

# Efficient and Incentive-Compatible Liver Exchange

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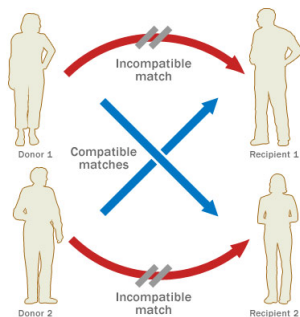
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- **Kidney Exchange** became a wide-spread modality of transplantation within the last decade.
- Around 800 patients a year receive kidney transplant in the US along through exchange, more than 12% of all living-donor transplants.
- In theory **living-donor organ exchange** can be utilized for any organ for which living donation is feasible.
- Liver is the second most transplanted organ after kidneys; moreover, living-donor lobar liver donation is feasible.
- Liver exchange utilized in S. Korea, Hong Kong, Turkey in small numbers

# Kidney Exchange

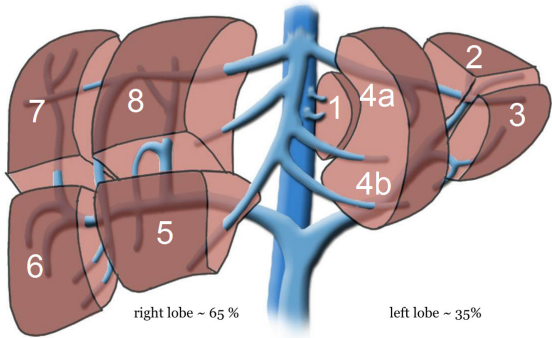


- Human organs cannot be received or given in exchange for "valuable consideration" (US, NOTA 1984, WHO)
- However, **living-donor kidney exchange** is not considered as "valuable consideration" (US NOTA amendment, 2007)

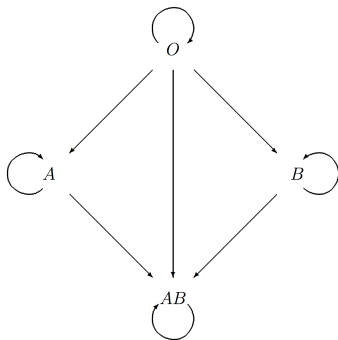
- **Kidney Exchange Literature:** Plenty . . .  
Two mostly related to this paper: Roth, Sönmez, & Ünver [2005] and Sönmez & Ünver [2014]
- **Liver Exchange Literature:**
  - Hwang et al. [2010] proposed the idea and documented the practice in South Korea since 2003
  - Chen et al. [2010] documented the program in Hong Kong
  - Dickerson & Sandholm [2014] asymptotic gains from liver+kidney exchange over isolated liver exchange and kidney exchange (first such exchange conducted recently)
  - Mishra et al. [2018] advocates for establishment for liver exchange clearinghouses in the US.
- **Dual-Donor Organ Exchange:**
  - Ergin, Sönmez, & Ünver [2017] proposed and modeled exchange for transplants each of that needs two living donors: lung, simultaneous liver+kidney, dual-graft liver

- We model liver exchange as a matching problem – different from kidney exchange due to size-compatibility requirement.
- We find the structure of feasible two-way exchanges and a sequential algorithm to find an efficient matching for two patient/donor sizes.
- The requirement of size compatibility induces an incentive problem for the pair/donor to donate
  - the larger/riskier/easier to match right lobe or
  - the smaller/safer/more difficult to match left lobe
- For a continuum of patient/donor sizes, we propose a Pareto-efficient and incentive-compatible mechanism that elicits willingness to donate right lobe truthfully.
  - A new class of bilateral exchange mechanisms for vector-partial-order-induced weak preferences.

# Medical Background: Lobar Liver Donation



- Blood-type compatibility is required.



- Size compatibility is required unlike kidneys: A patient requires a graft relatively large to survive.
- Tissue-type compatibility is **not** required unlike kidneys.

