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Incentivized Kidney Exchange[†]

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Over the last 15 years, kidney exchange has become a mainstream paradigm to increase transplants. However, compatible pairs do not participate, and full benefits from exchange can be realized only if they do. We propose incentivizing compatible pairs to participate in exchange by insuring their patients against future renal failure via increased priority in deceased-donor queue. We analyze equity and welfare benefits of this scheme through a new dynamic continuum model. We calibrate the model with US data and quantify substantial gains from adopting incentivized exchange, both in terms of access to living-donor transplants and reduced competition for deceased-donor transplants. (*JEL* D47, I11, I12, I18)

Transplantation is the best remedy for end-stage renal disease. However, there is a severe shortage of transplant kidneys, which can be harvested from either deceased donors or living donors. As of January 2019, more than 290,000 kidney transplants from deceased donors and more than 150,000 transplants from living donors have been performed in the United States. The number of willing living donors has been considerably higher than the number of living-donor transplants performed, yet a large fraction of intended gifts have not materialized due to biological incompatibilities. More than 30 percent of potential living donors are blood-type incompatible, and at least 7 percent are tissue-type incompatible, with their intended recipients. Blood-type *O* patients are especially disadvantaged by these biological barriers because they are only blood-type compatible with blood-type *O* donors. In contrast, blood-type *A* patients are blood-type compatible with donors of blood types *A* and *O*, blood-type *B* patients are blood-type compatible with donors of blood

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types *B* and *O*, and blood-type *AB* patients are blood-type compatible with donors of all blood types.[1] The resulting disadvantage to blood-type *O* patients is mitigated in deceased-donor transplants by a policy that reserves blood-type *O* kidneys for blood-type *O* patients, but a similar policy is not possible for living-donor transplants since a living-donor kidney is typically intended as a gift for a specific patient.

Kidney exchange has become more popular over the last 15 years as a way to circumvent the biological barriers to living-donor transplantation. In its most basic form, a kidney exchange is a swap of donors between two patients who are each incompatible with their own donor but compatible with the other patient's donor. Both donors' intended gifts are realized through the exchange, providing each patient with a transplant. However, blood-type *O* patients are again less likely to benefit from this transplantation modality. Consider a blood-type *O* patient unable to receive a transplant from his blood-type-incompatible *A* donor. The pair can potentially swap donors with a blood-type *A* patient who has a blood-type *O* donor. But since the blood-type *A* patient and blood-type *O* donor are compatible with each other, they are unlikely to enter a kidney exchange, only arriving when they are tissue-type incompatible. Hence, a large number of "underdemanded" blood-type *O* patients with blood-type *A* donors compete for a relatively scarce population of "overdemanded" blood-type *A* patients with blood-type *O* donors.[2] These pairs with highly sought-after blood-type *O* donors become available for exchange only because of a tissue-type incompatibility. Ironically, a biological barrier to transplantation increases in the number of living-donor transplants by facilitating a welfare-increasing utilization of living donors.

Obviously, the competition for an exchange would not be so unfavorable for blood-type *O* patients with blood-type-incompatible donors if all pairs participated in kidney exchange. Indeed, when a clearinghouse for organized kidney exchange was initially proposed, market designers advocated a mechanism where all pairs would participate in exchange, whether they are compatible or not (Roth, Sönmez, and Ünver 2004). However, since patients with compatible donors do not need an exchange, the practice of kidney exchange evolved mostly without them. Despite the resulting suboptimal utilization of living donors, none of the main kidney exchange systems currently offers any incentives for compatible pairs to participate in exchange.[3] This shortcoming is the motivation of our paper. Our main contribution is introducing and analyzing an incentive scheme that encourages compatible pairs to participate in kidney exchange. The incentive we propose is priority in the deceased-donor queue if the patient needs a repeat transplant, thus serving as insurance against a future kidney failure.[4] This insurance is valuable because the median lifespan of a living-donor transplant kidney is less than 16 years (Matas et al. 2015, conditional on one year survival), and 16 percent of all living-donor transplants fail within the first 5 years (United States Renal Data System 2018).

[1] For the United States, 45.6 percent of the population is blood type *O*, 37.8 percent is blood type *A*, 12.6 percent is blood type *B*, and 4 percent is blood type *AB*.

[2] Based on 2012–2014 data from the three largest kidney-exchange clearinghouses in the United States, the percentage of pairs with blood-type *O* patients was in the range 58.4–60.7 and the percentage of pairs with blood-type *O* donors was in the range 30.8–33 (Agarwal et al. 2019).

[3] The only exception we are aware of is the single-center kidney exchange program at the Methodist Hospital in San Antonio, where compatible pairs are incentivized with higher quality donors (Bingaman et al. 2012).

[4] A living donor already receives priority in the deceased-donor queue in the event of a kidney failure.

While our proposed incentive scheme can be offered to all compatible pairs, we analyze a version where the target group is the set of "overdemanded" pairs. These are compatible pairs where the blood types of the donor and the patient differ and either the donor is of blood type *O* or the patient is of blood type *AB*. For these pairs, the donor has a more highly sought-after blood type than the patient, and the patient is more likely to have lower tissue-type incompatibility chance with a random donor than the average patient, and their participation in exchange directly results in an additional transplant to the patient of an "underdemanded" pair.

Our incentive scheme has considerable potential for increasing welfare. Using data from the United States, our numerical analysis in Table 4 suggests that incentivized exchange can substantially increase the number of living-donor transplants even for modest participation rates from compatible pairs. In the absence of kidney exchange, 44.7 percent of patients with living donors fail to receive a transplant from their donors. With kidney exchange, the percentage of unutilized living donors falls to 32.1. Every 10 percent increase in participation of the target group in incentivized kidney exchange further decreases the percentage of unutilized living donors by around 2 percent, and leads to around 180 additional patients receiving a transplant each year.[5]

While the primary role of incentivized exchange is to increase the number of living-donor transplants, it also improves equity in access both for living-donor and deceased-donor transplants. Equity in access is one of the main objectives of the Organ Procurement and Transplantation Network (OPTN), the body that oversees the allocation of transplant organs in the United States.[6]

In the November 2016 OPTN report on equity in access (OPTN and UNOS 2016), patient blood type was identified as one of the three main contributors to inequity in deceased-donor transplantation.[7] Based on this report (and consistent

[5] While it is not clear what a "reasonable" participation rate of the target group in incentivized exchange might be, Kranenburg et al. (2006) reports a 25 percent rate in a survey from the Netherlands when no additional benefit is offered to the compatible pair, Ratner et al. (2010) reports a rate of 50 percent or more when there is some benefit to the patient of the compatible pair based on a survey from New York City, and Hendren et al. (2015) reports a participation willingness rate exceeding 90 percent from Canadian population of previous donors and current patients (willingness gets stronger when there is some benefit to the patient of the pair; however, this latter survey had a 42 percent response rate among donors and 100 percent among patients). We observe a positive time trend in these estimates correlated with the wider publicity of kidney exchange practice although their methodologies were somewhat different from each other. These estimates are from self-reported survey studies based on a hypothetical question. The only empirical estimate we could find from a natural experiment is about the substitution willingness between a compatible living donor and an immediate deceased-donor transplant. Choi (2019) reports this rate as 17.3 percent. On the other hand, the rate we really need is the substitution willingness between the direct compatible living donor and another compatible living donor plus future priority on the deceased-donor transplant list.

[6] Effective March 16, 2000, the US Department of Health and Human Services (HHS) implemented a *Final Rule* establishing a regulatory framework for the structure and operations of the OPTN. The primary goal of the OPTN is "to increase and ensure the effectiveness, efficiency, and equity of organ sharing in the national system of organ allocation," and "to increase the supply of donated organs available for transplantation" (Duda 2005, p. 604). Initially, the Final Rule only regulated allocation of deceased-donor organs. Since June 2006, its scope has been extended to include living-donor organs: "Under 42 CFR 121.4(a)(6), the Secretary directs the OPTN to develop policies regarding living organ donors and living organ donor recipients, including policies for the equitable allocation of living-donor organs, in accordance with section 121.8 of the final rule." See Health Resources and Services Administration (2006).

[7] The other two are donor service area and patient PRA (panel reactive antibodies), which indicates the likelihood of tissue-type incompatibility for a patient. The primary way to reduce inequity to high PRA patients is to increase the pool size. Hence, incentivized exchange should contribute to this objective as well. Moreover, incentivizing all compatible pairs rather than only overdemanded-type pairs can have a more pronounced benefit for high PRA patients.

with our numerical analysis in Section IV), patients of blood types *O* and *B* are disadvantaged in the United States compared to patients of blood types *A* and *AB*. Incentivized exchange improves equity in access for living-donor transplantation, mainly by increasing transplants to blood-type *O* patients through donor exchanges with incentivized pairs. This in turn improves equity in access for deceased-donor transplantation, since blood-type *O* patients who benefit from incentivized exchange no longer compete for deceased-donor transplants. Blood types *O* and *B* are more common among minority groups, therefore any disadvantage to patients of these blood types leads to inequity in access for patients of minority ethnic backgrounds. As such, incentivized exchange also reduces disparities across ethnic groups. To our knowledge, our proposed policy is the first to enhance both the efficiency and equity of the system.[8]

To analyze the efficiency and equity implications of incentivized exchange, we introduce a new and analytically tractable dynamic large-market model of kidney transplantation.[9] Unlike previous models that focus on a single organ-allocation technology, our model can be used to analyze the impact of various technologies and policies that are often used together and that interact with each other. Through our model, we analytically show that, while all primary technologies increase overall access to kidney transplants, living-donor transplantation and kidney exchange reduce equity in access. In contrast, incentivized exchange increases both overall access and equity in access to transplants.

