Weighted Quasi-optimal and Recursive Quasi-optimal Satellite Selection Techniques for GNSS

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ABSTRACT

The interoperability of Global Navigation Satellite Systems (GNSS) such as GPS, GLONASS and Galileo in near future will permit access to more number of satellite constellations. Dilution of Precision (DOP) is the parameter used to measure the effect of satellite geometry on the positional accuracy. Lower the DOP better the positional accuracy. In general, more than four satellites are selected as a subset to increase the estimation robustness and to minimize the degradation in the estimation accuracy. The conventional techniques such as highest elevation satellite selection algorithm, Kihara's maximum volume method and four-step satellite selection method impose huge computational load with the increase of satellites being tracked. Therefore, satellite selection techniques with minimal Floating Point Operations (FLOPs) are required to improve the performance of GNSS systems in real-time. In view of this, two prominent fast satellite selection techniques namely, weighted Quasi-optimal and Recursive quasi-optimal techniques that provide quasi geometries are analyzed. In this work, to obtain near-optimal geometries with fast satellite selection techniques, appropriate weight functions are applied. Two types of parametric weight functions namely satellite elevation angle $\left(w_{EL}\right)$ and a combined form of elevation and Carrier to Noise ratio (CNR) with multipath scaling factor $\left(w_{ELCNR}\right)$ are used to improve DOP. The multipath scaling factor is calculated using reflection coefficient parameter. The results obtained due to this approach are encouraging.

1. INTRODUCTION

GNSS is widely used in positioning and timing applications. Since the interoperability of GNSS (GPS(U.S), GLONASS (Russia), GALILEO (Europe), COMPASS (China)) in near future will permit access to more number of satellite constellations, GNSS receivers are being designed with more number of channels in order to track satellite signals from multiple constellations. Among the total number of visible satellites, a subset of satellites should be selected such that it gives near optimal or optimal DOP. The user satellite geometry and the ranging errors under the assumption of uniform, uncorrelated, zero-mean ranging error statistics affect the performance of GNSS. The most popular metric commonly used for GNSS application is Geometric Dilution of Precision (GDOP). To determine the point positioning accuracy of GNSS in terms of Distance Root Mean Square (DRMS) error, the standard deviation of range measurement errors (σ_{UERE}) with respect to DOP are considered and is given as [1],

$$RMS\ position\, error = GDOP \times \sigma_{UERE} \tag{1}$$

where, 'σ_{UERE}' is the root sum square of all the sources of errors

As DOP is the multiplicative factor, lower the DOP value the better the positional accuracy. Although the conventional techniques give optimal DOP, they impose huge computational load with the increase of satellites being tracked [2][3][4]. Hence, there is a necessity for the development of fast and efficient satellite selection techniques in order to reduce the computational load and also to give near optimal or optimal DOP. In view of this, two fast satellite selection techniques 'Weighted Quasi-optimal' and 'Recursive quasi-optimal' are presented in this paper and their performance is evaluated using appropriate weight functions for GPS and combined GPS and GLONASS (dual-constellation) constellations.

2. WEIGHTED QUASI-OPTIMAL SATELLITE SELECTION ALGORITHM

Quasi-optimal satellite selection technique selects near optimal geometries with significantly less computational load when compared to the other conventional techniques. In this technique, there is no restriction on the number of satellites to be selected in the subset. The method involves the computation of cost function based on the line-of-sight vectors. The direction cosine matrix 'G' is obtained from line-of-sight matrix 'R' as [5],

$$R = \begin{pmatrix} los_1 & 1 \\ los_2 & 1 \\ \vdots & \\ los_n & 1 \end{pmatrix} \tag{2}$$

$$G_{n\times n} = R\ .R^T = \begin{pmatrix} \cos\alpha_{11} & \cos\alpha_{12} & \dots\dots & \cos\alpha_{1n} \\ \cos\alpha_{21} & \cos\alpha_{22} & \dots\dots & \cos\alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \cos\alpha_{n1} & \cos\alpha_{n2} & \dots\dots & \cos\alpha_{nn} \end{pmatrix} \tag{3}$$