- Right-lobe transplant has been utilized for size compatibility despite its heightened donor mortality risk.
  - Patient needs roughly at least 40% of his own liver size to survive.
  - Donor needs at least 30% remnant liver volume to survive.
  - Usually right lobe is  $\sim 65\%$ , left lobe is  $\sim 35\%$  of liver.
  - In many occasions, size compatibility is only satisfied through right-lobe donation.



# Institutions: Living Donor Deaths

Trotter et al. [2006] documented living liver donor deaths due to donation.

TABLE 1. Deaths of Living Donors

Reference	Date	Location	Description
<b>Donor deaths "definitely" related to donor hepatectomy</b>			
11	2003	Japan	A mother in her late 40s donated a <b>right lobe</b> and died 9 months later from complications of hepatic failure.
12	2002	USA	A 57-year-old brother donated a <b>right lobe</b> and developed gastric gas gangrene and <i>Clostridium perfringens</i> infection 3 days after surgery and died.
13	2005	Brazil	A 31-year-old female <b>right lobe</b> donor of unknown relationship to the recipient died 7 days after surgery from a subarachnoid hemorrhage.
14	2003	India	A donor of unknown age and unknown relationship to the recipient donated an <b>unknown lobe</b> and died 10 days after surgery of unknown causes.
15	2003	India	A 52-year-old wife donated an <b>unknown lobe</b> and became comatose 48 hours after surgery from unknown causes and remains in chronic vegetative state.
16-18	1993	Germany	A 29-year-old mother donated a <b>left lateral lobe</b> and died of a pulmonary embolus 48 hours after surgery.
18, 19	2000	Germany	A 38-year-old father donated a <b>right lobe</b> and 32 days after developing progressive hepatic failure, died during transplantation of acute cardiac failure. The cause of the donor's death was attributed to Bernardini-Scip syndrome, a lipodystrophy syndrome characterized by loss of body fat, diabetes, hepatomegaly, and acanthosis nigricans.
18, 20	2000	France	A 32-year-old brother donated a <b>right lobe</b> and developed sepsis and multiple organ system failure 11 days after surgery and died of septic shock 3 days later.
18	2000	Europe	A 57-year-old wife donated a <b>right lobe</b> and died of sepsis and multiple organ system failure 21 days after surgery.
21, 22	1999	USA	A 41-year-old half-brother donated a <b>right lobe</b> and died of pancreatitis and sepsis 1 month later.
22, 23	1997	USA	A mother of unknown age donated an <b>unknown lobe</b> to a pediatric recipient and died 3 days after surgery of unknown causes.
24	2005	Asia	A 50-year-old mother donated a <b>right hepatic lobe</b> . She had no history of peptic ulcer disease and received a 2-week course of H2 antagonist. She died 10 weeks after surgery from an autopsy-proven duodenal ulcer with a duodenocaval fistula causing air embolism.
25	2006	Asia	A 39-year-old male "close relative" who donated an <b>unknown lobe</b> died of a myocardial infarction 4 days after donation. The patient reportedly had a preoperative electrocardiogram and treadmill test.
26	2005	Egypt	A brother of unknown age who donated a <b>right lobe</b> died of complications of sepsis from a bile leak 1 month after donation.
<b>Donor deaths "possibly" related to donor hepatectomy</b>			
27	2005	USA	A 35-year-old brother donated a <b>right lobe</b> and died of a self-induced drug overdose 23 months later.
27	2005	USA	A 50-year-old uncle donated a <b>right lobe</b> and died of a self-inflicted gunshot wound to the head 22 months after donation.
<b>Donor deaths "unlikely" to be related to donor hepatectomy</b>			
28	2003	Asia	A donor of unknown age and relationship to the recipient who donated an <b>unknown lobe</b> died of unknown causes during exercise 3 years after donation.
27, 29	2002	USA	A 35-year-old boyfriend donated a <b>right lobe</b> and died in a non-suicidal occupational pedestrian-train accident 2 years after donation. A lone railroad car rolling at high speed struck and killed the donor while he was on duty at his job for the railroad.
16	2003	Germany	A 30-year-old father donated a <b>left lateral segment</b> and died of complications of amyotrophic lateral sclerosis 11 years after successful

- Right hepatectomy is commonly reported to have fivefold **mortality** rate of that of left hepatectomy (0.4% vs 0.1%).
- Mishra et al. [2018] reports that the **morbidity** rates are 28% for right hepatectomy and 7.5% for left hepatectomy.
- A high profile death of a living right-lobe donor in 2002 decreased living donation steadily not only for livers, but for other organs including kidneys in the US.
- About half of living donations right lobe.