Literature Review.—Kidney exchange was originally proposed by Rapaport (1986) and later formulated and analyzed as a market-design problem by Roth, Sönmez, and Ünver (2004, 2005b, 2007). The idea of including compatible pairs in kidney exchange was initially evaluated by Ross and Woodle (2000) and further explored by Roth, Sönmez, and Ünver (2004, 2005a); Sönmez and Ünver (2014); and Nicolò and Rodríguez-Álvarez (2017) through a market-design lens. Although this idea was immediately dismissed by Ross and Woodle (2000) on ethical grounds, it has received wider acceptance in recent years (see, for example, Veatch 2006, Kranenburg et al. 2006, Gentry et al. 2007, Ratner et al. 2010, Steinberg 2011, Bingaman et al. 2012, and Ferrari et al. 2017). The proof of concept involving exchanges with compatible pairs is documented in Ratner et al. (2010). Moreover, Ratner et al. (2010), Kranenburg et al. (2006), and Hendren et al. (2015) report the results of surveys conducted among patients and living donors. They document

[8] The policy of reserving deceased-donor kidneys for same blood-type patients, called ABO-identical allocation policy, treats different blood types the same way. Therefore, it can be viewed as a procedurally or ex ante egalitarian policy. However, because of the interaction of the deceased-donor queue with the other transplantation technologies and because the donor-to-patient ratio for different blood types are not the same in practice, the waiting times for deceased-donor kidneys can vary across different blood types. Hence, this policy results in unequal waiting times. Our proposal reduces the difference between the longest and shortest waiting times for different blood type deceased-donor queues relative to the regular exchange (see online Appendix Table A-4). Therefore, the incentivized exchange may be better than the regular exchange for a social planner who exhibits ex post inequality aversion. See Grant et al. (2012) for a study of an ex post egalitarian social welfare function.

[9] While traditional matching models mostly focus on static, discrete settings, large-market and continuum models have become increasingly common over the last decade, especially in the context of market-design applications. These models include Kojima and Pathak (2009); Che and Kojima (2010); Lee (2017); Azevedo and Budish (2019); Azevedo and Leshno (2016); Kojima, Pathak, and Roth (2013); Liu and Pycia (2016); and Ashlagi and Roth (2014). See also Ünver (2010); Baccara, Lee, and Yariv (2016); Anderson et al. (2017); and Akbarpour, Li, and Gharan (2020) for dynamic matching models.

that patient and donor attitudes toward exchange are largely positive when the patient benefits from the exchange in some form. From a medical ethics perspective, Veatch (2006) and Steinberg (2011) also advocate for incentives. The literature also explores providing incentives through exchanging the donor of a compatible pair with a younger or genetically closer donor (see Roth, Sönmez, and Ünver 2004; Ferrari et al. 2017; and Nicolò and Rodríguez-Álvarez 2017). Bingaman et al. (2012) reports the implementation of this proposal by providing younger donors to patients of compatible pairs in a small sample. Such schemes have two drawbacks: they can only incentivize a limited number of compatible pairs, and they can also deter participation by extending waiting times. Our proposal is the first one that can globally and ex ante provide incentives to compatible pairs using tools that are already acceptable within the transplantation community.[10]

I. A Dynamic Model of Kidney Transplantation

Consider patients who need a kidney transplant, where each patient has a blood type $X \in \{O,A,B,AB\}$. Let $\pi_X > 0$ be the inflow rate of first-time blood-type X patients; that is $\pi_X dt$ is the measure of first-time blood-type X patients who arrive to the patient pool in a small time interval dt. Suppose that the expected lifetime while living with kidney disease is distributed with a continuous and strictly increasing distribution function $F(\cdot)$ on the interval $[0,T]$, and let $S(\cdot) = 1 - F(\cdot)$ denote the survival function on the same interval. Then the measure of blood-type X patients who are alive after t years is given by $\pi_X S(t)$. In the steady state of this model without transplantation, the total mass of blood-type X patients is $\int_0^T \pi_X S(t)\, dt$.

A. *Biological Barriers to Kidney Transplantation*

The best remedy for kidney failure is transplantation. There are two potential biological barriers to this procedure. A patient must be both blood-type and tissue-type compatible with a potential donor in order to receive his kidney. Blood-type O donors are blood-type compatible with patients of all four blood types, blood-type A donors are blood-type compatible with patients of blood types A and AB, blood-type B donors are blood-type compatible with patients of blood types B and AB, and blood-type AB donors are blood-type compatible with patients of blood type AB. Hence, other things being equal, blood-type O patients are at a disadvantage in finding a blood-type-compatible kidney donor. We denote blood-type compatibility through a "donation" relation $\triangleright$ over blood

[10] Indeed, our intertemporal insurance scheme found acceptance within the medical community after its associated NSF grant outline (Sönmez and Ünver 2014–2017) and paper draft (Sönmez and Ünver 2015) became publicly available in 2014 (https://www.nsf.gov/awardsearch/showAward?AWD_ID=1426440) and 2015 (https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2551344), respectively. Gill et al. (2017) makes a similar proposal to ours for incentivizing compatible pairs to participate in exchange. In practice, Veale et al. (2017) reports three uses of a variant of our proposal, leading to 25 transplants through chain exchanges. This scheme is utilized as follows. The old living donor of a younger patient, who likely will need a kidney transplant in the future, initiates a chain of exchanges in the present by donating her kidney to an incompatible pair. In return, the patient receives priority for a kidney at the end of a similar future chain when his kidney fails. The donor has a short donation window due to her old age, and the insurance scheme helps other pairs receive transplants through chain exchanges in the present, in addition to insuring the potential patient originally paired with the donor.

types, such that $X \rhd Y$ means that blood-type X donors are blood-type compatible with blood-type Y patients.

The second potential biological barrier to kidney transplantation is a tissue-type incompatibility. Transplantation is not possible if the patient has preformed antibodies against the donor DNA. To simplify the exposition in the main text, we assume that the probability of tissue-type incompatibility between a donor and a random patient is uniform at θ where $0 < \theta < 1$.[11] Hence, a patient can receive a kidney transplant from a blood-type-compatible donor with probability $(1 - \theta)$. The average θ for kidney deceased-donor queue arrivals in the United States, given in Table 1, is in the range 0.047–0.068 according to OPTN and SRTR (2009–2018) from the last several years.[12]

B. *Deceased-Donor Transplantation*

The most common source of transplant kidneys in the United States (and in much of the western world) is deceased donors. The United Network for Organ Sharing (UNOS) is the federal contractor in charge of allocating deceased-donor organs in the United States, and it uses a points system for kidneys. Since deceased-donor organs perish within a very short period, they are allocated as soon as they are harvested. The UNOS deceased-donor kidney-allocation system has two important features: (i) the waiting time in the queue is the most significant part of the points system, and (ii) kidneys are reserved for patients with the same blood type, with the exception of blood-type A kidneys which can also be allocated to blood-type AB patients.[13]

Reserving organs for same blood-type patients is referred to as an **ABO-identical** (**ABO-i**) allocation policy. Since blood type AB is relatively rare, ABO-i policy is a good approximation for the allocation of deceased-donor kidneys in the United States. Furthermore, given the strong influence of waiting time in the deceased-donor queue, we assume that deceased-donor kidneys are allocated with a **first-in-first-out** (**FIFO**) matching technology.[14]

Let δ_X be the inflow rate of blood-type X deceased-donor kidneys. There is a shortage of deceased-donor kidneys throughout the world, so we assume that $\delta_X < \pi_X$ for each blood type X. The median lifespan of a transplanted deceased-donor kidney is almost 12 years (Matas et al. 2015, conditional on one year survival). When a transplanted kidney eventually fails, the recipient reenters the patient pool as if he

[11] We relax this assumption in online Appendix Section F, allowing a non-uniform probability of tissue-type incompatibility between a donor and patients of different tissue types. This online Appendix also provides micro foundations for our results.

[12] This is according to the average calculated panel reactive antibody (CPRA) data of kidney deceased-donor-queue registrations. See online Appendix Section B for details of this calculation. CPRA measures the percentage of the US population against which the patient would have tissue-type incompatibility. The 2018 data point was added later in 2019 while the rest of the data was retrieved on October 30, 2018.

[13] Starting December 2014, deceased-donor kidneys that are in the highest-quality quintile are first offered to the top quintile of patients ranked according to long-term survival chances. A matching protocol similar to first-in-first-out is used to allocate these kidneys to their target group of patients and to allocate other kidneys to all patients. See also footnote 14.

[14] The use of FIFO in modeling deceased-donor kidney allocation only affects the calculation of waiting times in online Appendix Section C. As long as the steady state is well defined and no transplant kidney is wasted, none of our other results is affected by this assumption. Notably, service rates and the total numbers of transplants are unaffected. Thus, these predictions are valid for the current, modified UNOS deceased-donor kidney allocation scheme.

Table 1—Average of the Reported Tissue-Type Incompatibility Probabilities for New Entrants and Reentrants in the United States

Flow average tissue-type incompatibility probability	
Years	Probability
2009	0.047
2010	0.054
2011	0.056
2012	0.056
2013	0.054
2014	0.062
2015	0.064
2016	0.063
2017	0.068
2018	0.068

were a new patient. We assume that repeat patients' survival function is the same as the new entrants'. Let $\phi^{\mathbf{d}}$ be the fraction of the steady-state flow of previous recipients who reenter the patient pool because their transplant failed. Then $\phi^{\mathbf{d}}\boldsymbol{\delta}_X$ is the steady-state flow of blood-type X repeat patients. Therefore, the service rate of blood-type X deceased-donor-queue participants, defined as the supply-to-demand ratio at the deceased-donor queue, is[15]

$$s_X^{\mathbf{d},dec} = \frac{\boldsymbol{\delta}_X}{\boldsymbol{\pi}_X + \phi^{\mathbf{d}}\boldsymbol{\delta}_X}.$$

C. *Living-Donor Transplantation*

Living-donor transplantation is the second major source of transplant kidneys. In 2017, 29 percent of kidney transplants in the United States were from living donors. Let α_X be the fraction of blood-type X patients with a living donor. Living donors are assumed to become available for donation as soon as the patient enters the patient pool. Patients with living donors are referred to as *paired patients*, whereas patients without living donors are referred to as *unpaired patients*. In Section IB, we assumed that the inflow of patients is higher than the inflow of deceased-donor kidneys for each blood type. In the rest of the paper, we strengthen this assumption: the inflow of unpaired patients alone is higher than the inflow of deceased-donor kidneys for each blood type. This assumption easily holds in practice. We assume that each patient has at most one living donor, who is of blood type X with probability $p_X > 0$, and to simplify the analysis we also assume that

[15] We will use boldface superscripts next to policy variables to denote different transplantation regimes, in particular:

- **d** to denote only deceased-donor transplantation regime,
- **l** to denote deceased-donor and direct living-donor transplantation,
- **e** to denote deceased-donor and living-donor transplantation including regular (i.e., unincentivized) exchange, and
- **i** to denote deceased-donor and living-donor transplantation including both regular and incentivized exchange.

