Two-Sided Matching via Balanced Exchange: Tuition and Worker Exchanges

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Design of Matching Markets

In the last decade and a half, economists have worked on the design of matching markets

- Some influenced policy makers for adoption of new policies and institutions:
  - Doctor-residency matching (Roth 84, Roth & Peranson 99)
  - School choice (Balinski & Sönmez 98, Abdulkadiroğlu & Sönmez 03)
  - Kidney exchange (Roth, Sönmez,Ünver 04, 05, 07)
  - Signaling in Econ PhD Market (Coles, Kushnir, Niederle 13)
  - Course allocation (Sönmez & Ünver 10, Budish 11, Budish & Kessler 14)
  - Adoption of children (Vaughn, Akan, Kesten, Ünver 14)
- Some have not influenced the policy yet:

  On-campus housing; Cadet-branch matching in the military; Dynamic daycare and public housing assignment; Lobar live-donor lung and liver exchange ...
- Deeper understanding of how matching markets work

In This Paper

- A new two-sided matching problem where eventual market outcome is linked to an initial status-quo matching, which may give firms and workers certain rights on how the future activity can play out.
- Two new classes of assignment problems which mimic **Tuition Exchanges** and **Student/Worker Exchanges**
  - The Tuition Exchange, Inc.
  - US National and EU Erasmus Student Exchange
  - Commonwealth Teacher Exchange
  - International Clinical Exchange
  - Employee rotation programs of departments of a company or institution: public school teacher rotation such as Turkey
- Maintaining one-to-one balance between the outgoing and incoming students is the central issue for colleges

In This Paper

- New axiom playing key role in success of these markets: **Balancedness**
- Procedure in use suffers from serious problems
  - Decentralized matching causing withdrawal of schools (tuition exchange): we identify the problems with stable market outcomes.
  - Bilateral agreements (student/worker exchanges) not being able to get all gains from exchange.

In This Paper

- Propose a new mechanism: **two-sided top-trading cycles (2S-TTC)**
  - Uses a variant TTC algorithm (Gale via Shapley and Scarf 74, and Abdulkadiroğlu and Sönmez 03), first use of TTC algorithm in a two-sided market
  - The unique mechanism that satisfies student–strategy-proofness, balanced-efficiency, individually rationality, and a fairness criterion respect for internal priorities.
  - Immune to admission and export quota manipulation by colleges.
  - Any individually rational mechanism that matches more students is manipulable by students.
  - When firms have 0-1 preferences over incoming workers, then it is also stable and strategy-proof.

What is Tuition Exchange?

- “The Tuition Exchange” (TuitionExchange.org) is a reciprocal scholarship program for children of faculty at more than 600 institutions.
- Dependent children of faculty are able to access tuition benefits in the other member institutions.
- Participating in tuition exchange programs enhances the packages at a nominal cost.
- Every year 20 new institutions join the program.
- On average 6,000 scholarships are awarded annually: $115 million awarded annually.
- No money transaction and tax.
- Schools prefer tuition exchange over direct compensation to protect themselves from “yearly demand shocks” (marginal cost of a student $\approx 1/4$ of tuition; fixed costs dominate)

What is Tuition Exchange?

Each institutions has agreed to maintain a balance between

- The number of awarded students sponsored by an institution: EXPORTS
- The number of scholarships awarded to students sponsored by other colleges: IMPORTS
- If EXPORTS exceed IMPORTS then

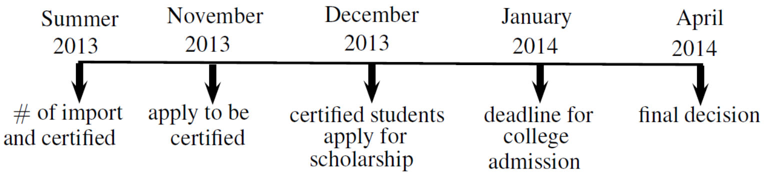
SUSPENDED

What is Tuition Exchange?

- Not all applicants are certified as eligible by the home institutions
  - Based on years of service
- Not all certified applicants are awarded
  - Scholarship receipts are chosen based on academic profile

What is Tuition Exchange?