- Medical Metric:

$$\frac{\text{Probability of Patient Survival at 5 years Given Donated Lobe}}{\text{Probability of Donor Death Given Donated Lobe}}$$

- Transplant if the metric is higher than a threshold

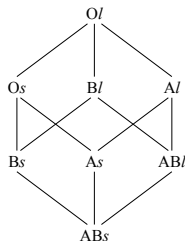
- Leaders in living donation: Turkey, South Korea (each more than 1000 per year), other East Asian countries, . . . , USA 400 per year
- Liver exchange first done in South Korea, followed by Hong Kong and Turkey.
- Liver exchange can have two benefits:
  - It can plainly increase the number of transplants.
  - It can decrease the share of right-lobe transplants (and increase donor safety) through matching with respect to size;
  - Also improves double equipose metric allowing more transplants.

# Liver Exchange Model

- $\underbrace{\{O, A, B, AB\}}_{\mathcal{B}} \times \underbrace{\{0, 1, \dots, S-1\}}_{\mathcal{S}}$ : Set of individual types
- Initially, we focus on living-donor left-lobe liver transplants.
- **Left-Lobe Compatibility:** A donor can donate to a patient if and only if
  - the patient is blood type compatible with the donor, and
  - the donor is not smaller than the patient.

Liver Donation Partial Order  $\triangleright$  on  $\mathcal{B} \times \mathcal{S}$

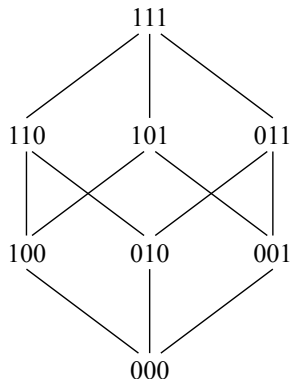
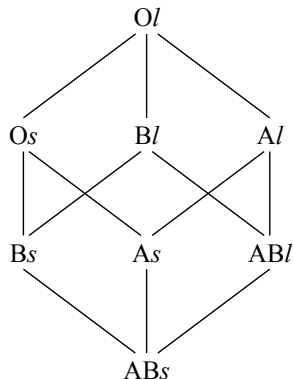
**Example:**  
2 Sizes Only  $\mathcal{S} = \{s, l\}$



# An Equivalent Representation

- Consider the following two partially ordered sets:
  - The liver donation partial order  $\triangleright$  on  $\mathcal{B} \times \mathcal{S}$ , and
  - the standard partial order  $\geq$  over the rectangular integer prism  $\{0, 1\}^2 \times \{0, 1, \dots, S-1\}$ .

**Example:** 2 Sizes Only  $\mathcal{S} = \{s, l\}$



- Individual patient/donor types:  $X, Y \in \{0, 1\} \times \{0, 1\} \times \mathcal{S}$
- Pair types:  $X - Y \in (\{0, 1\}^2 \times \mathcal{S})^2$
- Right-lobe donation function: A non-decreasing function  $\rho : \mathcal{S} \rightarrow \mathcal{S}$  such that  $\rho(s) > s$  for all  $s \in \mathcal{S} \setminus \{S - 1\}$

A donor of size  $s$  size can donate his right lobe to a blood-type compatible patient of any size  $s' \leq \rho(s)$ .

## Definition

A **liver exchange problem** is a list  $\mathcal{E} = \{\mathcal{I}, \tau\}$  where  $\mathcal{I} = \{1, 2, \dots, K\}$  is a set of pairs, for each  $i \in \mathcal{I}$ ,  $\tau(i) = X - Y$  is the type of pair  $i$ .

