing scenarios also elicit increased math thinking, then one might expect sharing contexts to also support a formal understanding of division. Future research will be important for further understanding how sharing scenarios may support children's understanding of symbolic arithmetic.

4.5. Limitations

In our current work, we opted for a between-subjects design in order to make our procedure a reasonable length of our youngest participants, as well as to limit practice, transfer, and fatigue effects. However, it is important for future work to investigate whether there may be individual differences in how children differentiate between social and non-social contexts using a within-subjects design.

Moreover, it is important to acknowledge the homogeneity of our high-SES sample of participants. Given that children's action-based responses seem more accessible than verbal responses, particuarlly for children from lower SES backgrounds, our results may be accentuated when working with a more diverse sample (see Jordan et al., 1994). Given that children from lower income communities may have relatively less exposure to formal schooling (and thus, less exposure to decontextualized arithmetic problems of the type that we used in our non-social context condition), our results may also be more pronounced or magnified in a lower SES community. Future work should investigate this directly.

4.6. Conclusion

In sum, our study points to both the benefits and limits of contextualizing division problems. We find that (a) sharing scenarios facilitate performance on division problems, but not other arithmetic operations, and (b) action-based responses are more accurate than verbal responses at this age. Given the difficulty children have learning formal division, it is important to continue investigating the contexts that best support young children's learning of these concepts. Together, our data not only demonstrate how to best foster early success in division, but also in how to present division problems to young children in an age-appropriate manner.

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Appendix A

Given that sharing scenarios were predicted to specifically facilitate division performance, children also completed a nonsymbolic division task (modeled after McCrink & Spelke, 2016) after completing the three math problems to explore the possibility of transfer. Twenty participants were excluded from the nonsymbolic division task for not completing the task ($n = 6$), experimenter error ($n = 3$), or biased responding ($n = 11$; described in the data coding section), this resulted in a final sample of 93 children ($M_{age} = 58.36$ months, $SD_{age} = 8.17$). We predicted that completing math problems in a sharing context may facilitate performance on a subsequent, nonsymbolic division task.

Methods

Nonsymbolic Division Task: Children were shown an animated "dividing machine" on a computer and told that the machine always divides balls into two boxes so that *half* the balls go in one box and *half* of them go into another box. During two familiarization trials, children saw two examples in which the machine divided two balls into two boxes (resulting with one in each box), and then four balls into two boxes (resulting in two balls in each box). The experimenter emphasized that after the dividing machine had moved the balls, each box had the same number of balls ("Look! This box has 1 ball, and this box has 1 ball!"). During the 16 test trials, children were shown an image of the dividing machine containing 6, 8, 12, or 16 balls (each number was presented four times in a random order). Next to the machine, children were shown two potential resulting boxes: one box contained half of the original amount (correct answer) and the other box contained a foil (incorrect answer; See Fig. A1). The experimenter asked, "When I turn on my dividing machine, how many balls will be in each box?" Children were then prompted to select one of the two answer choices and the experimenter pressed a corresponding key on the keyboard to record their response and advance to the next trial. On half of the trials, the foil provided was larger than the correct answer (i.e., if the answer was 6 and the foil was 12) and on half of the trials the foil provided was smaller than the correct answer (i.e., if the answer was 6 and the foil was 3). The ratios between the correct answer and the foil were chosen to be approximately 1:2 and 2:3. Children had an unlimited amount of time to respond on each trial and were not prevented from counting the balls if they wished. The stimuli for the nonsymbolic division task were displayed and responses recorded using a Xojo program on a Mac laptop.

Data Coding

We coded the percent of correct responses across all test trials presented during the nonsymbolic division task. To ensure that children's responses were not biased by always picking the larger (or smaller) value, we removed data from any children who chose only the larger (or only the smaller) value on 12 or more of 16 trials (12/16, *Binomial*, $p < 0.05$) from all analyses on the division task ($n = 11$).