We will use superscript *dec* to denote variables related to patients who receive/wait for deceased-donor transplants and *liv* to denote variables related to patients who receive/wait for living-donor transplants directly or through exchange under the transplant regime in question.

blood types of the patient and his donor are uncorrelated. Then a blood-type X patient with a living donor is (both blood-type and tissue-type) compatible with his donor with probability $p_X^{\mathbf{l}}$, where

$$\begin{aligned} p_O^{\mathbf{l}} &= (1-\theta)p_O, \\ p_A^{\mathbf{l}} &= (1-\theta)(p_O+p_A), \\ p_B^{\mathbf{l}} &= (1-\theta)(p_O+p_B), \\ p_{AB}^{\mathbf{l}} &= (1-\theta)(p_O+p_A+p_B+p_{AB}) = (1-\theta). \end{aligned}$$

We assume that a patient with a compatible living donor receives the kidney as soon as he needs a transplant without ever entering the deceased-donor queue. Therefore, the service rate of paired blood-type X patients to receive a transplant, defined as the fraction of paired patients who can receive a living-donor transplant, is given as

$$s_X^{\mathbf{l},liv} = p_X^{\mathbf{l}},$$

and the flow of paired blood-type X patients who receive a transplant from their donors is given as

$$\boldsymbol{\lambda}_X = p_X^{\mathbf{l}}\alpha_X\boldsymbol{\pi}_X.$$

Although they last longer than deceased-donor transplants (with a median lifespan of almost 16 years conditional on one year survival, Matas et al. 2015), living-donor transplants can also fail. Let $\phi^{\mathbf{l}} \leq \phi^{\mathbf{d}}$ be the fraction of the flow of steady-state living-donor transplant recipients who reenter the patient pool because their transplants fail. We assume that reentrants no longer have a paired living donor.

For each blood type X, the availability of living donors decreases the flow of arrivals to the deceased-donor queue by $\boldsymbol{\lambda}_X$, but a fraction of that flow, $\phi^{\mathbf{l}}\boldsymbol{\lambda}_X$, reenter due to failure of living-donor transplants. Therefore, the net steady-state flow of patients entering the blood-type X deceased-donor queue is[16]

$$\boldsymbol{\pi}_X^{\mathbf{l}} = \boldsymbol{\pi}_X + \phi^{\mathbf{d}}\boldsymbol{\delta}_X + \phi^{\mathbf{l}}\boldsymbol{\lambda}_X - \boldsymbol{\lambda}_X = \boldsymbol{\pi}_X + \phi^{\mathbf{d}}\boldsymbol{\delta}_X - \left(1-\phi^{\mathbf{l}}\right)\boldsymbol{\lambda}_X,$$

and the service rate of blood-type X deceased-donor-queue participants is given as

$$s_X^{\mathbf{l},dec} = \frac{\boldsymbol{\delta}_X}{\boldsymbol{\pi}_X^{\mathbf{l}}} = \frac{\boldsymbol{\delta}_X}{\boldsymbol{\pi}_X + \phi^{\mathbf{d}}\boldsymbol{\delta}_X - \left(1-\phi^{\mathbf{l}}\right)\boldsymbol{\lambda}_X}.$$

Observe that, for each blood type X, the availability of living-donor transplantation reduces the steady-state flow of patients entering the deceased-donor queue

[16] Thus, the steady-state flow of patients entering the blood-type X deceased-donor queue takes into account not only the patients without living donors but also the patients with living donors who cannot receive living-donor transplants.

by $(1-\phi^{\mathbf{l}})\boldsymbol{\lambda}_X$. Hence, living-donor transplantation not only benefits the paired patients, but also the unpaired patients by increasing service rates for deceased-donor kidneys.

The total service rate for blood-type X patients is

$$s_X^{\mathbf{l}} = \frac{\boldsymbol{\delta}_X + \boldsymbol{\lambda}_X}{\boldsymbol{\pi}_X + \phi^{\mathbf{d}}\boldsymbol{\delta}_X + \phi^{\mathbf{l}}\boldsymbol{\lambda}_X}.$$

II. Kidney Exchange

While the availability of living-donor transplantation benefits all patient groups, not all willing living donors can donate to their intended recipient. Despite this difficulty, an increasing number of patients with incompatible living donors have been receiving kidney transplants through an exchange with other incompatible patient-donor pairs.

Formally, a two-way *kidney exchange* matches two "mutually compatible" patient-donor pairs: the patient of the first pair is compatible with the donor of the second pair, and the patient of the second pair is compatible with the donor of the first pair. Through an exchange of donors, both patients receive a kidney transplant. While patients with compatible donors can also participate in such exchanges, their participation so far has been very limited since they can directly receive a transplant from their own donors. In this section, we restrict our attention to kidney exchanges between incompatible pairs.

We consider a kidney-exchange program that operates in parallel with the deceased-donor allocation scheme. A patient with a compatible donor immediately receives a transplant from his donor without entering either the deceased-donor queue or the kidney-exchange pool. A patient with an incompatible donor, on the other hand, joins both the deceased-donor queue and the kidney-exchange pool. The patient accepts the first available kidney from either program.

We refer to a pair with a blood-type X patient and a blood-type Y donor as a type $X-Y$ pair. In real life, there are far fewer type $A-O$ pairs in kidney-exchange pools than their reciprocal type $O-A$ pairs. Pairs of the former type are blood-type compatible, so they do not need a kidney exchange unless they are tissue-type incompatible. This is a relatively rare event with small θ. Pairs of the latter type, on the other hand, are blood-type incompatible, and thus they must rely on kidney exchange for a living-donor transplant. This motivates the following assumption.

ASSUMPTION 1: *For any two distinct blood types X, Y with $X \rhd Y$, $\theta p_X \alpha_Y \boldsymbol{\pi}_Y \leq p_Y \alpha_X \boldsymbol{\pi}_X$.*

That is, the inflow of type $X-Y$ pairs (who always join the kidney-exchange pool) is at least as large as the inflow of type $Y-X$ pairs (who only join the kidney-exchange pool when they are tissue-type incompatible). Since θ is small, this assumption easily holds in practice.[17]

[17] Based on 2012–2014 data from the three largest kidney-exchange clearinghouses in the United States, the percentage of "underdemanded" $O-A$, $O-B$, $O-AB$, $A-AB$, and $B-AB$ pairs was in the range 41.9–43.1

To simplify the presentation of our analytical results, we also assume that the inflow of type $B - A$ pairs is at least as large as the inflow of type $A - B$ pairs. This assumption is superfluous and symmetric results hold if the inequality is reversed.

ASSUMPTION 2: $p_A \alpha_B \pi_B \geq p_B \alpha_A \pi_A$.

Since there are fewer type $A - O$ pairs in the pool than their reciprocal type $O - A$ pairs (by Assumption 1), it is possible to match every $A - O$ pair as soon as they arrive. While the patient of an arriving type $A - O$ pair is tissue-type incompatible with a θ fraction of the donors of type $O - A$ pairs in the pool, he is compatible with a much larger fraction $(1 - \theta)$, and mutually compatible with a fraction $(1 - \theta)^2$. Similarly, for any two distinct blood types X, Y with $X \rhd Y$, it is possible to match every type $Y - X$ pair as soon as they arrive. This is also the case for any type $A - B$ pair by Assumption 2. It turns out that this simple observation forms the basis for an optimal exchange mechanism.

THEOREM 1 (ABO-Identical Exchange Is Optimal): *Suppose Assumptions 1 and 2 hold. Then an exchange policy where an arriving incompatible pair is immediately matched with a mutually compatible pair of its reciprocal type maximizes the measure of transplants to pairs arriving at that moment. Moreover, any such policy maximizes the mass of pairs arriving in an interval that can be matched within that interval.*

Observe that the optimal exchange described in Theorem 1 can accommodate FIFO matching where, whenever possible, an arriving type $X - Y$ pair is matched with the longest waiting mutually compatible pair of its reciprocal type $Y - X$. This is the kidney-exchange mechanism we consider in our theoretical analysis.

The following grouping of transplants is helpful in explaining the effect of exchange on paired patients and highlighting the welfare loss that results from excluding compatible pairs from exchange:

(i) For each blood type X, transplants due to tissue-type incompatible pairs of type $X - X$: while these patients are blood-type compatible with their donors, they are tissue-type incompatible. Kidney exchange renders tissue-type incompatibility immaterial for them, since each one can be matched with a mutually compatible pair of identical type as soon as they join the kidney-exchange pool. The resulting net increase in the flow of transplants at steady state is $\theta p_X \alpha_X \pi_X$ for each blood type X.

(ii) For each pair of distinct blood types X, Y with $X \rhd Y$, transplants due to tissue-type-incompatible pairs of type $Y - X$: tissue-type incompatibility becomes immaterial for these patients as well, since they too can be matched with a mutually compatible pair as soon as they join the kidney-exchange pool. The resulting net increase in the flow of transplants at steady state is

and the percentage of "overdemanded" $A - O$, $B - O$, $AB - O$, $AB - A$, and $AB - B$ pairs was in the range 14–15.2 (Agarwal et al. 2019).