- **Donation Possibilities:**
  - Left-lobe donation: Less risky for the donor.  
Blood-type compatible donor should be at least as large as the patient.
  - Right-lobe donation: More risky for the donor.  
Blood-type compatible donor can donate to a larger patient.
- **Pair Preferences:**
  - Donating left lobe is always preferable to donating right lobe or not donating at all
  - The pair may prefer donating right lobe to not donating at all: Type **willing** ( $w$ ).
  - The pair may prefer not donating at all to donating right lobe: Type **unwilling** ( $u$ ).



- We focus only on **individually rational exchanges**:
  - A left-lobe compatible pair does not join in any exchange, but only in a direct transplant.
  - A right-lobe-only compatible pair participates in an exchange only if its donor donates his left lobe; otherwise, it participates in a direct right-lobe transplant.
- A **matching** is a collection of mutually exclusive individually rational exchanges and direct transplants.
- Willingness type of a pair is private information.
- We inspect direct revelation **mechanisms** to elicit willingness types.

Willing preferences  $R_i^w$ :

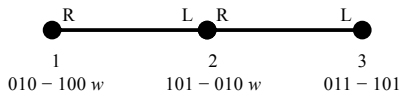
- Left Direct
- Left Exchange
- Right Direct
- Right Exchange
- $\emptyset$
- $\vdots$

Unwilling preferences  $R_i^u$ :

- Left Direct
- Left Exchange
- $\emptyset$
- $\vdots$

- A **mechanism** is a systematic procedure that finds a matching for each willingness type profile reported.
- A mechanism is **incentive compatible** if it is a weakly dominant strategy for each pair to reveal its willingness truthfully.
- Our mechanism will be based on a sequential priority algorithm.
- Two questions:
  - How do we take their different options, left lobe vs right lobe donation, into account?
  - How do we prioritize pairs?

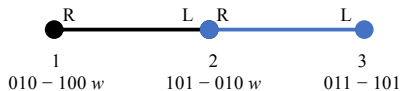
## Example: Wrong Priority Order



Priority order:

3 - 2 - 1

## Example: Wrong Priority Order

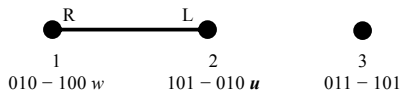


Outcome:

$$M = \{\{2, 3\}\}$$

## Example: Wrong Priority Order

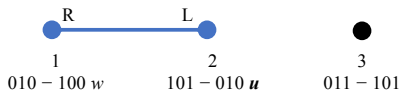
However, Pair 2 can declare itself as being (unwilling)  $u$  and benefit:



Priority order:

3 - 2 - 1

## Example: Wrong Priority Order



Outcome:

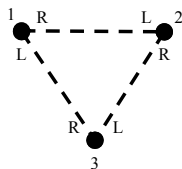
$$M' = \{\{1, 2\}\}$$

Pair 2 is matched by donating left lobe instead of right lobe.

- We need to respect each willing pair's left lobe donation options before right lobe for **incentive compatibility**.
  - Start assuming all pairs are **left-lobe donors**
  - We will consider a willing pair as a **right-lobe donor** only if its all current and future left-lobe donation options are exhausted.
- Questions:
  - Is it always possible to find a priority order that will achieve Pareto efficiency?
  - In general, does a PE, IR, IC mechanism always exist?



# A PE, IR, IC Mechanism Exists?



- Suppose priority order is  $1 - 2 - 3$
- When no pair is transformed, pair 1 has no exchange in which it donates left lobe.
- But we cannot transform it yet, as in the future  $\{1, 3\}$  can become feasible, and it donates left lobe in this exchange.
- The same is true for each pair as we go in order.
- No pairs are matched. However, any matching with an exchange Pareto dominates it.
- What is wrong? **There is a Left-Robe – Right-Lobe exchange cycle**

## Proposition

*For an exchange pool  $(\mathcal{I}, \tau)$  suppose the compatibility graph consists of a Left-Robe – Right-Lobe exchange cycle. Then there is no Pareto-efficient, individually rational, and incentive-compatible mechanism.*

Can such a cycle exist for liver exchange?