Timeline



- Lewis & Clark, Reed, Puget Sound, Whitman, Willamette
- Children of faculty members were allowed to attend one of the members tuition free upon admission.
  - Balancedness was not required.
  - Huge imbalances between the colleges.
- It will stop accepting new applicants after Fall of 2015.

Why Balancedness?

Bilateral Agreements

- In student/worker exchange programs bilateral agreements are signed
- If balancedness fails after a period of time the agreement is nullified.

Why Balancedness?

Time banks and favor currency holdings

- In "time banks" people make favors of each other.
- Marginal rate of substitution is one favor is equal to one favor (but not must).
- Baby-sitting, dog-sitting exchanges.
- Sweeney & Sweeney (77) reports the shutting down of a baby–sitting coop, as people are averse to spending their accumulated favor currencies (negative balance aversion). (See also Möbius 01 on dynamic favor exchange.)

A **tuition exchange market** consists of

- a set of **colleges** $C = \{c_1, ..., c_m\}$
- a set of **students** $S = \bigcup_{c \in C} S_c$ where S_c is the set of students who are applying to be sponsored by college c
- an **admissions quota** vector $q = (q_c)_{c \in C}$ where q_c is the maximum number of students who will be imported by college c
- an **eligibility quota** vector $e = (e_c)_{c \in C}$ where e_c is the number of students certified as eligible by college c
- a list of **college internal priorities** $\rhd = (\rhd_c)_{c \in C}$ ($\rhd_c$ is a linear order over S_c); let $r_c(s)$ be the ranking of student $s \in S_c$ in $\rhd_c$.
- a list of **student and college preferences** $\succsim = (\succsim_C, \succsim_S) = ((\succsim_c)_{c \in C}, (\succsim_s)_{s \in S})$ over matchings. Students only care about their assignments.

Fixing $C, \{S_c\}_{c \in C}, \rhd$, a tuition exchange market is defined by $[q, e, \succsim]$.

Model: Matchings

An outcome of a market $[q, e, \succsim]$ is a *matching*.

- A **matching** is a correspondence $\mu : C \cup S \twoheadrightarrow C \cup S \cup c_{\emptyset}$ such that:
  - $\mu(c) \subseteq S$ where $|\mu(c)| \leq q_c$ for all $c \in C$,
  - $\mu(s) \subseteq C \cup c_{\emptyset}$ where $|\mu(s)| = 1$ for all $s \in S$,
  - if $r_c(s) > e_c$ then $\mu(s) = c_{\emptyset}$ for all $s \in S_c$ (i.e., a student is eligible if and only if its internal priority does not exceed the cutoff.)
- Set of matchings $\mathscr{M}$.
- A ***(direct) mechanism*** φ is a systematic way of selecting a matching for each market $[q, e, \succsim]$.

Property: Balancedness

Given a matching μ,

- X_c^μ: **set of exports** of college c; the eligible students in S_c matched with other colleges.
- M_c^μ**:** **set of imports** the eligible students of other colleges matched with c
- $b_c^\mu = |M_c^\mu| - |X_c^\mu|$: **net balance** of college c.
- μ is **balanced** if $b_c^\mu = 0$ for all $c \in C$.

Model: College Preferences

- College preferences over admitted (groups of) students are **responsive** (Roth, 1985) (to ranking over individual students) and are denoted by a linear order P_c:
  For any $J \subset S$ with $|J| < q_c$ and any $i, j \in S \setminus J$,
  - $(J \cup \{i\}) P_c J \iff iP_c\emptyset$
  - $(J \cup \{i\}) P_c (J \cup \{j\}) \iff iP_c j$
- Colleges possibly also care about their net balance in the matching in addition to the admitted students.
  - For any two matchings ν and μ such that $b_c^{\nu} = b_c^{\mu}$ we have $\nu(c) P_c \mu(c) \implies \nu \succ_c \mu$

Other Desired Properties: Efficiency

- A matching μ is **Pareto efficient** if it is not possible to find an alternative matching that makes
  - all agents at least as well off,
  - at least one agent better off.
- A balanced matching is ***balanced-efficient*** if it is not Pareto dominated by another balanced matching.