Results

On average, children performed significantly above chance (i.e., chose the correct response more than 50 % of the time; $M = .57$,

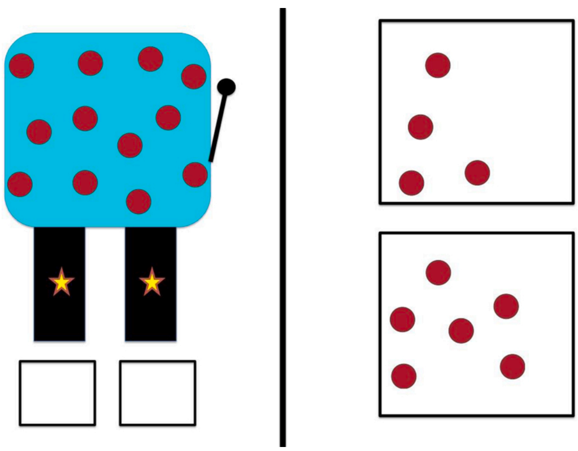

Fig. A1. Example of Nonsymbolic Division Task.

$SD = .15$) on the nonsymbolic division task, $t(92) = 4.56$, $p < .001$, Cohen's $d = .47$. There was no difference in performance between children in the sharing context ($M = .56$, $SD = .14$) and the non-social context ($M = .58$, $SD = .15$), $t(91) = .810$, $p = .42$, Hedges' $g = 0.17$[8].

Next, we determined whether children's performance on the nonsymbolic division task was related to their performance on the arithmetic task. Controlling for Age, nonsymbolic division did not correlate with the number of correct behavioral, $r = .141$, $p = .182$, or verbal, $r = .037$, $p = .755$, accuracy on the arithmetic task. Moreover, the strength of the correlation between nonsymbolic division and behavioral accuracy (number correct out of 3) on the arithmetic task was comparable for the sharing ($r = .191$, $p = .198$) and the non-social ($r = .096$, $p = .541$) contexts when controlling for Age, $z = .45$, $p = .65$. Given the theorized link between sharing and division, we tested for a unique relation between nonsymbolic division and performance on the division problem in each Context separately. When controlling for Age, neither correlation reached significance (Sharing Context: $r = .073$, $p = .633$, Non-Social Context: $r = .186$, $p = .243$). Finally, to explore whether performance on the nonsymbolic division task was impacted by experience during our arithmetic trials, we entered Context and Set Size into a linear regression with nonsymbolic division performance as the outcome variable. The model did not reach significance, $F(2, 90) = .338$, $p = .714$, suggesting that completing a social sharing task did not result in enhanced performance on the subsequent nonsymbolic division task.

Discussion

Because of the theorized relation between sharing and division, we predicted that completing math problems in a sharing context might facilitate performance on a subsequent, nonsymbolic division task. Counter to our predictions, there was no evidence of transfer from the sharing context to the nonsymbolic division task, nor was there a relation between children's performance on the arithmetic task and the subsequent nonsymbolic division task. We believe there are several possibilities for why this may be the case. First, children's overall performance on the nonsymbolic division task, while above chance, was relatively poor, which may indicate that the task was simply too challenging or the concept of a dividing machine was not transparent to children of this age. As such, it may be that performance on our task was not the best indicator of implicit nonsymbolic division abilities. Moreover, there was limited structural commonality between the two tasks, thus hindering any potential transfer. The nonsymbolic division task was presented in a different medium (on a computer screen compared to with real toys); used different, non-social stimuli; and was explicitly introduced as a separate game. Most relevant, our nonsymbolic division task was not presented as a sharing scenario, and so children in the sharing condition may have failed to identify the commonalities across the two tasks. Additionally, children completed only three arithmetic trials in the sharing or non-social context and thus the time period during which children could learn from the context was relatively brief. Future research investigating whether repeated exposure to division in a sharing scenario helps children develop a deeper understanding of division that transfers to non-social division tasks is needed. Most critically, the two tasks may rely on fundamentally distinct cognitive systems for representing number: while the arithmetic task involved relatively small sets and needed be solved *precisely*, the nonsymbolic division task involved using larger sets and could be solved *approximately* (Feigenson, Dehaene, & Spelke, 2004). In sum, although we included the nonsymbolic division task to assess potential transfer effects, we did not find any evidence of improved performance for children in the sharing condition of the arithmetic task.

References

Alibali, M. W., & Goldin-Meadow, S. (1993). Gesture speech mismatch and mechanisms of learning: What the hands reveal about the child's state of mind. *Cognitive Psychology, 25*, 468–523.

Ashcraft, M. H. (1992). Cognitive arithmetic: A review of data and theory. *Cognition, 44*(1–2), 75–106.