## Definition

Define the following **precedence digraph** on the set of pair types, where for any pair types  $X - Y$  and  $U - V$

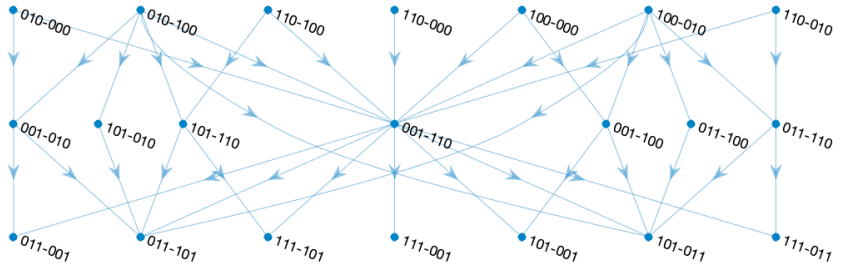
$$X - Y \longrightarrow U - V \iff X \leq V, U \not\leq Y \text{ \& } U \leq \rho(Y).$$

That is,  $X - Y \longrightarrow U - V$ , if

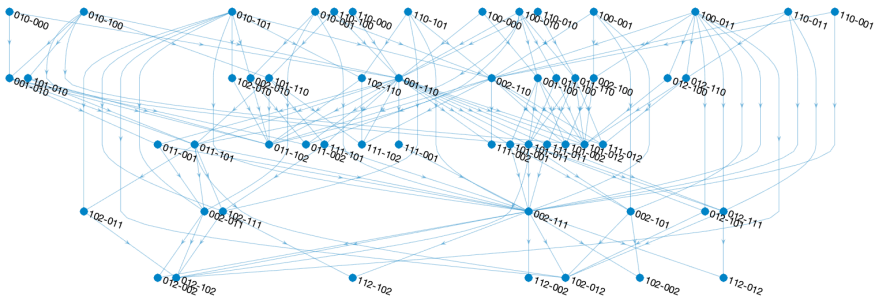
- an  $X - Y$  pair can donate only **right lobe** to a  $U - V$  pair,
- while the  $U - V$  pair can donate **left lobe** to the  $X - Y$  pair.

If  $X - Y \longrightarrow U - V$ , we say that  $X - Y$  **precedes**  $U - V$ .

# Precedence Digraph: 2 Sizes $\mathcal{S} = \{0, 1\}$



# Precedence Digraph: 3 Sizes $\mathcal{S} = \{0, 1, 2\}$



## Lemma (from graph theory)

Given an *acyclic digraph*, there exists a linear order of all nodes, known as a *topological order*,  $\mathbb{L}$ , that is consistent with the digraph:

$$x \rightarrow y \implies x \mathbb{L} y$$

## Lemma

*The precedence digraph on pair types is acyclic.*

Thus, a topological order of pair types, as well as a topological order of all pairs exist.

The latter can be used as a priority order over transformations.

## Proof of Lemma.

Suppose for a contradiction that the precedence digraph has a cycle:

$$X^0 - Y^0 \longrightarrow X^1 - Y^1 \longrightarrow \dots \longrightarrow X^{n-1} - Y^{n-1} \longrightarrow X^0 - Y^0$$

where  $n \geq 2$ .