$2\theta p_X \alpha_Y \pi_Y$, since each tissue-type-incompatible pair of type $Y - X$ facilitates a transplant for a patient of its (blood-type-incompatible) reciprocal type $X - Y$.

(ii) Transplants due to pairs of types $A - B$ and $B - A$: for patients of type $A - B$ (which has a lower inflow than type $B - A$ by Assumption 2), both blood-type and tissue-type incompatibility become immaterial; they can all be immediately matched with a pair of type $B - A$. The resulting net increase in the flow of transplants at steady state is $2p_B \alpha_A \pi_A$, since each pair of type $A - B$ also facilitates a transplant for a patient of type $B - A$.

Intuitively, kidney exchange eliminates tissue-type incompatibility as a barrier to living-donor transplantation, and, in doing so, it facilitates an additional transplant to a patient with a blood-type-incompatible donor. Furthermore, it also facilitates transplants to all pairs of type $A - B$, and as many transplants to pairs of type $B - A$. For pairs with blood-type O or AB patients, kidney exchange is directly tied to tissue-type incompatibility. Pairs with blood-type AB patients in the kidney-exchange pool join the pool only because they are tissue-type incompatible with their own donors. Pairs with blood-type O patients in the pool, on the other hand, can only receive a transplant if a mutually compatible pair of their reciprocal type becomes available for exchange through a tissue-type incompatibility. As a result, the effect of kidney exchange on patient groups of blood types O and AB is modest compared to its effect on patient groups of blood types A and B.[18] Indeed, in the absence of tissue-type incompatibility (i.e., for $\theta = 0$), the effect of kidney exchange would be exclusively limited to patients of blood types A and B.

Let ϵ_X denote the steady-state flow of blood-type X patients who receive a living-donor transplant through kidney exchange. For blood type O and any blood type Y, the flow of tissue-type-incompatible type $Y - O$ pairs is $\theta p_O \alpha_Y \pi_Y$. Therefore, a flow $\theta p_O \alpha_Y \pi_Y$ of type $O - Y$ pairs are matched with type $Y - O$ pairs, and

$$\epsilon_O = \theta p_O(\alpha_O \pi_O + \alpha_A \pi_A + \alpha_B \pi_B + \alpha_{AB} \pi_{AB}).$$

Similarly,

$$\epsilon_A = \theta p_A(\alpha_A \pi_A + \alpha_{AB} \pi_{AB}) + \theta p_O \alpha_A \pi_A + p_B \alpha_A \pi_A,$$

$$\epsilon_B = \theta p_B(\alpha_B \pi_B + \alpha_{AB} \pi_{AB}) + \theta p_O \alpha_B \pi_B + p_B \alpha_A \pi_A,$$

and

$$\epsilon_{AB} = \theta(p_{AB} + p_A + p_B + p_O)\alpha_{AB} \pi_{AB} = \theta \alpha_{AB} \pi_{AB}.$$

[18] See Theorem A-2 in online Appendix Section A for a comparative result regarding the effect of kidney exchange across blood types, formalizing this observation for a homogeneous population.

Since the availability of kidney exchange increases the steady-state flow of living-donor transplants by $\boldsymbol{\epsilon}_X$ for any blood type X, the service rate of paired blood-type X patients receiving a living-donor transplant increases by $\boldsymbol{\epsilon}_X/(\alpha_X \boldsymbol{\pi}_X)$ to

$$s_X^{\mathbf{e},liv} = \frac{\boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X}{\alpha_X \boldsymbol{\pi}_X}.$$

We can summarize the effect of kidney exchange on pairs with living donors as follows:

(i) Type $A-B$ and each type $X-Y$ with $Y \rhd X$: each patient of these pairs either immediately receives a transplant from his own donor, or immediately receives a transplant through kidney exchange. In either case, they do not wait in the deceased-donor queue or exchange pool.

(ii) Type $B-A$ and each type $X-Y$ with $X \neq Y$ and $X \rhd Y$: patients of these pairs join both the kidney-exchange pool and the deceased-donor queue. They all wait for a transplant and some do not survive.

(a) For any of these types $X-Y$, if the wait in the kidney-exchange pool is less than the wait in the blood-type X deceased-donor queue, then all surviving pairs of type $X-Y$ receive a transplant through exchange, while none of them receives a transplant from the deceased-donor queue.

(b) Since all patients of type $X-Y$ receive the first available kidney, the wait in the kidney-exchange pool cannot be more than the wait in the blood-type X deceased-donor queue. If the wait for the kidney-exchange pool $X-Y$ is the same as the blood-type X deceased-donor queue, then patients of type $X-Y$ pool with unpaired patients of blood-type X. Among those who survive, some receive a transplant through exchange and the rest receive a transplant from the deceased-donor queue.

For each blood type X, the availability of kidney exchange along with living-donor transplantation decreases the flow of patients who utilize the deceased-donor queue by $\boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X$; however, a fraction of that flow, $\phi^{\mathbf{l}}(\boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X)$, reenter the patient pool due to the failure of living-donor transplants and they exclusively join the deceased-donor queue. Therefore, the net steady-state flow of patients entering the blood-type X deceased-donor queue is

$$\begin{aligned}\boldsymbol{\pi}_X^{\mathbf{e}} &= \boldsymbol{\pi}_X + \phi^{\mathbf{d}} \boldsymbol{\delta}_X + \phi^{\mathbf{l}}(\boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X) - (\boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X) \\ &= \boldsymbol{\pi}_X + \phi^{\mathbf{d}} \boldsymbol{\delta}_X - (1 - \phi^{\mathbf{l}})(\boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X),\end{aligned}$$

and the service rate of blood-type X deceased-donor-queue participants increases to

$$s_X^{\mathbf{e},dec} = \frac{\boldsymbol{\delta}_X}{\boldsymbol{\pi}_X^{\mathbf{e}}} = \frac{\boldsymbol{\delta}_X}{\boldsymbol{\pi}_X + \phi^{\mathbf{d}}\boldsymbol{\delta}_X - \left(1 - \phi^{\mathbf{l}}\right)\left(\boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X\right)}.$$

The total service rate for blood-type X patients is

$$s_X^{\mathbf{e}} = \frac{\boldsymbol{\delta}_X + \boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X}{\boldsymbol{\pi}_X + \phi^{\mathbf{d}}\boldsymbol{\delta}_X + \phi^{\mathbf{l}}\left(\boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X\right)}.$$

III. A New Proposal: Incentivized Exchange

In Section II, we have seen that kidney exchange increases transplants from living donors. However, real-life applications of kidney exchange are almost exclusively utilized by incompatible pairs, limiting its welfare gains. To see how excluding compatible pairs limits the gains from exchange, it is helpful to focus on the grouping in Section II.

The critical exchanges are those in group 2: for any distinct blood types X, Y with $X \rhd Y$, a tissue-type-incompatible type $Y - X$ pair exchanges its donor with the donor of a type $X - Y$ pair. To simplify the discussion, let $X = O$ and $Y = A$. Through this exchange, the patient of the tissue-type-incompatible $A - O$ pair immediately receives a transplant from the donor of the type $O - A$ pair. Hence, with kidney exchange, whether a type $A - O$ pair is tissue-type incompatible or not does not affect when or if its patient receives a transplant. More importantly, this exchange also benefits a type $A - O$ pair. In a way, kidney exchange transforms the "misfortune" of the tissue-type-incompatible type $A - O$ pair to a life-saving opportunity for the type $O - A$ pair. Since the type $A - O$ pair is blood-type compatible, they would not have participated in exchange if they were tissue-type compatible. Kidney exchange not only eliminated tissue-type incompatibility as an obstacle for the transplantation, but it also facilitated a transplant for an additional patient. Put differently, from a social-welfare point of view, there is a welfare loss when a blood-type A patient receives a transplant from a blood-type O donor. When exchange is possible, tissue-type incompatibility avoids this welfare loss and more efficiently utilizes living donor kidneys. But why depend on a relatively rare tissue-type incompatibility to avoid this kind of welfare loss? Any pair of type $A - O$, whether they are tissue-type incompatible or not, can participate in kidney exchange, facilitating a transplant for an additional patient. The challenge here is that a tissue-type-compatible pair of type $A - O$ has no reason to participate in exchange.

As our main contribution, we propose incentivizing compatible pairs to participate in exchange by giving the patient "insurance" against future transplant failure. The insurance takes the form of prioritizing the patient in the deceased-donor queue in the event of a repeat kidney failure.[19] To incentivize their participation in kidney

[19] While UNOS is the sole federal contractor in charge of allocating deceased-donor organs in the United States, it also administers one of the three main kidney-exchange platforms. There are also several much smaller (typically single-center) systems. If, in the future, UNOS implements our proposed incentivized exchange, it could either offer the incentives exclusively to compatible pairs that register with the UNOS-administered kidney-exchange system, or to any pair that joins kidney exchange regardless of where they register. The first option has the advantage that it

exchange, these *prioritized reentrants* are placed at the top of the deceased-donor queue altering its FIFO structure.[20] Since in our theoretical dynamic large market model welfare gains only occur with the inclusion of tissue-type-compatible pairs of any type $Y - X$ such that $Y \neq X$ and $X \rhd Y$ to exchange, we use the incentive scheme only for these pairs. For each such pair of type $Y - X$, let ρ_{Y-X} be the fraction of compatible pairs who are willing to take up the *incentivized-exchange* option. In online Appendix Section D, we use simulations to show that incentivizing compatible pairs of types $A - A$, $B - B$, $O - O$, and $AB - AB$ can also provide nonnegligible welfare gains in environments with finite arrivals. Hence, for real-life implementation, we propose providing the incentivized-exchange option to all compatible pairs.

In the absence of incentivized exchange, there is an abundance of type $O - A$ pairs compared to type $A - O$ pairs. For high values of ρ_{A-O}, this may change with incentivized exchange. We assume that compatible pairs only take the incentivized-exchange option if they can immediately participate in exchange, ensuring that type $A - O$ remains "overdemanded."