Other Desired Properties: Non-manipulability

- A mechanism is **immune to preference manipulation by students (or colleges)** if it is always a weakly dominant strategy for each student (or college) to truthfully reveal her (or its) preferences over matchings for fixed quotas.
- A mechanism is **immune to quota manipulation by colleges** if for fixed college preferences, it is a weakly dominant strategy for each college to reveal its true admission and eligibility quotas.
- A mechanism is **student–strategy-proof** if it is immune to preference manipulation by students.
- A mechanism is **college–strategy-proof** if it is a weakly dominant strategy for a college to truthfully reveal its preferences and admission and eligibility quotas.
- A mechanism is **strategy-proof** if it is student–strategy-proof and college–strategy-proof.

Other Desired Properties: Fairness

- The by-laws of many colleges regarding tuition exemption and exchange use priorities based on the seniority of the dependent faculty member/staff. This needs to be somehow respected.
- A mechanism φ ***respects internal priorities*** if for all colleges c, whenever a student $i \in S_c$ is assigned to a college in problem $[(q_c, q_{-c}), (e_c, e_{-c}), \succsim]$ then i is also assigned to a college in the problem $[(\tilde{q}_c, q_{-c}), (\widetilde{e}_c, e_{-c}), \succsim]$ where $\widetilde{e}_c > e_c$ and $\widetilde{q}_c \geq q_c$.

Decentralized Market and Stability

- The current decentralized market works as follows
  - Eligible students applications are sent to the colleges listed in their preference list
  - Each college ranks its applicants and sends acceptance letter to the best students without exceeding its quota
  - Students receive acceptance letter and reject all the offers except the best one
  - Each rejected colleges sends acceptance letter to the best students in the waiting list without exceeding its quota
  - Students receive acceptance letter and reject all the offers except the best one.
- This procedure works like the first few steps of the college-proposing deferred acceptance algorithm.
- **Benchmark decentralized market mechanism: stable mechanisms.**

Stability and Market Shutting Down

- We say a matching μ is ***blocked by a college*** $c \in C$ if there exists some $\mu' \in \mathscr{M}$ such that $\mu' \succ_c \mu$, $\mu'(s) = \mu(s)$ for all $s \in S \setminus \mu(c)$ and $\mu'(c) \subset \mu(c)$.
- A matching μ is ***blocked by a student*** $s \in S$ if $c_\emptyset \; P_s \; \mu(c)$.
- A matching μ is ***individually rational*** if it is not blocked by any individual college or student.
- A matching $\mu \in \mathscr{M}$ is ***blocked by college-student pair*** (c, s) if $c \; P_s \; \mu(s)$ and $\mu' \succ_c \mu$ where $\mu' \in \mathscr{M}$ is obtained from μ by the *mutual deviation* of college c and student s, that is, $s \in \mu'(c) \subseteq \mu(c) \cup s$, and $\mu'(s') = \mu(s')$ for all $s' \in S \setminus (\mu(c) \cup s)$.
- A matching μ is ***(pairwise) stable*** if it is individually rational and not blocked by any college-student pair.

Assumption (1)

For any $c \in C$ and $\mu, \nu \in \mathcal{M}$,
*(1) (**Preference increases with better admitted class and non-deteriorating balance**) if $b_c^\mu \geq b_c^\nu$ and $\mu(c) P_c^* \nu(c)$ then $\mu \succ_c \nu$,*
*(2) (**Awarding unacceptable students exchange scholarships is not preferable**) if there exists $s \in \nu(c) \setminus \mu(c)$, $\emptyset P_c s$ and $\nu(s') = \mu(s')$ for all $s' \in S \setminus s$ then $\mu \succ_c \nu$, and*
*(3) (**Unacceptability of own students for exchange scholarships**) $\emptyset P_c s$ for all $s \in S_c$.*

Assumption (2)

*(**Negative Net Balance Aversion**) College c prefers $\mu \in \mathcal{M}$ with $b_c^\mu = 0$ to all $\nu \in \mathcal{M}$ with $b_c^\nu < 0$.*

Stability and Market Shutting Down

Theorem

Under Assumption 1,

- *A stable matching exists.*
- *All stable matchings have the same net balance for all colleges.*
- *There may not be a stable and balanced matching in general.*
- *In a quota reporting game (when preferences are common knowledge) where market outcome is found by a stable mechanism:*
  - *If Assumption 2 also holds, the only best responses for a negative net balance college (under true quota revelation) dictate to decrease its eligibility quota.*
  - *When a college decreases its eligibility quota, the negative net balance of no college gets closer to zero.*

Two-Sided Top-Trading-Cycles (2S-TTC) Mechanism works via the following variant of A&S TTC algorithm: Consider a problem $[q, e, \succsim]$: Assign two counters for import and eligible students to each college $c \neq c_\emptyset$ and set them equal to q_c and e_c.