Each individual element in the symmetric matrix G, represents the cosine of the angle between the two line-of-sight vectors $los_i \text{ and } los_j$. The cost for the i^{th} measurement C_i is given as,

$$C_i = \sum_{j=1}^{n} (\cos 2\alpha_{ij}) = \sum_{j=1}^{n} (2\cos^2(\alpha_{ij}) - 1) \tag{4}$$

The function indicates that the cost is highest if the two vectors are nearly collinear i.e., $(\alpha_{ij} \approx 0° \text{ or } \alpha_{ij} \approx 180°)$. The cost for the i^{th} measurement is obtained by the summation of the squares of all the elements in the i^{th} row of the matrix G. As the measurement with highest cost represents redundant information, it is eliminated and the procedure is repeated until the desired number of satellites is obtained in the subset. The obtained subset is used for DOP estimation. The weighted cost function of quasi-optimal technique is expressed as,

$$C_{ij} = W_i \sum_{j=1}^{n} \cos(2\alpha_{ij}) \tag{5}$$

3. RECURSIVE QUASI-OPTIMAL SATELLITE SELECTION ALGORITHM

This technique gives quasi-optimal geometries for any number of desired satellites as a subset [6]. The important steps of this method are as follows,

i.) The line-of-sight vector for each visible satellite is calculated as,

$$L_i = [(x_i, y_i, z_i), 1] \tag{6}$$

ii.) The line-of-sight vectors for all the visible satellite are expressed in matrix form (Eq.6). The line-of-sight matrix (L_i) is used to compute co-factor matrix 'Q_n' and this is given as,

$$Q_n = L_i^T L \tag{7}$$

iii.) Initialize, k=n and for ‘n’ visible satellites generate subsets with ‘k-1’ satellites.

iv.) The line-of-sight vector of the satellite, which is not a part of the subset is identified. The following mathematical relation is derived to exclude this satellite measurement from co-factor matrix,

$$Q_{k-1,i} = Q_k - L_i^T L_i \tag{8}$$

v.) The obtained co-factor matrix is used to compute GDOP and is given as,

$$GDOP = sqrt(trace(Q_{k-1,i})) \tag{9}$$

vi.) Step iv and v are repeated for all the subsets generated. The subset that corresponds to minimum GDOP value is picked out.

vii.) The satellites in this subset are considered and the same procedure, from step iii to v is repeated until desired numbers of satellites are obtained as a subset.

The weighted recursive quasi optimal technique is expressed as,

$$Q_n^w = R^T W_i R \tag{10}$$

$$Q_{k-1,i}^w = Q_k^w - W_i L_i^T L_i \tag{11}$$

In the open literature it is stated that any factor that determine the quality of signal such as elevation angle, signal strength, covariance of carrier phase bias etc., can be used as weighting factor, such that the performance of these techniques may be improved [6]. Therefore, an effort is made in this aspect.

4. WEIGHT FUNCTIONS

Investigations are carried out using different weight functions namely satellite elevation angle (W_{EL_i}) and elevation angle with signal strength including multipath scaling factor (W_{ELCNR_i}).

4.1 ELEVATION ANGLE

The cosine function of satellite elevation angle, which is widely used for calculation of accuracy of GPS measurements, is considered and given as [7],

$$W_{EL_i} = cos^2\left(\theta_{el}\right) \tag{12}$$

4.2 COMBINATION OF ELEVATION ANGLE, SIGNAL STRENGTH AND MULTIPATH

The impact of atmosphere, multipath and orbit error on the satellite signal can affect the signal strength. These effects can be correlated with elevation angle, multipath and Carrier to Noise Ratio (CNR). Therefore, weight function using these parameters is considered and is given as [8],

$$W_{ELCNR_i} = \frac{\theta_{el_i}}{\theta_{el_{\max}}} + \alpha_m \cdot \frac{CNR_i}{CNR_{\max}} \tag{13}$$

Where,

$\theta_{el_{\max}}$: Maximum elevation angle among the visible satellites at an epoch (deg.)

α_m : Multipath scaling factor

$CNR_{\max}$: Maximum signal strength among the visible satellites at an epoch

In Eq.13 the multipath scaling factor (α_m) is assumed as ‘1’ in multipath free environment and greater than ‘1’ in strong multipath environment. In attitude DOP computation, the value of scaling factor is not defined. Also the multipath effect is satellite specific. Fig.1 shows typical multipath scenario at antenna ‘A1’ due to reflector.