ASSUMPTION 3: *For any two distinct blood types X, Y with* $X \rhd Y$,

$$\left[\rho_{Y-X}(1 - \theta) + \theta\right] p_X \alpha_Y \pi_Y \leq p_Y \alpha_X \pi_X.$$

Due to differences in the estimated value of α across different blood types, this assumption holds even when $\rho = 1$. (See Table 2 for an estimate of parameter α for each blood type.) For any two distinct blood types X, Y with $X \rhd Y$ (and as in the case of kidney exchange), Assumption 3 ensures that it is possible to match every pair of type $Y - X$ at steady state as soon as they arrive. Moreover, replacing Assumption 1 with Assumption 3 ensures that the optimality result in Theorem 1 continues to hold under incentivized exchange. Hence, we proceed with an optimal exchange mechanism where an arriving type $X - Y$ pair, whenever possible, is matched with the longest-waiting mutually compatible pair of its reciprocal type $Y - X$.

Since incentivized exchange simply increases the scope of kidney exchange, the analysis in this section parallels the analysis in Section II. Recall that in our theoretical analysis the target group for incentivized exchange is tissue-type-compatible pairs of types $A - O$, $B - O$, $AB - O$, $AB - A$, and $AB - B$. Consider such a pair that takes the incentivized-exchange option. The patient of this pair could have received a transplant from his own donor, and, hence, his own transplant does not directly increase the total number of transplants. The increase is due to the patient of the blood-type-incompatible reciprocal type pair, with whom the compatible pair

could encourage the growth of the UNOS system, which may reduce the inefficiencies that result from several small kidney-exchange platforms. (See Section 8 in Sönmez and Ünver 2015 for a formal result.) The second option, on the other hand, may result in higher participation in incentivized exchange. In our paper we abstract away from this issue and focus on a single kidney-exchange program.

[20] This incentive can be provided in other ways as well, such as by giving additional points to the patient of the incentivized pair (rather than absolute priority) or even by giving them in-kind incentives. How compatible pairs are incentivized is immaterial to the analysis in the main text, although the analysis on the waiting times in online Appendix Section A.2 relies on prioritizing them in the deceased-donor queue.

exchanges. Therefore, at steady state, one more transplant occurs for each compatible pair that takes the incentivized-exchange option.

Let $Y - X$ be any type targeted for incentivized exchange. The flow of all $Y - X$ pairs is $p_X \alpha_Y \boldsymbol{\pi}_Y$, the flow of tissue-type-compatible $Y - X$ pairs is $(1 - \theta) p_X \alpha_Y \boldsymbol{\pi}_Y$, and the flow of $Y - X$ pairs who take the incentivized-exchange option is $\rho_{Y-X}(1 - \theta) p_X \alpha_Y \boldsymbol{\pi}_Y$.

For each blood type X, let $\boldsymbol{\iota}_X$ denote the steady-state flow of the contribution of incentivized exchange on blood-type X living-donor transplants. Each blood-type AB patient with a living donor already receives a living-donor transplant once kidney exchange becomes available, so living-donor transplants to blood-type AB patients do not change with the introduction of incentivized exchange. Therefore,

$$\boldsymbol{\iota}_{AB} = 0.$$

In contrast, patients of the following five types will benefit from incentivized exchange through increased living-donor transplantation: $A - AB$, $B - AB$, $O - A$, $O - B$, and $O - AB$. Living-donor transplants to blood-type A patients with blood-type AB donors increase due to incentivized pairs of type $AB - A$; living-donor transplants to type B patients with AB donors increase due to incentivized pairs of type $AB - B$; and living-donor transplants to blood-type O patients with blood-type-incompatible donors increase due to incentivized pairs of types $A - O$, $B - O$, and $AB - O$. Therefore,

$$\boldsymbol{\iota}_A = \rho_{AB-A}(1 - \theta) p_A \alpha_{AB} \boldsymbol{\pi}_{AB},$$

$$\boldsymbol{\iota}_B = \rho_{AB-B}(1 - \theta) p_B \alpha_{AB} \boldsymbol{\pi}_{AB},$$

and

$$\boldsymbol{\iota}_O = \rho_{A-O}(1 - \theta) p_O \alpha_A \boldsymbol{\pi}_A + \rho_{B-O}(1 - \theta) p_O \alpha_B \boldsymbol{\pi}_B + \rho_{AB-O}(1 - \theta) p_O \alpha_{AB} \boldsymbol{\pi}_{AB}.$$

Since the availability of incentivized exchange weakly increases the steady-state flow of living-donor transplants by $\boldsymbol{\iota}_X$ for any blood type X, the service rate of paired blood-type X patients to receive a living-donor transplant weakly increases by $\boldsymbol{\iota}_X / (\alpha_X \boldsymbol{\pi}_X)$ to

$$s_X^{\mathbf{i},liv} = \frac{\boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X + \boldsymbol{\iota}_X}{\alpha_X \boldsymbol{\pi}_X}.$$

Observe that the service rate of living-donor transplants strictly increases for blood types A, B, and O, and it remains at the maximum rate of one for blood type AB. Moreover, since blood type AB is rare, the flow of arriving pairs is modest for types $AB - A$, $AB - B$, and $AB - O$. Therefore, most of the incentivized pairs are of types $A - O$ or $B - O$, so the primary beneficiaries of incentivized exchange are paired patients of blood type O.

For each blood type X, the availability of incentivized exchange along with living-donor transplantation and kidney exchange decreases the flow of patients waiting in the deceased-donor queue by $\boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X + \boldsymbol{\iota}_X$; however, a fraction of that

flow, $\phi^{\mathbf{l}}(\boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X + \boldsymbol{\iota}_X)$, reenter the patient pool due to the failure of living-donor transplants. Therefore, the net steady-state flow of patients entering the blood-type X deceased-donor queue is

$$\begin{aligned}\boldsymbol{\pi}^{\mathbf{i}}_X &= \boldsymbol{\pi}_X + \phi^{\mathbf{d}}\boldsymbol{\delta}_X + \phi^{\mathbf{l}}(\boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X + \boldsymbol{\iota}_X) - (\boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X + \boldsymbol{\iota}_X)\\ &= \boldsymbol{\pi}_X + \phi^{\mathbf{d}}\boldsymbol{\delta}_X - (1 - \phi^{\mathbf{l}})(\boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X + \boldsymbol{\iota}_X),\end{aligned}$$

and the service rate of blood-type X deceased-donor-queue participants weakly increases to

$$s^{\mathbf{i},dec}_X = \frac{\boldsymbol{\delta}_X}{\boldsymbol{\pi}^{\mathbf{i}}_X} = \frac{\boldsymbol{\delta}_X}{\boldsymbol{\pi}_X + \phi^{\mathbf{d}}\boldsymbol{\delta}_X - (1 - \phi^{\mathbf{l}})(\boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X + \boldsymbol{\iota}_X)}.$$

Then the total service rate for blood-type X patients is

$$s^{\mathbf{i}}_X = \frac{\boldsymbol{\delta}_X + \boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X\boldsymbol{\iota}_X}{\boldsymbol{\pi}_X + \phi^{\mathbf{d}}\boldsymbol{\delta}_X + \phi^{\mathbf{l}}(\boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X + \boldsymbol{\iota}_X)}.$$

IV. Numerical Model Predictions

In this section, we inspect the predictions of our model by calibrating it with the US patient and donor characteristics. We estimate the proportion of each group served and the number of transplants under various transplantation regimes, including current policy as well as our proposed incentivized exchange.[21] We also run simulations with discrete arrivals using the US population characteristics with either two-way or two-and-three-way exchange technologies. These simulations give us comparable results to the numerical predictions and serve as a robustness check for our theoretical analysis (see Section IVB for more on this issue). The simulations are reported in online Appendix Section D.

We report the calibration parameters for our model in Table 2. We explain in online Appendix Sections B, C, and D how we obtain these parameters. The second row of Table 2, *De facto deceased-donor flows* ($\boldsymbol{\delta}'_X$), requires some further explanation. Deceased-donation regulations in the United States explicitly dictate that blood-type *O* and *B* deceased-donor kidneys are to be transplanted to their respective blood-type patients. However, blood-type *O* kidneys are occasionally transplanted to blood-type *B* patients and less frequently to patients of other blood types (see also Subsection IVA). Moreover, blood-type *AB* patients occasionally receive kidneys from other blood types. For these reasons, in addition to the strict ABO-i allocation policy, we calculate our model's predictions as if deceased donors arrived according to this observed transplantation allocation across blood types. This is what we refer

[21] In online Appendix Section A, we conduct a theoretical analysis of our model and find waiting times for different patient groups. We also estimate the welfare consequences for different policy proposals if living-donation and deceased-donation rates across different blood groups are homogeneous in a population (see Theorems A-2 and A-3) using service rates. Using the calibrated parameters in this section, we also calculate predicted waiting times in online Appendix Section C.

TABLE 2

	Benchmark calibration parameters			
	O	*A*	*B*	*AB*
ABO-i deceased-donor flows $(\boldsymbol{\delta}_X)$	5,589	4,343	1,386	395
De facto deceased-donor flows $(\boldsymbol{\delta}'_X)$	5,357	4,188	1,548	621
New patient flows $(\boldsymbol{\pi}_X)$	15,241	10,218	4,626	1,176
Paired-donor blood-type probability (p_X)	0.456	0.378	0.126	0.040
Paired-donor fractions (α_X)	32.84%	24.90%	26.57%	18.93%
Tissue-type incompatibility probability θ		0.0679		
Reentry fraction of the recipients $\phi^{\mathbf{l}} = \phi^{\mathbf{d}}$		24.10%		
Incentivized-exchange participation fraction (ρ)		10%, 20%, 30%, 50%, 100%		

Notes: Benchmark calibration parameters for the numerical policy experiments; time unit is one year. Benchmark survival function $S(t)$ is given in online Appendix Section C.

to as the de facto deceased-donor flow for each blood type X, denoted as $\boldsymbol{\delta}'_X$. We conduct all of our analyses using both ABO-i and de facto deceased-donor flows.