Ashcraft, M. H., & Guillaume, M. M. (2009). Mathematical cognition and the problem size effect. *The Psychology of Learning and Motivation, 51*, 121–151.

Bailey, D. H., Siegler, R. S., & Geary, D. C. (2014). Early predictors of middle school fraction knowledge. *Developmental Science, 17*(5), 775–785. https://doi.org/10.1111/desc.12155

Berkowitz, T., Schaeffer, M. W., Maloney, E. A., Peterson, L., Gregor, C., Levine, S. C., & Beilock, S. L. (2015). Math at home adds up to achievement in school. *Science, 350*(6257), 196–198.

Booth, J. L., & Newton, K. J. (2012). Fractions: Could they really be the gatekeeper's doorman? *Contemporary Educational Psychology, 37*(4), 247–253. https://doi.org/10.1016/j.cedpsych.2012.07.001

Bruner, J. (1987). Life as narrative. *Social Research, 54*, 11–32.

Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology, 105*(2), 380.

Carpenter, T. P., & Moser, J. M. (1982). The development of addition and subtraction problem solving skills. In T. P. Carpenter, J. M. Moser, & T. A. Romberg (Eds.), *Addition and subtraction: A cognitive perspective* (pp. 9–24). Hillsdale, NJ: Erlbaum.

Carraher, T. N., Carraher, D. W., & Schliemann, A. D. (1985). Mathematics in the streets and in schools. *The British Journal of Developmental Psychology, 3*(1), 21–29.

Casey, B., Erkut, S., Ceder, I., & Young, J. M. (2008). Use of a storytelling context to improve girls' and boys' geometry skills in kindergarten. *Journal of Applied Developmental Psychology, 29*(1), 29–48. https://doi.org/10.1016/j.appdev.2007.10.005

Chernyak, N. (2018). Number-based sharing: Conversation about quantity in the context of resource distribution. *Early Childhood Research Quarterly*.

Chernyak, N., Sandham, B., Harris, P. L., & Cordes, S. (2016). Numerical cognition explains age-related changes in third-party sharing. *Developmental Psychology, 52*, 1555–1562. https://doi.org/10.1037/dev0000196

Chernyak, N., Harris, P. L., & Cordes, S. (2019). Explaining early moral hypocrisy: Numerical cognition promotes equal sharing behavior in preschool-aged children. *Developmental Science, 22*(1), Article e12695.

[8] All of the following analyses held when removing children who performed below chance levels on the nonsymbolic division task ($n = 45$).

Cordova, D. I., & Lepper, M. R. (1996). Intrinsic motivation and the process of learning: Beneficial effects of contextualization, personalization, and choice. *Journal of Educational Psychology, 88*, 715–730. https://doi.org/10.1037//0022-0663.88.4.715

Correa, J., Nunes, T., & Bryant, P. (1998). Young children's understanding of division: The relationship between division terms in a noncomputational task. *Journal of Educational Psychology, 90*(2), 321.

DeWolf, M., Bassok, M., & Holyoak, K. J. (2015). From rational numbers to algebra: Separable contributions of decimal magnitude and relational understanding of fractions. *Journal of Experimental Child Psychology, 133*, 72–84. https://doi.org/10.1016/j.jecp.2015.01.013

Dubé, A. K., & Robinson, K. M. (2018). Children's understanding of multiplication and division: Insights from a pooled analysis of seven studies conducted across 7 years. *The British Journal of Developmental Psychology, 36*, 206–219.

Empson, S. B. (1999). Equal sharing and shared meaning: The development of fraction concepts in a first-grade classroom. *Cognition and Instruction, 17*(3), 283–342.

Empson, S. B. (2001). Equal sharing and the roots of fraction equivalence. *Teaching Children Mathematics, 7*(7), 421.

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8*(7), 307–314. https://doi.org/10.1016/j.tics.2004.05.002

Frydman, O., & Bryant, P. (1988). Sharing and the understanding of number equivalence by young children. *Cognitive Development, 3*(4), 323–339.

Geary, D. C. (1996). The problem-size effect in mental addition: Developmental and cross- national trends. *Mathematical Cognition, 2*(1), 63–94.