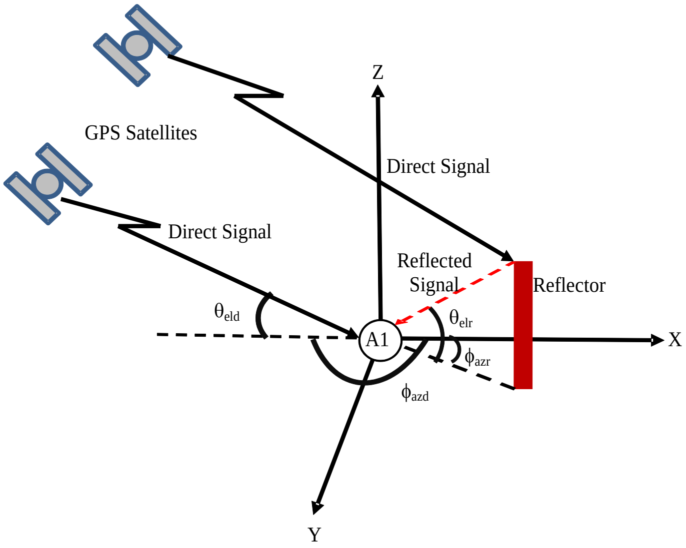


Figure 1. Illustration of multipath scenario

The elevation and azimuth angles of direct signal are denoted as θ_{eld}, ϕ_{azd} and for the reflected signal θ_{elr}, ϕ_{azr} are used. The signal power of multipath signal is a function of reflection coefficient (R_{coef}) which is related to the CNR and is given as [9],

$$\alpha_m = \frac{\sqrt{R_{coef}} - 1}{\sqrt{R_{coef}} + 1} \tag{14}$$

where, reflection coefficient is expressed as,

$$R_{coef} = \frac{10^{\left(\frac{(C/N_0)_{\max}}{20}\right)}}{10^{\left(\frac{(C/N_0)_i}{20}\right)}} \tag{15}$$

C/N_0: GPS signal strength in dB-Hz

The typical GPS receiver has a minimum ‘C/N_0’ of 28-32 dB-Hz and a maximum ‘C/N_0’ of 50-51 dB-Hz. For practical reasons, Eq.13 is modified and given as,

$$W_{ELCNR_i} = \frac{\theta_{el_i}}{\theta_{el_{\max}}} + (1 + \alpha_m).\frac{CNR_i}{CNR_{\max}} \tag{16}$$

In Eq.15, ‘$(C/N_0)_{\max}$’ is taken as 51dB-Hz. ‘$(C/N_0)_i$’ corresponds to the signal strength specific to the ‘ith’ satellite at that epoch. Thus, the reflection coefficient will be ‘1’ for multipath free signal, then the multipath scaling factor 'α_m' becomes zero, this will not affect the generality of Eq.(13). In case of multipath effect, the 'α_m' value is non-zero and is added to '1' as given in Eq.16.

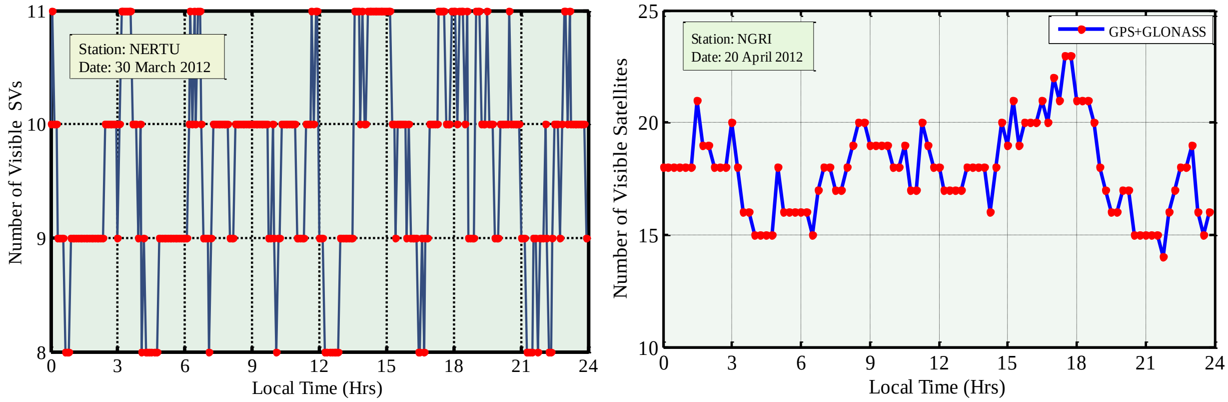
These weight functions are used to eliminate the satellites from the subset to obtain good satellite geometry.