- Each student points to her favorite college, which considers her acceptable, each college $c \neq c_\emptyset$ points to the highest internal priority student, and $c_\emptyset$ points to all students pointing to it.
- Every student in a cycle is assigned a seat at the college she is pointing to removed.
- The eligible student counter of each college whose student is in a cycle is reduced by one.
- The import counter of each college in a cycle is reduced by one only if the cycle includes at least two colleges.
  if either counter falls to zero, the college is removed.
- Continue with the remaining colleges and students.

Two-Sided Top Trading Cycles

Example

Let $C = \{a, b, c, d, e\}$, $S_a = \{\mathbf{1}, \mathbf{2}\}$, $S_b = \{\mathbf{3}, \mathbf{4}\}$, $S_c = \{\mathbf{5}, \mathbf{6}\}$, $S_d = \{\mathbf{7}, \mathbf{8}\}$, $S_e = \{\mathbf{9}\}$. Let $e = (2, 2, 2, 2, 1)$ and $q = (2, 2, 2, 1, 1)$. The internal priority order is given as

a	*b*	*c*	*d*	*e*
1	**3**	**6**	**7**	**9**
2	**4**	**5**	**8**	

The preference profiles of colleges and students are given as

a	*b*	*c*	*d*	*e*
3	**5**	**2**	**2**	**2**
4	**1**	**3**	**3**	**3**
5	**6**	**4**	**4**	**8**
9	**2**	**9**	**9**	**7**
7	**7**	**7**	**5**	**5**

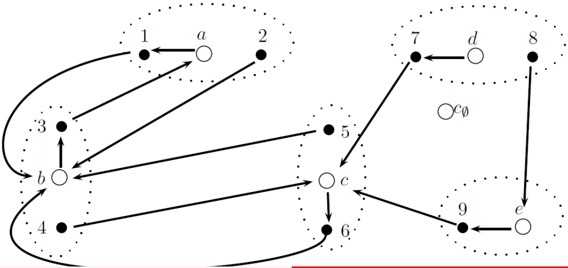
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
b	*b*	*a*	*c*	*b*	*a*	*c*	*e*	*c*
c	*c*	*c*	*a*	*a*	*b*	*a*	*c*	*d*
$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$

$\emptyset P_a \mathbf{6}$, Counters: $e = (2,2,2,2,1)$ $q = (2,2,2,1,1)$

a	*b*	*c*	*d*	*e*
1	**3**	**6**	**7**	**9**
2	**4**	**5**	**8**	

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
b	*b*	*a*	*c*	*b*	*a*	*c*	*e*	*c*
c	*c*	*c*	*a*	*a*	*b*	*a*	*c*	*d*
$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$

Round 1

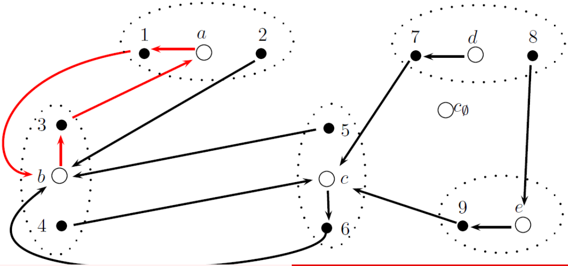


$\emptyset P_a \mathbf{6}$, Counters: $e = (1,1,2,2,1)$ $q = (1,1,2,1,1)$

a	*b*	*c*	*d*	*e*
1	**3**	**6**	**7**	**9**
2	**4**	**5**	**8**	

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
b	*b*	*a*	*c*	*b*	*a*	*c*	*e*	*c*
c	*c*	*c*	*a*	*a*	*b*	*a*	*c*	*d*
$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$