Gilmore, C. K., McCarthy, S. E., & Spelke, E. S. (2007). Symbolic arithmetic knowledge without instruction. *Nature, 447*(7144), 589–591. https://doi.org/10.1038/nature05850

Ginsburg, H. P., Klein, A., & Starkey, P. (1998). *The development of children's mathematical thinking: Connecting research with practice*.

Goldin-Meadow, S., Alibali, M. W., & Church, R. B. (1993). Transitions in concept acquisition: Using the hand to read the mind. *Psychological Review, 100*(2), 279.

Hughes, M. (1986). *Children and number*. Oxford, England: Blackwell.

Jennings, C. M., Jennings, J. E., Richey, J., & Dixon-Krauss, L. (1992). Increasing interest and achievement in mathematics through children's literature. *Early Childhood Research Quarterly, 7*(2), 263–276. https://doi.org/10.1016/0885-2006(92)90008-M

Jordan, N. C., Huttenlocher, J., & Levine, S. C. (1994). Assessing early arithmetic abilities: Effects of verbal and nonverbal response types on the calculation performance of middle-and low-income children. *Learning and Individual Differences, 6*(4), 413–432.

Kibbe, M. M., & Feigenson, L. (2015). Young children 'solve for x' using the approximate number system. *Developmental Science, 18*(1), 38–49.

Lave, J., Murtaugh, M., & De La Rocha, O. (1984). The dialectic of arithmetic in grocery shopping. In B. Rogoff, & J. Lave (Eds.), *Everyday cognition: Its development in social context* (pp. 67–94). Cambridge, MA: Harvard University Press.

Le Corre, M., & Carey, S. (2007). One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. *Cognition, 105*(2), 395–438. https://doi.org/10.1016/j.cognition.2006.10.005

Manches, A., O'Malley, C., & Benford, S. (2010). The role of physical representations in solving number problems: A comparison of young children's use of physical and virtual materials. *Computers & Education, 54*(3), 622–640.

Martin, T., & Schwartz, D. L. (2005). Physically distributed learning: Adapting and reinterpreting physical environments in the development of fraction concepts. *Cognitive Science, 29*(4), 587–625.

McCrink, K., & Spelke, E. S. (2016). Non-symbolic division in childhood. *Journal of Experimental Child Psychology, 142*, 66–82. https://doi.org/10.1016/j.jecp.2015.09.015

McNeil, N., & Jarvin, L. (2007). When theories don't add up: Disentangling the manipulatives debate. *Theory Into Practice, 46*(4), 309–316.

McNeil, N. M., Uttal, D. H., Jarvin, L., & Sternberg, R. J. (2009). Should you show me the money? Concrete objects both hurt and help performance on mathematics problems. *Learning and Instruction, 19*(2), 171–184.

Montessori, M. (1964). *The montessori method, trans*. AE George. Cambridge, Mass: Robert Bentley.

Muldoon, K. P., Lewis, C., & Berridge, D. (2007). Predictors of early numeracy: Is there a place for mistakes when learning about number? *The British Journal of Developmental Psychology, 25*(4), 543–558. https://doi.org/10.1348/026151007X174501

Muldoon, K. P., Lewis, C., & Freeman, N. (2009). Why set-comparison is vital in early number learning. *Trends in Cognitive Science, 13*(5), 203–208. https://doi.org/10.1016/j.tics.2009.01.010

Mulligan, J. (1992). Children's solutions to multiplication and division word problems: A longitudinal study. *Mathematics Education Research Journal, 4*(1), 24–41.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards (Mathematics)*. Washington D.C: National Governors Association Center for Best Practices, Council of Chief State School Officers.

Olson, K. R., & Spelke, E. S. (2008). Foundations of cooperation in young children. *Cognition, 108*(1), 222–231. https://doi.org/10.1016/j.cognition.2007.12.003

Perry, M., Church, R. B., & Goldin-Meadow, S. (1988). Transitional knowledge in the acquisition of concepts. *Cognitive Development, 3*, 359–400.

Petersen, L. A., & McNeil, N. M. (2013). Effects of perceptually rich manipulatives on preschoolers' counting performance: Established knowledge counts. *Child Development, 84*(3), 1020–1033.

Posid, T., & Cordes, S. (2015a). Verbal counting moderates perceptual biases found in children's cardinality judgments. *Journal of Cognition and Development, 16*(4), 621–637.