6. DATA ACQUISITION AND PROCESSING

The quasi-optimal and recursive quasi-optimal techniques with aforementioned weight functions are evaluated for GPS constellation and also for combined GPS and GLONASS constellations. The GPS data is obtained from the receiver (make: Novatel, model: DL4 plus) located at Research and Training Unit for Navigational Electronics (17.29^o N, 78.51^o E), Hyderabad, India. GPS and GLONASS data is obtained from the receiver (make: Leica, model: GRX1200GGPRO) located at National Geophysical Research Institute (17.30^o N, 78.55^o E), Hyderabad, India. Two days typical data one corresponds GPS only receiver (30th March 2012) and the other one corresponds to GPS plus GLONASS data (20th April 2012) are used for the analysis.

7. RESULTS AND DISCUSSION

As GDOP is the combination of all other DOPs (HDOP, VDOP, PDOP and TDOP), the value of GDOP is used as measure for the evaluation of fast satellite selection techniques.



(a) (b)

Figure 2. Number of visible SVs with respect to local time at a) NERTU and b) NGRI

Fig.2 shows the total number of satellites visible over NERTU and NGRI stations, Hyderabad. Usually four satellites are sufficient for estimation of position but to increase robustness in estimation more than four can be used. It can be observed that the number of SVs is varying from a minimum of 8 to maximum of 11 at NERTU (Fig.2a) and a minimum of 14 to maximum of 23 at NGRI (Fig.2b).

7.1 SATELLITE SELECTION TECHNIQUES COMPARISION FOR GPS CONSTELLATIONS

The accuracy of the navigation solution using fixed number of satellites essentially depends on the quality of the subset (GDOP) considered. As the minimum number of SVs visible is 8, the subset with seven satellites is considered for DOP estimation (Fig.2a). Fig.3 shows the variations in GDOP due to best 'seven', quasi-optimal and quasi-optimal with weight functions (W_{EL} and W_{ELCNR}). Though, combinations method (Best-7

SVs) gives optimal GDOP values it imposes huge computational load. The minimum and maximum GDOP values due to quasi-optimal are 1.60 and 5.93 respectively. The 'W_{EL}' when used as weight function with quasi-optimal the minimum and maximum DOP values are 1.89 and 10.29 respectively. When 'W_{ELCNR}' is used as weight function the minimum and maximum DOP values are 1.59 and 5.90 respectively. It is noticed that, the weight functions W_{EL} and W_{ELCNR} for quasi-optimal technique did not aid in significant improvement of DOP estimation. Further, the W_{EL} as a weight function led to degraded performance.