Round 1



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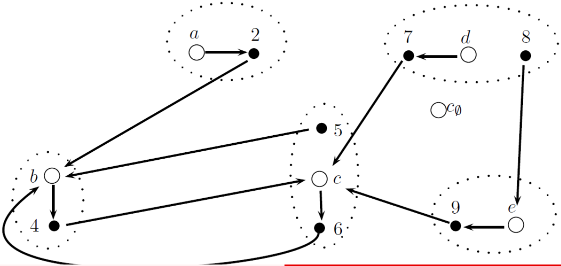
a	*b*	*c*	*d*	*e*
1	**3**	**6**	**7**	**9**
2	**4**	**5**	**8**	

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
b	*b*	*a*	*c*	*b*	*a*	*c*	*e*	*c*
c	*c*	*c*	*a*	*a*	*b*	*a*	*c*	*d*
$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$

Round 2

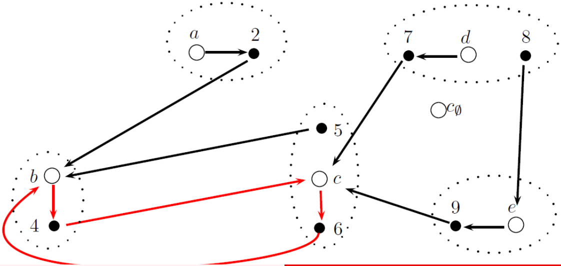


$\emptyset P_a \mathbf{6}$, Counters: $e = (1,0,1,2,1)$ $q = (1,0,1,1,1)$

a	*b*	*c*	*d*	*e*
1	**3**	**6**	**7**	**9**
2	**4**	**5**	**8**	

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
b	*b*	*a*	*c*	*b*	*a*	*c*	*e*	*c*
c	*c*	*c*	*a*	*a*	*b*	*a*	*c*	*d*
$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$

Round 2

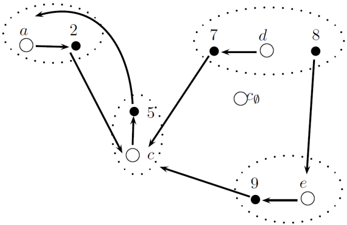


$\emptyset P_a \mathbf{6}$, Counters: $e = (1,0,1,2,1)$ $q = (1,0,1,1,1)$

a	*b*	*c*	*d*	*e*
1	**3**	**6**	**7**	**9**
2	**4**	**5**	**8**	

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
b	*b*	*a*	*c*	*b*	*a*	*c*	*e*	*c*
c	*c*	*c*	*a*	*a*	*b*	*a*	*c*	*d*
$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$

Round 3



Two-Sided Top Trading Cycles

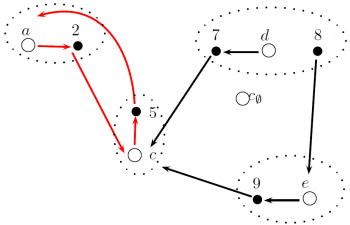
Example

$\emptyset P_a \mathbf{6}$, Counters: $e = (0,0,0,2,1)$ $q = (0,0,0,1,1)$

a	b	c	d	e
1	**3**	**6**	**7**	**9**
2	**4**	**5**	**8**	

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
b	b	a	c	b	a	c	e	c
c	c	c	a	a	b	a	c	d
$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$

Round 3

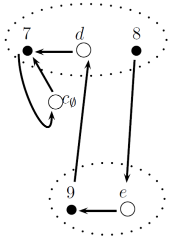


$\emptyset P_a \mathbf{6}$, Counters: $e = (0,0,0,2,1)$ $q = (0,0,0,1,1)$

a	b	c	d	e
1	**3**	**6**	**7**	**9**
2	**4**	**5**	**8**	

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
b	b	a	c	b	a	c	e	c
c	c	c	a	a	b	a	c	d
$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$

Round 4

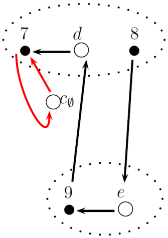


$\emptyset P_a \mathbf{6}$, Counters: $e = (0,0,0,1,1)$ $q = (0,0,0,1,1)$

a	*b*	*c*	*d*	*e*
1	**3**	**6**	**7**	**9**
2	**4**	**5**	**8**	

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
b	*b*	*a*	*c*	*b*	*a*	*c*	*e*	*c*
c	*c*	*c*	*a*	*a*	*b*	*a*	*c*	*d*
$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$