Posid, T., & Cordes, S. (2015b). The small-large divide: A case of incompatible numerical representations in infancy. *Mathematical Cognition and Learning, 1*, 253–276.

Pouw, W. T., Van Gog, T., & Paas, F. (2014). An embedded and embodied cognition review of instructional manipulatives. *Educational Psychology Review, 26*(1), 51–72.

Riley, M., Greeno, J. G., & Heller, J. I. (1983). Development of children's problem solving abilities in arithmetic. In H. Ginsburg (Ed.), *The development of mathematical thinking* (pp. 153–196). New York: Academic Press.

Robinson, K. M., & Dubé, A. K. (2009). Children's understanding of the inverse relation beween multiplication and division. *Cognitive Development, 24*(3), 310–321.

Robinson, K. M., & Ninowski, J. E. (2003). Adults' understanding of inversion concepts: How does performance on addition and subtraction inversion problems compare to performance on multiplication and division inversion problems? *Canadian Journal of Experimental Psychology/Revue Canadienne de psychologie experimental, 57*(4), 321.

Robinson, K. M., Ninowski, J. E., & Gray, M. L. (2006). Children's understanding of the arithmetic concepts of inversion and associtivity. *Journal of Experimental Child Psychology, 94*(4), 349–362.

Sarnecka, B. W., & Wright, C. E. (2013). The idea of an exact number: Children's understanding of cardinality and equinumerosity. *Cognitive Science, 37*(8), 1493–1506. https://doi.org/10.1111/cogs.12043

Saxe, G. B. (1988). Candy selling and math learning. *Educational Researcher, 17*(6), 14–21.

Schank, R. C., & Abelson, R. P. (1995). Knowledge and memory: The real story. In R. S. Wyer (Ed.), *Knowledge and memory: The real story* (pp. 1–85). Hillsdale, NJ: Lawrence Erlbaum Associates.

Schmidt, M. F., & Sommerville, J. A. (2011). Fairness expectations and altruistic sharing in 15-month-old human infants. *PloS One, 6*(10), e23223. https://doi.org/10.1371/journal.pone.0023223

Siegler, R. S., & Pyke, A. A. (2013). Developmental and individual differences in understanding of fractions. *Developmental Psychology, 49*(10), 1994. https://doi.org/10.1037/a0031200

Siegler, R. S., & Ramani, G. B. (2008). Playing linear numerical board games promotes low-income children's numerical development. *Developmental Science, 11*(5), 655–661.

Siegler, R. S., & Ramani, G. B. (2009). Playing linear number board games—But not circular ones—Improves low-income preschoolers' numerical understanding. *Journal of Educational Psychology, 101*(3), 545. https://doi.org/10.1037/a0014239

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., … Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological Science, 23*(7), 691–697. https://doi.org/10.1177/0956797612440101

Simmons, J. P., Nelson, L. D., & Simonsohn, U. (2011). False-positive psychology: Undisclosed flexibility in data collection and analysis allows presenting anything as significant. *Psychological Science, 22*(11), 1359–1366. https://doi.org/10.1177/09567976114417632

Smith, C. E., Blake, P. R., & Harris, P. L. (2013). I should but I won't: Why young children endorse norms of fair sharing but do not follow them. *PloS One, 8*(3), e59510. https://doi.org/journal.pone.0059510.
Squire, S., & Bryant, P. (2002a). The influence of sharing on children's initial concept of division. *Journal of Experimental Child Psychology, 81*(1), 1–43. DOI: 10.1006. jecp.2001.2640.
Squire, S., & Bryant, P. (2002b). From sharing to dividing. Young children's understanding of division. *Developmental Science, 5*(4), 452–466. https://doi.org/10.1111/1467-7687.00240
Stafylidou, S., & Vosniadou, S. (2004). The development of students' understanding of the numerical value of fractions. *Learning and Instruction, 14*(5), 503–518. https://doi.org/10.1016/j.learninstruc.2004.06.015
Vergnaud, G. (1982). A classification of cognitive tasks and operations of thought involved in addition and subtraction problems. In T. P. Carpenter, J. M. Moser, & T. A. Romberg (Eds.), *Addition and subtraction: A cognitive perspective* (pp. 39–59). Hillsdale, NJ: Erlbaum.