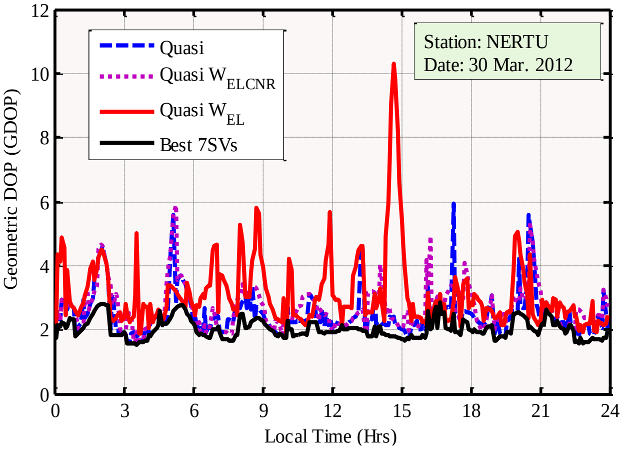
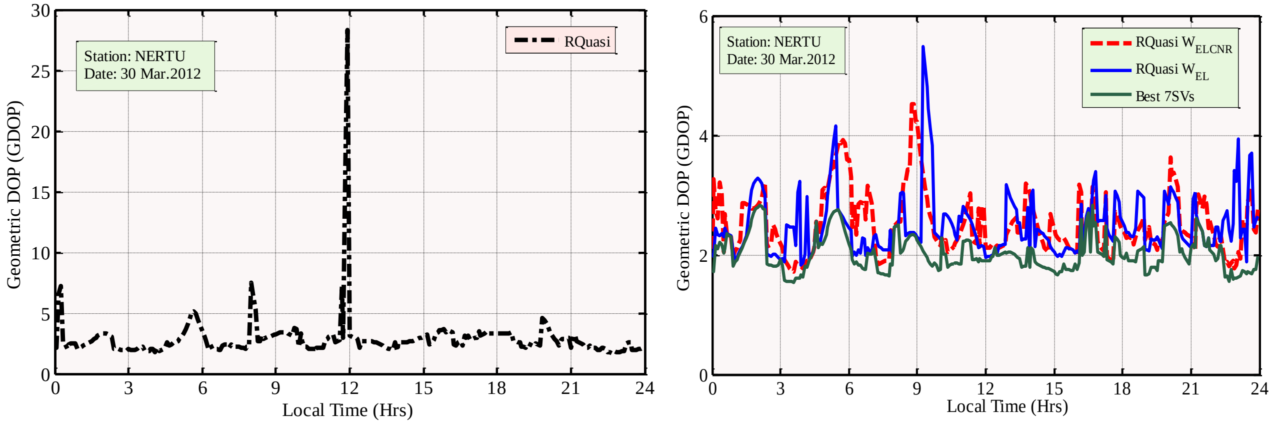


Figure 3. GDOP variations due to Best-7SVs, quasi-optimal and weighted quasi-optimal



(a) (b)

Figure 4. GDOP variations due to (a) Recursive quasi-optimal (b) Weighted recursive quasi-optimal techniques and Best-7 SVs at NERTU

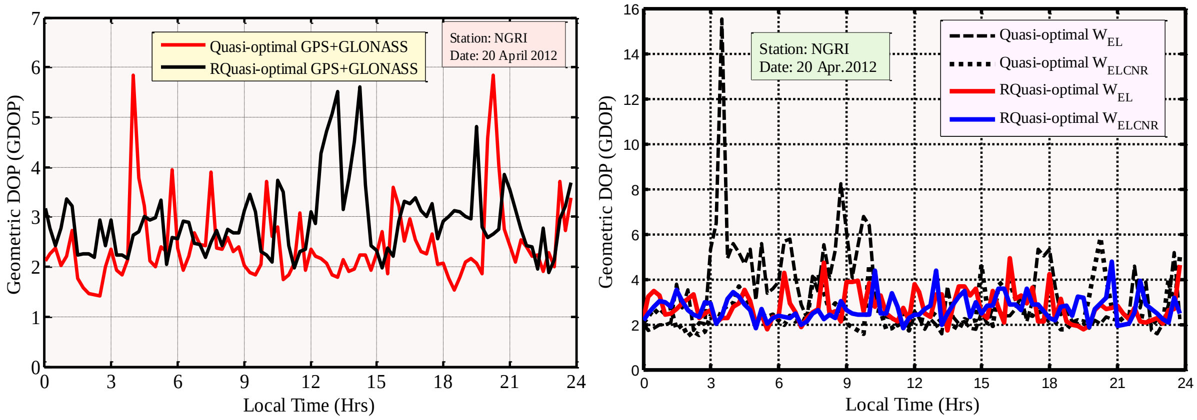
Further, the analysis is carried out to evaluate the GDOP due to recursive quasi-optimal technique (Fig.4a). It can be observed that the GDOP varies from a minimum of 1.72 to a maximum of 28.30 (11.92 Hrs). Fig.4b shows GDOP variations due to recursive quasi-optimal technique using weight functions. It can be noticed from Fig.4 (a) and (b) that, using weight functions aid in better DOP realisation for recursive quasi-optimal technique. The GDOP estimates with the technique at 11.92 Hrs due to weights W_{EL} and W_{ELCNR} are 2.13 and 2.83 respectively. Table 1 shows the maximum, mean and standard deviation of GDOP. The maximum GDOP is 4.51 due to the weight function, W_{ELCNR} , which is less when compared to the maximum GDOP values of 5.47 due to W_{EL}. From the maximum GDOP values of Fig.3 and Fig.4b, it is noticed that with weight functions, the performance of recursive quasi technique is to be better.