Note that for each  $k \in \{0, 1, \dots, n-1\}$  in modulo  $n$ :

$$X^k - Y^k \longrightarrow X^{k+1} - Y^{k+1} \longrightarrow X^{k+2} - Y^{k+2}$$

implies that  $X_3^k \leq Y_3^{k+1}$ . It also implies that  $Y_3^{k+1} < X_3^{k+2}$  since  $Y^{k+1} \not\geq X^{k+2}$  and  $\rho(Y^{k+1}) \geq X^{k+2}$ . Therefore,  $X_3^k < X_3^{k+2}$ . That is, a patient along the cycle has a smaller size than the patient two steps ahead in the cycle. This can be used to obtain a contradiction in two separate cases:

**Case 1 “ $n$  is even”:**  $X_3^0 < X_3^2 < \dots < X_3^{n-2} < X_3^0$ .

**Case 2 “ $n$  is odd”:**

$X_3^0 < X_3^2 < \dots < X_3^{n-1} < X_3^1 < X_3^3 < \dots < X_3^{n-2} < X_3^0$ . □

- We will attain incentive compatibility by **gradually transforming** willing pairs as their left-lobe transplant prospects are **fully exhausted**.
- Fix a willingness profile  $R = (R_i)_{i \in \mathcal{I}} \in \{R_i^u, R_i^w\}^K$
- A pair of type  $X_1 X_2 X_3 - Y_1 Y_2 Y_3 w$  is treated as if it is of type  $X_1 X_2 X_3 - Y_1 Y_2 \rho(Y_3)$  when it donates a right lobe. We refer to this transition as a **transformation**.
- $G' = (\mathcal{I}, E')$  is a **reduced compatibility graph** between pairs if  $E' \subseteq E_{IR}$ , i.e., a subset of **the individually rational matches**:

$$\{i, j\} \in E' \implies \begin{cases} j R_i i & \& j R_i \emptyset \\ i R_j j & \& i R_j \emptyset \end{cases}$$



- We will rely on a **priority** approach, based on matchability arguments.
- A set of pairs  $\mathcal{I}_0 \subseteq \mathcal{I}$  is **matchable** in a compatibility graph  $G$  of all pairs  $\mathcal{I}$ , if there exists a matching of  $G$  such that all pairs in  $\mathcal{I}_0$  receive a transplant.

- For left-lobe donation commitment/transformation: Fix a topological order  $\mathbb{L}$  over pairs.
- For right-lobe donation commitment: Fix a priority order  $\mathbb{R}$  over pairs.
- Let preference profile  $R$  be reported.

# Precedence-Induced Adaptive-Priority Mechanism

Step Left Suppose  $\mathbb{L} = i_1 - i_2 - \dots - i_K$  is the topological order.

- Initially, every patient can only donate left lobe. Let  $G_0$  be this reduced compatibility graph.
- Let  $\mathcal{L}_0 := \emptyset$  be the set of left-lobe committed pairs.

Step Left.k for  $k = 1, \dots, K$

- If pair  $i_k$  together with  $\mathcal{L}_{k-1}$  are **matchable** in  $G_{k-1}$ , then let left-lobe committed set  $\mathcal{L}_k := \mathcal{L}_{k-1} \cup \{i_k\}$  and  $G_k := G_{k-1}$
- Otherwise, let  $\mathcal{L}_k := \mathcal{L}_{k-1}$ , and
  - if  $i_k$  is **willing**, then **transform**  $i_k$  to obtain a new reduced compatibility graph  $G_k$  from  $G_{k-1}$ ,
  - otherwise let  $G_k := G_{k-1}$

This step ends at Step Left.K with **graph**  $G_K$  and **left-lobe committed pairs**  $\mathcal{L}_K$ .

Step Right Suppose  $\mathbb{R}|_{\mathcal{I} \setminus \mathcal{L}_K} = i_1^* - i_2^* \cdots - i_N^*$  is the second step priority order restricted to the remaining uncommitted pairs  $\mathcal{I} \setminus \mathcal{L}_K$ .

- Let  $G_0^* := G_K$  be the reduced compatibility graph.
- Let  $\mathcal{R}_0 := \emptyset$  be the initial set of right-lobe committed pairs.