We calculate our model's steady state using these calibration parameters and report outcome variables, such as deceased-donation recipient flows and living-donation recipient flows, $\boldsymbol{\lambda}_X, \boldsymbol{\epsilon}_X, \boldsymbol{\iota}_X$, for different transplantation regimes (see Table 3). We also find the service rate of paired blood-type X patients receiving a living-donor transplant (see Table 4), the service rate of blood-type X deceased-donor-queue participants (see Table 5), and the overall service rate of blood-type X patients receiving either kind of transplant (see Table 6). The overall service rate of blood-type X patients determines what percentage of the patient population receives either kind of transplant and is the ratio of the flow of all transplants to the inflow of all patients, new and reentering, for each blood type X (see the Table 6 notes for a formal definition). We also pool service rates among all blood types for all patients in this table according to participation in living donation, deceased-donor queue, and either kind of transplant.

A. *Welfare Consequences*

In terms of overall impact, 37.5 percent of patients receive deceased-donor transplants (measured as a fraction of new entrants, $\boldsymbol{\pi}_X$; see the last column in the *Total Transplants* panel of Table 3). An additional 15.9 percent receive direct living-donor transplants. An additional 3.6 percent of patients benefit from regular exchange, resulting in 1,135 more transplants annually. Our policy proposal, incentivized exchange, helps provide transplants for an additional 0.6 percent of patients (or about 180 additional patients) for each 10 percentage point increase in participation of eligible, compatible pairs annually.[22]

We also consider how each blood type is affected by the introduction of different transplantation regimes. Blood-type B patients are at a disadvantage even when only deceased-donor transplantation is available; they have the lowest service rate

[22] We can test the external validity of our model by looking at overall service rates. For example, our model predicts that when regular exchange is available only 50.1 percent of all patients, new arrivals and reentrants, will be served (see the last column of Table 6). This rate is 47.3 percent when only deceased-donor and direct living-donor transplantation is available. Indeed, Hart et al. (2018) reports that less than 50 percent of the patients of cohort entering since 2005 has received a transplant.

TABLE 3—NUMERICAL PREDICTIONS OF THE MODEL: PATIENTS RECEIVING TRANSPLANT

	O		*A*		*B*		*AB*		All	
	Number	Percent	Number	Percent	Number	Percent	Number	Percent	Number	Percent
Living-donor transplants										
Direct (**l**)	2,127	14.0	1,978	19.4	667	14.4	208	17.6	4,979	15.9
Direct and exchange (**e**)	2,406	15.8	2,448	24.0	1,038	22.4	223	18.9	6,115	19.6
Direct and $\rho = 10\%$	2,576	16.9	2,456	24.0	1,040	22.5	223	18.9	6,295	20.1
Regular $\rho = 20\%$	2,746	18.0	2,464	24.1	1,043	22.5	223	18.9	6,475	20.7
Exchange and $\rho = 30\%$	2,916	19.1	2,472	24.2	1,045	22.6	223	18.9	6,655	21.3
Incentivized $\rho = 50\%$	3,255	21.4	2,487	24.3	1,051	22.7	223	18.9	7,016	22.4
Exchange (**i**) $\rho = 100\%$	4,105	26.9	2,527	24.7	1,064	23.0	223	18.9	7,918	25.3
Total transplants										
Deceased-donor transplantation only (**d**)										
ABO-i	5,589	36.7	4,343	42.5	1,386	30.0	395	33.6	11,713	37.5
de facto	5,357	35.2	4,188	41.0	1,548	33.5	621	52.8	11,714	37.5
Deceased-/direct living-donor transplantation (**l**)										
ABO-i	7,716	50.6	6,321	61.9	2,053	44.4	603	51.2	16,693	53.4
de facto	7,484	49.1	6,166	60.3	2,215	47.9	828	70.4	16,694	53.4
Deceased-/direct living-donor transplantation and regular exchange (**e**)										
ABO-i	7,995	52.5	6,791	66.5	2,424	52.4	618	52.5	17,828	57.0
de facto	7,763	52.5	6,636	66.5	2,586	52.4	844	52.5	17,829	57.0
Deceased-/direct living-donor trans., regular and $\rho = 10\%$-incentivized exchange (**i**, $\rho = 10\%$)										
ABO-i	8,165	53.6	6,799	66.5	2,427	52.5	618	52.5	18,008	57.6
de facto	7,933	53.6	6,644	66.5	2,588	52.5	844	52.5	18,009	57.6
Deceased-/direct living-donor trans., regular and $\rho = 20\%$-incentivized exchange (**i**, $\rho = 20\%$)										
ABO-i	8,335	54.7	6,806	66.6	2,429	52.5	618	52.5	18,188	58.2
de facto	8,103	54.7	6,652	66.6	2,591	52.5	844	52.5	18,189	58.2
Deceased-/direct living-donor trans., regular and $\rho = 30\%$-incentivized exchange (**i**, $\rho = 30\%$)										
ABO-i	8,505	55.8	6,814	66.7	2,432	52.6	618	52.5	18,369	58.8
de facto	8,273	55.8	6,660	66.7	2,594	52.6	844	52.5	18,370	58.8
Deceased-/direct living-donor trans., regular and $\rho = 50\%$-incentivized exchange (**i**, $\rho = 50\%$)										
ABO-i	8,844	58.0	6,830	66.8	2,437	52.7	618	52.5	18,729	59.9
de facto	8,613	58.0	6,675	66.8	2,599	52.7	844	52.5	18,730	59.9
Deceased-/direct living-donor trans., regular and $\rho = 100\%$-incentivized exchange (**i**, $\rho = 100\%$)										
ABO-i	9,694	63.6	6,869	67.2	2,450	53.0	618	52.5	19,631	62.8
de facto	9,462	63.6	6,715	67.2	2,612	53.0	844	52.5	19,632	62.8

Notes: Numerical predictions of the model for the flow of patients receiving transplant (measured in numbers per year) for different patient blood types. The *Percentage* columns to the right of each *Number* column is the total transplant rate with respect to the new patient flow (π_X), i.e., $\#/\pi_X$.

of deceased-donor-queue participants (see Table 5). Blood type *B* is at least twice more common among Asian and African minorities of the US population than among Americans of European descent (see online Appendix Table A-1). African Americans are known to be relatively more prone to kidney disease, while the blood-type *B* deceased-donation rate is not much different from that of other blood types. This explains the lower service rates for Asian and African minorities. Thus, the treatment of blood type *B* under our proposed policies, as well as blood-type *O* patients, bears additional importance in equity considerations.

We summarize our main findings regarding the consequences of different transplantation regimes on service rates under the de facto deceased-donor allocation policy.

- The largest benefits from deceased-donor and direct living-donor transplantation go to blood-type *AB* patients, with blood-type *A* patients, blood-type

Table 4—Numerical Predictions of the Model: Service Rate for Paired Patients to Receive Living-Donor Transplants (Percent)

	O	*A*	*B*	*AB*	All
Deceased-donor transactions only (**d**)	0	0	0	0	0
Deceased/direct living (**l**)	42.5	77.7	54.2	93.2	55.3
Deceased/direct living and exchange (**e**)	48.1	96.2	84.4	100.0	67.9
All plus $\rho = 10\%$-incentivized exchange (**i**)	51.5	96.5	84.6	100.0	69.9
All plus $\rho = 20\%$-incentivized exchange (**i**)	54.9	96.8	84.8	100.0	71.9
All plus $\rho = 30\%$-incentivized exchange (**i**)	58.3	97.1	85.0	100.0	73.9
All plus $\rho = 50\%$-incentivized exchange (**i**)	65.0	97.8	85.5	100.0	77.9
All plus $\rho = 100\%$-incentivized exchange (**i**)	82.0	99.3	86.5	100.0	88.0

Notes: Numerical predictions of the model for each regime $\mathbf{t} \in \{\mathbf{d}, \mathbf{l}, \mathbf{e}, \mathbf{i}\}$ service rate of paired patients to receive living-donor transplants ($s_X^{\mathbf{t},liv} = (\boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X + \boldsymbol{\iota}_X)/(\alpha_X \boldsymbol{\pi}_X)$ for each blood type X and $s^{\mathbf{t},liv} = \sum_X (\boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X + \boldsymbol{\iota}_X)/\sum_X (\alpha_X \boldsymbol{\pi}_X)$ for total) where $\boldsymbol{\lambda}_X = \boldsymbol{\epsilon}_X = \boldsymbol{\iota}_X = 0$ in regime **d** (deceased-donor transplantation only), $\boldsymbol{\epsilon}_X = \boldsymbol{\iota}_X = 0$ in regime **l** (deceased-donor/direct living-donor transplantation), and $\boldsymbol{\iota}_X = 0$ in regime **e** (deceased-donor/direct living-donor transplantation and regular exchange). Every rate is reported in percents.

O patients, and blood-type *B* patients receiving successively smaller welfare gains. While the overall service rate is 60.2 percent for blood-type *AB* patients and 52.7 percent for blood-type *A* patients, the overall service rate from these two modalities falls to less than 44 percent for blood-type *O* patients and to less than 43 percent for blood-type *B* patients (see Table 6).