Round 4

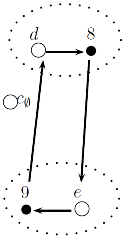


$\emptyset P_a \mathbf{6}$, Counters: $e = (0,0,0,1,1)$ $q = (0,0,0,1,1)$

a	b	c	d	e
1	**3**	**6**	**7**	**9**
2	**4**	**5**	**8**	

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
b	b	a	c	b	a	c	e	c
c	c	c	a	a	b	a	c	d
$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$

Round 5

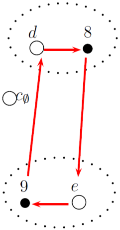


$\emptyset P_a \mathbf{6}$, Counters: $e = (0,0,0,0,0)$ $q = (0,0,0,0,0)$

a	*b*	*c*	*d*	*e*
1	**3**	**6**	**7**	**9**
2	**4**	**5**	**8**	

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
b	*b*	*a*	*c*	*b*	*a*	*c*	*e*	*c*
c	*c*	*c*	*a*	*a*	*b*	*a*	*c*	*d*
$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$

Round 5



Theorem

2S-TTC is balanced-efficient, respecting internal priorities, and individually rational.

Theorem

There does not exist a mechanism which is balanced-efficient, individually rational, and immune to preference manipulation by colleges (even under Assumption 1).

Theorem

2S-TTC is student–group–strategy-proof.

Relax Assumptions 1 and 2:

Assumption (3)

For any $\mu, \nu \in \mathscr{M}$ and $c \in C$, if $b_c^{\mu} = 0$, $b_c^{\nu} \leq 0$, and $\mu(c) P_c \nu(c)$ then $\mu \succ_c \nu$.

Theorem

Under Assumption 3 and when $e_c = |S_c|$ for all c, 2S-TTC is immune to quota manipulation by colleges.

- Given an acceptable set of students, colleges are indifferent between any of their rankings.
- Hence, the mechanism can be run through colleges only reporting acceptable students.

Theorem

Under Assumption 3, 2S-TTC is the ***unique*** *mechanism that is balanced-efficient, respecting internal priorities, individually rational, and student–strategy-proof.*

Related: Ma (94), Pápai (00), Pycia & Ünver (09), Morrill (11), Abdulkadiroğlu & Che (11), Dur (12)

Proposition

Any balanced and individually rational mechanism, which

- *assigns at least the same number of students as 2S-TTC*
- *selects an allocation in which more student is assigned whenever exists,*

is not strategy-proof for students.

Temporary Student and Worker Exchanges

Compatibility-Based (0-1) Firm Preferences

- In temporary exchange programs, firm preferences can be coarser.
- Suppose firms find workers either acceptable or not:
  - US National exchange and EU Erasmus Exchange
  - International Clinical Exchange: Medical students
  - Commonwealth Tuition Exchange
  - Staff Rotation Programs: Teacher rotation under Ministry of Education; employee rotation for multinationals.
- Initial employees are acceptable.
- If a firm's employee is not matched to a different firm in the market then she remains matched to her home firm.

Temporary Student and Worker Exchanges

Assumption (3*)

For any $c \in C$ and $\mu, \nu \in \mathscr{M}$, if $b_c^{\mu} = b_c^{\nu}$ and $|\{s \in \mu(c) : sP_c\emptyset\}| \geq |\{s \in \nu(c) : sP_c\emptyset\}|$ then $\mu \succsim_c \nu$.

Theorem

Under Assumption 3, 2S-TTC is*

- *a balanced–efficient, individually rational, **strategy-proof**, and **stable** mechanism that also respects internal priorities; and*
- *the **unique** balanced–efficient, individually rational, and student–strategy-proof mechanism that respects internal priorities.*

Future Work

- Implementation
- Dynamic tuition exchange 2S-TTCC
- Erasmus student exchange and diversity, 2S-TTCC (Dur, Kesten, Ünver, in progress)