*Step Right.n* for  $n = 1, \dots, N$

- If pair  $i_n^*$  together with  $\mathcal{L}_K \cup \mathcal{R}_{n-1}$  are **matchable** in  $G_0^*$ , then let right-lobe committed set of pairs  $\mathcal{R}_n := \mathcal{R}_{n-1} \cup \{i_n^*\}$ .
- Otherwise, let  $\mathcal{R}_n := \mathcal{R}_{n-1}$ .

This step ends at Step Right.N with **right-lobe committed pairs**  $\mathcal{R}_N$ .

The mechanism picks a matching that matches pairs in  $\mathcal{L}_K \cup \mathcal{R}_N$ .

## Theorem

*The precedence-induced adaptive-priority mechanism's outcome is utility-wise uniquely defined and the mechanism satisfies*

- *individual rationality,*
- *Pareto efficiency, and*
- *incentive compatibility.*

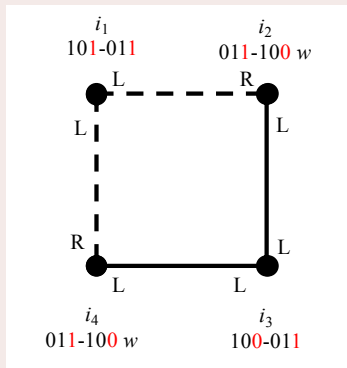
There is also a polynomial algorithm to find its outcome.

In the paper, we have the generalized model and mechanism:

- Patients have medically determined **weak preferences** over compatible received transplants (not necessarily one indifference class)
- Donors with the same left-lobe size may have different right-lobe sizes.
- **Compatible pairs** can also participate in exchanges if they find it beneficial for them.
- 4 privately known pair preference relations instead of 2:  
 $R_i^{e/w}, R_i^{e/u}, R_i^{d/w}, R_i^{d/u}$

## Proposition

*There is no incentive-compatible mechanism that maximizes the number of transplants or the number of left-lobe donations.*



Any total-transplant or left-lobe-donation maximizing matching generates 4 transplants with 3 left-lobe donations:

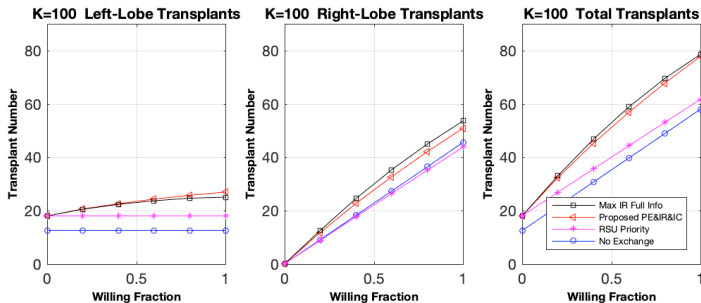
$100 - 011$  &  $011 - 100w$  (1)     $101 - 011$  &  $011 - 100w$  (2)

One type  $011 - 100w$  pair is matched to donate left lobe in exchange type (1), and the other one is matched to donate right lobe in exchange type (2). If the one matched in exchange type (2) declares  $u$  instead of  $w$ , then it will be the one matched in exchange type (1) and donate left lobe instead.  $\square$



# Simulations Using South Korean Stats

- % of left-lobe transplants higher under PE&IR&IC than no exchange, RSÜ Priority for left-lobe exchanges, and hypothetical full-info maximum IR.
- PE&IR&IC generates 44%-34% more transplants than no exchange.
- PE&IR&IC generates 20%-28% more transplants than RSÜ Priority.
- PE&IR&IC is within 3% of the total matches of full-info maximum IR.



- We model **living-donor liver exchange** as a market design problem. Information/incentive problems are modeled and solved through a **PE&IR&IC mechanism**.
- Size incompatibility increases the benefit from exchange, more gains plausible with respect to kidney exchange.
- Off-the-shelf-implementable mechanism in Turkey and East Asia: Liver transplants are more complex, two-way may be the way to start the exchange.
- **Implications for matching theory in general:** A new class of bilateral exchange mechanisms for  $n$ -dimensional vector partial-order induced weak preferences:
  - Other examples: vacation house exchanges, time/favor exchanges