- At the margin, blood-type *B* patients benefit the most from regular exchange. Blood-type *A* patients benefit second most, while blood-type *O* and blood-type *AB* patients benefit the least. An additional 6.4 percent of all blood-type *B* patients receive a kidney due to regular exchange, taking into consideration the increase in reentries to the patient pool caused by the additional steady-state living-donor transplants. The corresponding service-rate increase for blood type *A* is 3.5 percent, and 1.5 and 0.9 respectively for blood-type *O* and blood-type *AB* patients. The widest service-rate gap, the gap between the service rates of blood-type *O* and *AB* patients as a result of deceased-donor/direct living-donor transplantation and regular exchange, is 15.7 percent.
- Blood type *O* patients are the main beneficiaries of incentivized exchange. For each $\Delta\rho = 10$ percent participation increase, incentivized exchange provides kidney to an additional 0.8–0.9 percent of all blood-type *O* patients. The overall service rates are unaffected for blood-type *AB*, and the increase is modest for blood types *A* and *B*. Thus, the widest service-rate gap, the gap between those of blood-type *AB* and *O* patients, decreases by 0.8–0.9 percent for each $\Delta\rho = 10$ percent increase.
- Service rates for deceased-donor-queue participants slightly increase with the introduction of new exchange technologies overall and across blood types. Although additional transplants under these technologies cause an increased number of patients to reenter the patient pool at steady state, an even higher number of additional paired patients receive living-donor transplants and drop out of competition for deceased-donor transplants. The service rate of deceased-donor-queue participants increases from 38.7 percent under deceased-donor/direct living-donor transplantation to 39.8 percent with the

TABLE 5—NUMERICAL PREDICTIONS OF THE MODEL: SERVICE RATE FOR DECEASED-DONOR-QUEUE PARTICIPANTS (PERCENT)

	O	*A*	*B*	*AB*	All
Deceased-donor trans. only (**d**)					
ABO-i	33.7	38.5	28.0	31.1	34.4
de facto	32.4	37.3	31.0	46.8	34.4
Deceased/direct living (**l**)					
ABO-i	37.3	44.5	31.1	35.5	38.7
de facto	35.9	43.1	34.5	53.1	38.7
Deceased/direct living and exchange (**e**)					
ABO-i	37.9	46.2	33.2	35.8	39.8
de facto	36.4	44.7	36.8	53.7	39.8
All plus $\rho = 10\%$-incentivized exchange (**i**)					
ABO-i	38.2	46.2	33.2	35.8	40.0
de facto	36.8	44.7	36.8	53.7	40.0
All plus $\rho = 20\%$-incentivized exchange (**i**)					
ABO-i	38.5	46.2	33.3	35.8	40.2
de facto	37.1	44.8	36.8	53.7	40.2
All plus $\rho = 30\%$-incentivized exchange (**i**)					
ABO-i	38.9	46.3	33.3	35.8	40.3
de facto	37.4	44.8	36.8	53.7	40.3
All plus $\rho = 50\%$-incentivized exchange (**i**)					
ABO-i	39.6	46.3	33.3	35.8	40.7
de facto	38.1	44.8	36.8	53.7	40.7
All plus $\rho = 100\%$-incentivized exchange (**i**)					
ABO-i	41.5	46.5	33.4	35.8	41.7
de facto	39.9	45.0	36.9	53.7	41.7

Notes: Numerical predictions of the model for each regime $\mathbf{t} \in \{\mathbf{d}, \mathbf{l}, \mathbf{e}, \mathbf{i}\}$ service rate of deceased-donor-queue participants $(s_X^{\mathbf{t},dec} = \boldsymbol{\delta}_X / (\boldsymbol{\pi}_X + \phi^{\mathbf{d}} \boldsymbol{\delta}_X - (1 - \phi^{\mathbf{l}})(\boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X + \boldsymbol{\iota}_X))$ for each X and $s^{\mathbf{t},dec} = \sum_X \boldsymbol{\delta}_X / \sum_X (\boldsymbol{\pi}_X + \phi^{\mathbf{d}} \boldsymbol{\delta}_X - (1 - \phi^{\mathbf{l}})(\boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X + \boldsymbol{\iota}_X))$ for total) where $\boldsymbol{\lambda}_X = \boldsymbol{\epsilon}_X = \boldsymbol{\iota}_X = 0$ in regime **d** (deceased-donor transplantation only), $\boldsymbol{\epsilon}_X = \boldsymbol{\iota}_X = 0$ in regime **l** (deceased-donor/direct living-donor transplantation), and $\boldsymbol{\iota}_X = 0$ in regime **e** (deceased-donor/direct living-donor transplantation and regular exchange). Every rate is reported in percents. For the de facto deceased-donor allocation policy, $\boldsymbol{\delta}_X$ is replaced by $\boldsymbol{\delta}'_X$ in each formula.

availability of regular exchange. With each $\Delta\rho = 10$ percent participation increase in incentivized exchange, this rate increases by about 0.2 percent. Among unpaired patient groups, blood-type *O* patients benefit the most from incentivized exchange (see Table 5).

Thus, incentivized exchange not only helps all patient groups by increasing transplants, but it also mitigates the inequities in access to deceased-donor and living-donor transplantation due to medical incompatibilities (as in the case of blood type *O* patients) and patient-arrival asymmetries (as in the case of blood type *B* patients).

B. *Stress Testing the Model*

The magnitudes of our findings depend on a few key parameters. One of these is the tissue-type incompatibility probability θ for arriving patients.[23] It is important to

[23] Different sources report higher θ values in the earlier literature (for example, see Zenios, Woodle, and Ross 2001, which cites θ as 0.11).

Table 6—Numerical Predictions of the Model: Overall Service Rate (Percent)

	O	*A*	*B*	*AB*	All
Deceased-donor trans. only (**d**)					
ABO-i	33.7	38.5	28.0	31.1	34.4
de facto	32.4	37.3	31.0	46.8	34.4
Deceased/direct living (**l**)					
ABO-i	45.1	53.8	40.1	45.6	47.3
de facto	43.9	52.7	42.9	60.2	47.3
Deceased/direct living and exchange (**e**)					
ABO-i	46.6	57.3	46.5	46.6	50.1
de facto	45.4	56.2	49.3	61.1	50.1
All plus $\rho = 10\%$-incentivized exchange (**i**)					
ABO-i	47.4	57.3	46.6	46.6	50.6
de facto	46.2	56.2	49.3	61.1	50.6
All plus $\rho = 20\%$-incentivized exchange (**i**)					
ABO-i	48.3	57.4	46.6	46.6	51.0
de facto	47.1	56.3	49.3	61.1	51.0
All plus $\rho = 30\%$-incentivized exchange (**i**)					
ABO-i	49.2	57.5	46.7	46.6	51.5
de facto	48.0	56.3	49.4	61.1	51.5
All plus $\rho = 50\%$-incentivized exchange (**i**)					
ABO-i	50.9	57.6	46.7	46.6	52.4
de facto	49.7	56.4	49.5	61.1	52.4
All plus $\rho = 100\%$-incentivized exchange (**i**)					
ABO-i	55.1	57.9	47.0	46.6	54.5
de facto	54.0	56.7	49.7	61.1	54.5

Notes: Numerical predictions of the model for each regime $\mathbf{t} \in \{\mathbf{d}, \mathbf{l}, \mathbf{e}, \mathbf{i}\}$ overall service rate $(s_X^{\mathbf{t}} = (\boldsymbol{\delta}_X + \boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X + \boldsymbol{\iota}_X)/(\boldsymbol{\pi}_X + \phi^{\mathbf{d}}\boldsymbol{\delta}_X + \phi^{\mathbf{l}}(\boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X + \boldsymbol{\iota}_X))$ for each X and $s^{\mathbf{t}} = \sum_X(\boldsymbol{\delta}_X + \boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X + \boldsymbol{\iota}_X)/\sum_X(\boldsymbol{\pi}_X + \phi^{\mathbf{d}}\boldsymbol{\delta}_X + \phi^{\mathbf{l}}(\boldsymbol{\lambda}_X + \boldsymbol{\epsilon}_X + \boldsymbol{\iota}_X))$ for total) where $\boldsymbol{\lambda}_X = \boldsymbol{\epsilon}_X = \boldsymbol{\iota}_X = 0$ in regime **d** (deceased-donor transplantation only), $\boldsymbol{\epsilon}_X = \boldsymbol{\iota}_X = 0$ in regime **l** (deceased-donor/direct living-donor transplantation), and $\boldsymbol{\iota}_X = 0$ in regime **e** (deceased-donor/direct living-donor transplantation and regular exchange). Every rate is reported in percents. For the de facto deceased-donor allocation policy, $\boldsymbol{\delta}_X$ is replaced by $\boldsymbol{\delta}_{\acute{X}}$ in each formula.

note that this probability is higher for the entire pool of patients. In real life, two separate mechanisms may cause the tissue-type incompatibility probability for the waiting mass of the patients to go up while they are waiting for a transplant. First, patients with higher intrinsic tissue-type incompatibility, who are referred to as highly sensitized patients, are less likely to receive transplants as they are less likely to be compatible with donors. Second, patients often need blood transfusions while they are waiting for a transplant. Blood transfusions cause foreign tissue-type antigens to enter the body. As a result, new antibodies are formed against these tissue types, causing the patient's intrinsic tissue-type incompatibility probability to go up.

The flow tissue-type incompatibility has been increasing in the United States since 2009 (see Table 1). For this reason, we use the 2017/2018 value $\theta = 0.068$ in our benchmark analysis. To assess the effects of changing θ, we conduct a sensitivity analysis for our model's numerical predictions when θ changes.

There is one unforeseen effect of changing θ in our model for the other underlying parameters. As we do not directly observe the living-donor pairing probabilities (α_X), we have to calibrate these using the actual direct living-donor transplant

flows ($\boldsymbol{\lambda}_X$) and new patient arrival flows ($\boldsymbol{\pi}_X$) as well as the probabilities of finding a compatible paired donor ($p_X^{\mathbf{l}}$) as

$$\alpha_X = \frac{\boldsymbol{\lambda}_X}{p_X^{\mathbf{l}} \boldsymbol{\pi}_X},$$

where $p_X^{\mathbf{l}}$ is a decreasing linear function of θ (see Section IC). Thus, each α_X is an increasing function of θ. Hence, as θ goes up, we expect more paired patients to arrive with living donors to match the data.