Table 1: Min., max., mean and std. of GDOP for recursive quasi-optimal technique (30th Mar. 2012**)**

Recursive Quasi-optimal (RQuasi) technique (Date:30th Mar. 2012)	**GDOP**			
	Min.	**Max.**	**Mean**	**Std.**
Recursive Quasi-optimal	1.72	28.31	2.89	1.91
Recursive Quasi-optimal with W_{EL}	1.83	5.47	2.50	0.52
Recursive Quasi-optimal with W_{ELCNR}	1.72	4.51	2.51	0.52

7.2 SATELLITE SELECTION TECHNIQUES COMPARISION FOR GPS AND GLONASS CONSTELLATIONS

Similar analysis is extended for a typical day (20th April 2012) with NGRI station data for combined GPS and GLONASS constellation. Fig.2b shows the number of satellites visible over the NGRI station, Hyderabad. As the minimum number of SVs visible is 14, the subset with thirteen satellites is considered for DOP estimation. The value of GDOP varies from a minimum of 1.42 to a maximum of 5.83 for Quasi-optimal and 1.89 to 5.60 for recursive quasi-optimal technique respectively (Fig.5a). Fig.5b shows the GDOP estimates of combined GPS and GLONASS due to weight functions W_{EL} and W_{ELCNR}.



(a) (b)

Figure 5. GDOP variations for combined GPS and GLONASS due to Quasi-optimal and Recursive quasi-optimal techniques a) without weight function and b) with weight function

It is observed that there is slight improvement in DOP due to weighted recursive quasi-optimal technique. Table 2 shows the minimum, maximum, and mean of GDOP. The maximum GDOP due to quasi-optimal technique using weight functions W_{EL} and W_{ELCNR} are 15.5 and 5.83, whereas in case of recursive quasi-optimal technique, it is 4.92 and 4.75 respectively. Table 2 shows that, the weight function W_{ELCNR} aids in the improvement of overall performance of recursive quasi-optimal technique.

Table 2: Min., max. and mean of GDOP for weighted quasi-optimal and recursive quasi-optimal techniques for dual constellation (GPS and GLONASS)

GDOP	**Weighted Quasi-optimal**		**Weighted Recursive Quasi-optimal**	
	W_{EL}	W_{ELCNR}	W_{EL}	W_{ELCNR}
Minimum	1.57	1.46	1.75	1.81
Maximum	15.5	5.83	4.92	4.75
Mean	3.37	2.56	2.78	2.69

8. CONCLUSIONS

The fast satellite selection techniques with the weight functions are evaluated for GPS constellation data and also for combined GPS and GLONASS constellations data. In both the cases, the weight functions W_{EL} and W_{ELCNR} with quasi-optimal technique did not aid in improvement in DOP. But it is observed that, the weight function W_{ELCNR}, when used with recursive quasi-optimal technique gives near optimal DOP. It is apparent from the maximum GDOP values of GPS constellation on 30th March 2012. Due to recursive quasi-optimal maximum GDOP is 28.31 and with weight functions W_{EL} and W_{ELCNR} the maximum GDOP is 5.47 and 4.51 respectively. Significant improvement in DOP is also noticed due to W_{ELCNR} in case of combined GPS and GLONASS. Even in this case, the maximum GDOP value observed on 20th April 2012 due to recursive quasi-optimal technique is 5.60 and with weight functions W_{ELCNR} the maximum GDOP is 4.75. Significant improvement is achieved when W_{ELCNR} is used as the weight function with recursive quasi-optimal technique.

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