Therefore, we conduct two sets of stress tests. We first fix each α_X at the benchmark level reported in online Appendix Table A-2 and adjust $\boldsymbol{\lambda}_X$ as we change θ according to the formula above. Then we allow changes in θ to affect each α_X by keeping $\boldsymbol{\lambda}_X$ at its benchmark level. We report the results of these tests in online Appendix Figures A-3 and A-4.

When only θ changes at fixed (α_X) with an average equal to 0.288, as ρ increases the importance of varying θ disappears. The service rate and living-donor transplant number changes are shown in online Appendix Figure A-3 for $\theta \in \{0.047, 0.068, 0.089, 0.11, 0.131\}$. The number of annual direct living-donor transplants ranges from 5,091 to 4,644 with increasing θ. The marginal gain through regular exchange ranges in 983–1,593 additional transplants per year, and this gain is increasing in θ. Each additional 10 percent participation in incentivized exchange adds an additional 184–168 transplants annually, and the increase is inversely proportional to θ.[24]

We also report how much the assumptions we made affect our model's predictions. Our theoretical model makes two important assumptions. First, it models the flow of patients and donors as a continuum process, which effectively means that the markets are large. This minimizes the role of tissue-type incompatibility in the model. Second, it assumes that tissue-type incompatibility is the same for each patient, while we know that patients are heterogeneous in their sensitization levels against tissue types of donors in real life. We inspect the implications of these two assumptions by contrasting the numerical predictions of the model with the simulations in online Appendix Section D. The simulation model considers a finite market about one-twentieth the size of the whole United States (roughly the size of a small transplant region with about 1,750 annual new patient arrivals) and inspects the outcomes after 15 years of running. It assumes the tissue-type incompatibility probabilities of patients are distributed according to the US patient flow statistics from OPTN (data from OPTN and SRTR 2009–2018). Simulations show that these two main simplifying assumptions only minimally impact the predicted number of additional patients benefiting from incentivized exchange, though they do impose costs on the prediction of the overall number of patients benefiting from regular exchange.

For example, Table 4 (in the *All* column) shows that our theoretical model predicts 2 percent of additional paired patients will receive living donor transplantation for every $\Delta\rho = 10$ percent increase in participation of compatible pairs in incentivized exchange. Online Appendix Table A-8 shows that this percentage gain is 1.9 or 2.1 (in column 5) in the simulations (depending on whether compatible $X - X$ type

[24] It is important to emphasize that a change in θ mostly affects the number of patients benefiting from regular exchange. Its impact on the benefit of incentivized exchange is much smaller.

pairs are incentivized). Although our model and simulations predict very similar benefits from incentivized exchange, our theoretical model predicts that 12.6 percent of pairs will benefit from regular exchange, while the simulation model only predicts 11 percent.

The main rationale for this difference is that, whenever a blood-type compatible pair is not tissue-type compatible, the patient of the pair is likely to have a higher tissue-type incompatibility probability than the average patient. These pairs constitute most of the incompatible overdemanded pairs that are utilized under regular kidney exchange. In contrast, whenever a blood-type compatible pair is also tissue-type compatible, the patient of the pair is likely to have a lower tissue-type incompatibility probability than the average patient. This observation reduces the role of tissue-type incompatibility in incentivized exchange compared to its role in regular kidney exchange. As a result, our model does a good job measuring the numbers of patients benefiting from incentivized exchange in finite markets. However, its precision is slightly inferior for measuring patients benefiting from regular exchange.[25]

We report additional stress test results regarding the effect of the availability of three-way exchange technology in online Appendix Section D.

V. Conclusion

As the need for transplant kidneys is at an all-time high, the efforts to increase living donation continue. For example, the Living Donor Protection Act of 2017 introduced in the US Congress is aimed at removing some of the disincentives to living donation by ensuring job protections for organ donors who need to take medical leave to recover from organ donation, and insurance protections so organ donors are not denied or charged higher premiums because they donated an organ (H.R.1270—115th Congress 2017). Similarly the National Kidney Foundation issued a statement in January 2019 calling on Congress and the US Administration to make organ transplantation a top priority, identifying living donation as a critical area to be addressed from a legislative and regulatory standpoint.[26] Most recently in July 2019, President Trump signed the *kidney care executive order*, which aims to remove financial barriers to living organ donation (Executive Office of the President 2019). Our proposal for incentivized exchange is in line with these recent efforts.

In practice, although our incentivization scheme can be made available to any compatible pair participating in exchange, prioritizing patients from these pairs for a possible repeat transplant requires the consent of the authorities that manage the deceased-donor queue. In the United States, deceased-donor allocation is managed by OPTN and its subsidiary UNOS, which also operates a nationwide living-donor kidney exchange program. Therefore, one possibility is using our proposed incentive scheme only for compatible pairs who participate in the UNOS kidney exchange program. Providing the incentives only to participants of the UNOS kidney exchange program may also lead to consolidation, mitigating the welfare loss

[25] Therefore, the implications of our large market assumptions are even more benign in this paper for policy purposes than Roth, Sönmez, and Ünver (2007), which ignored tissue-type incompatibility and primarily focused on regular exchange as the policy question.

[26] National Kidney Foundation, "NKF Statement: A Path Forward for Increasing Kidney Transplantation," https://www.kidney.org/news/nkf-statement-path-forward-increasing-kidney-transplantation (accessed January 1, 2019).

from a fragmented kidney exchange market. In a recent paper, Agarwal et al. (2019) observes that the US kidney exchange programs are fragmented and as a result there is a significant transplant loss from (i) having multiple small programs running in parallel instead of a unified large one and (ii) some small hospitals not being able to participate in any program due to prohibitive administrative costs. By encouraging participation of compatible pairs, the UNOS program can become a focal program that attracts the largest number of pairs, partially mitigating this welfare loss.[27]

While restricting the incentives only to the participants of the UNOS kidney exchange system is one way to implement incentivized exchange, it is not the only way. This is important because restricting the incentives only to participants of the UNOS kidney exchange program may face some opposition. An alternative implementation could provide the incentives to any compatible pair regardless of which kidney exchange program they join. Any such pair provides valuable benefits to the entire pool of patients by reducing the demand for deceased-donor kidneys, and thus providing incentives to them is in the spirit of providing similar incentives for prior living donors.

Two key aspects of our proposal are inclusion of compatible pairs in exchange (to better utilize living donors) and an adjusted priority ranking in the deceased-donor queue (to incentivize them to participate in exchange). Incentivized exchange is related to three sparsely practiced variants of kidney exchange. In conclusion, we compare and contrast incentivized exchange with these variants.

An *altruistically unbalanced exchange* involves a kidney exchange between one compatible and one incompatible pair. Ross and Woodle (2000) dismisses these exchanges on ethical grounds. Their concern is potential coercion of compatible pairs who have nothing to gain from exchange. In contrast, exchange is no longer "altruistically unbalanced" under incentivized exchange, since patients of participating pairs are insured against a repeat kidney failure.

Under an *indirect exchange*, the donor of an incompatible pair donates a kidney to the deceased-donor queue in exchange for increased priority for his patient in the deceased-donor queue. Hence, this variant involves an exchange between an incompatible pair and the deceased-donor queue. Ross and Woodle (2000) objects to indirect exchange for blood-type-incompatible pairs, but supports it for blood-type-compatible (but tissue-type-incompatible) pairs. Consider a blood-type *O* patient with a blood-type *A* donor. Under an indirect exchange, the pair donates a blood-type *A* kidney to the donor queue in exchange for priority in the blood-type *O* deceased-donor queue. That is, they receive priority for a more highly sought-after blood-type kidney than the kidney they donate. This is the basis of the Ross and Woodle (2000, p. 1539) objection:

> *The indirect ABO-incompatible exchange does create a new ethical concern because it may increase the vulnerability of O blood group recipients. If mechanisms can be developed to avoid increasing the waiting time for blood group O recipients, we would support the implementation of the indirect ABO-incompatible exchange.*

[27] See an earlier draft of the current paper, Sönmez and Ünver (2015), for a formal analysis of such a participation game.

In contrast, they support indirect exchange for blood-type-compatible pairs because those pairs either donate the same blood-type or a more highly sought-after blood-type kidney than the one for which they receive priority. While incentivized exchange is also based on priority in the deceased-donor queue, there are two key differences. First, an incentivized pair donates a kidney of a more highly sought-after blood type than that for which its patient receives priority. Indeed, incentivized kidney mainly benefits the blood-type *O* patient population. Second, unlike indirect exchange, the priority is only used if the patient needs a repeat transplant. Both factors address Ross and Woodle's ethical considerations.

A *voucher for a chronologically incompatible* pair (Veale et al. 2017) involves priority for a (typically young) patient of a pair for a future transplant in exchange for a donation from an older donor today. The donor will be too old to donate when the patient is expected to need a transplant. Observe that this variant is very similar to indirect exchange, and indeed it can be interpreted as an intertemporal version of indirect exchange. Therefore, the same ethical considerations from Ross and Woodle (2000) apply. That is, the case for these exchanges is stronger when the pair is blood-type compatible than when they are blood-type incompatible. Unlike an incentivized exchange or an indirect exchange, the first three of these intertemporal exchanges were organized by the National Kidney Registry, which arranges kidney chains initiated by good Samaritan donors.[28] The older donor starts a chain today, and the younger patient receives priority for a kidney at the end of a chain when he needs a transplant in the future. However, these chains almost never end with a blood-type *O* kidney, and indeed they are likely to end with a blood-type *AB* kidney. Hence, honoring the voucher may require artificially terminating a kidney chain, especially if the patient is of blood type *O*. Perhaps motivated by these concerns, Veale et al. (2017) suggests that patients also be prioritized in the deceased-donor queue in case the patient cannot be placed at the end of a kidney chain. Conceptually, incentivized exchange is similar, but it avoids the shortcomings mentioned above since incentivized pairs are blood-type compatible. In summary, incentivized exchange harbors all the positive elements of the variants of kidney exchange above without suffering from their shortcomings.

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