# Transplant Center Incentives in Kidney Exchange

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There are $k$ transplant centers each of which can carry out feasible exchanges among its own pairs.[1] Let $T$ be the set of transplant centers and for any transplant center $t \in T$, let $I_t$ denote the set of incompatible pairs registered at the center $t$. A pair can only register at one center. Therefore, $I = \cup_{t \in T} I_t$ is the set of incompatible pairs in the exchange pool.

Suppose each transplant center wants to maximize the number of its own patients who receive a transplant (or equivalently to maximize the number of its own incompatible pairs that are matched). A transplant center may feel it does not receive its fair share from the collaborative effort if it receives fewer transplants through the multi-center kidney exchange than it would receive by arranging its own exchanges.

A mechanism is **individually rational** if each center always receives as many transplants as it would receive by itself. While a priority mechanism induced by an exogenous priority ordering is not individually rational, it can be easily modified to satisfy this mild requirement. One way this can be done is via endogenizing the priority ordering.

A **priority ordering of** $I$ is a one-to-one and onto function $\pi : \{1, 2, ..., |I|\} \to I$. Pair $\pi(k)$ denotes the $k^{\text{th}}$ highest priority pair under $\pi$. For any priority ordering $\pi$ and $J \subseteq I$, let $\pi_J$ be the priority ordering of $J$ that is obtained by restricting $\pi$ to $J$, that is, for all $i, j \in J$,

$$\pi_J^{-1}(i) < \pi_J^{-1}(j) \iff \pi^{-1}(i) < \pi^{-1}(j).$$

Fix a priority ordering $\pi$ of all incompatible pairs. For any transplant center $t \in T$, let $\pi_t$ be the restriction of $\pi$ to incompatible pairs in center $t$ (i.e., for any $i, j \in I_t$, $\pi^{-1}(i) < \pi^{-1}(j)$ if and only if $\pi_t^{-1}(i) < \pi_t^{-1}(j)$). For each transplant center $t$, let $\tilde{I}_t$ denote the set of incompatible pairs

[1] See Rapaport (1986) for the medical background and Roth, Sönmez, and Ünver (2004a, 2004b, 2005) for economic background of the problem.

who are matched via the priority mechanism induced by $\pi_t$ when only $I_t$, $C_t$, $G_t$ participate in exchange. Let $\tilde{I} = \cup_{t \in T} \tilde{I}_t$ and construct a priority ordering $\tilde{\pi}$ from $\pi$ as follows:

1. $\tilde{\pi}^{-1}(i) < \tilde{\pi}^{-1}(j)$ for any $i \in \tilde{I}$ and $j \in I \setminus \tilde{I}$,
2. $\tilde{\pi}^{-1}(i) < \tilde{\pi}^{-1}(j)$ if and only if $\pi^{-1}(i) < \pi^{-1}(j)$ for any $i, j \in \tilde{I}$, and
3. $\tilde{\pi}^{-1}(i) < \tilde{\pi}^{-1}(j)$ if and only if $\pi^{-1}(i) < \pi^{-1}(j)$ for any $i, j \in I \setminus \tilde{I}$.

That is,

1. Any incompatible pair in $\tilde{I}$ has higher priority under $\tilde{\pi}$ than any incompatible pair in $I \setminus \tilde{I}$,
2. the relative priority ordering among incompatible pairs in $\tilde{I}$ under $\tilde{\pi}$ is the same as their relative priority ordering under $\pi$, and
3. the relative priority ordering among incompatible pairs in $I \setminus \tilde{I}$ under $\tilde{\pi}$ is the same as their relative priority ordering under $\pi$,

This uniquely determines a priority ordering.

Let $\phi^{\pi}$ denote a mechanism that chooses a priority matching induced by the endogeneous priority ordering $\tilde{\pi}$ derived from $\pi$ as explained above.We refer to $\phi^{\pi}$ as the **multi-center priority mechanism induced by** $\pi$.

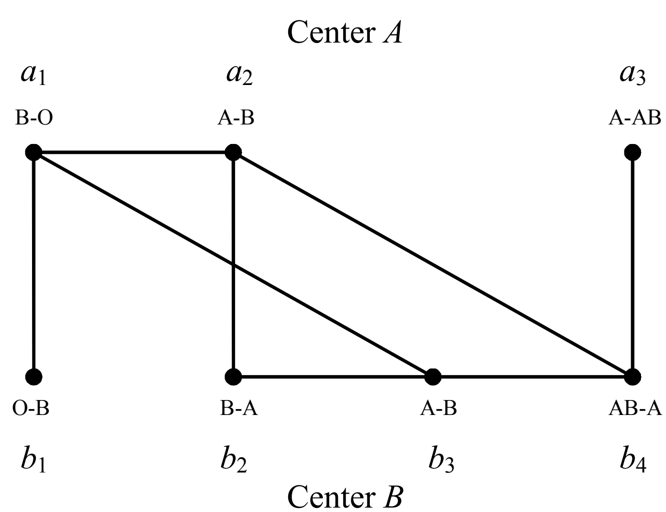
**Proposition 1:** Given a priority ordering $\pi$, the induced multi-center priority mechanism $\phi^{\pi}$ is individually rational.

**Proof:** For each transplant center $t$ and given ordering $\pi_t$ let $\mu_t$ be a resulting priority matching when only $I_t$ is available for exchange. Let $\mu = \cup_{t \in T} \mu_t$. Observe that $M^{\mu} \cap I = \tilde{I}$ and recall that incompatible pairs in $\tilde{I}$ are the highest priority $|\tilde{I}|$ pairs under $\tilde{\pi}$. Consider the priority mechanism induced by $\tilde{\pi}$ and observe that $\mu \in \mathcal{E}_{\tilde{\pi}}^{|\tilde{I}|}$ (since $\mu$ matches all of the highest priority $|\tilde{I}|$ pairs). Therefore all incompatible pairs in $\tilde{I}$ are matched under any matching in $\mathcal{E}_{\tilde{\pi}}^{|\tilde{I}|}$ and since $\mathcal{E}_{\tilde{\pi}}^{|I|} \subseteq \mathcal{E}_{\tilde{\pi}}^{|\tilde{I}|}$, they are all matched under the priority mechanism induced by $\tilde{\pi}$. Hence for any transplant center $t$, at least $|\tilde{I}_t|$ incompatible pairs are matched under the priority mechanism induced by $\tilde{\pi}$ (that is the multi-center priority mechanism induced by $\pi$). $\Diamond$

Each mechanism induces a participation game among transplant centers where each center decides which of its pairs and Good Samaritan donors to enroll in the collaborative kidney exchange and which ones to keep to itself to be matched within the center. Individual rationality is a relatively mild requirement and it requires full participation strategy to dominate the no participation strategy for each center. A more demanding and plausible requirement would be full participation to be a dominant strategy of the induced participation game. Unfortunately there is no such mechanism that is also Pareto efficient. Actually, this negative result is a very strong result. It holds even if there is no tissue-type incompatibility between patients and donors.

**Proposition 2:** Even if there is no tissue-type incompatibility between patients and donors of different pairs, there exists no Pareto efficient mechanism where full participation is always a dominant strategy for each transplant center.

**Proof:** The proof is through an example. There are two transplant centers $A, B$, three pairs $a_1, a_2, a_3 \in I_A$ in center $A$, and four pairs $b_1, b_2, b_3, b_4 \in I_B$ in center $B$. Suppose that there is no tissue-type incompatibility between patients and donors of different pairs. The blood types of patients and donors in each pair are given as follows: In transplant center $A$, pairs $a_1, a_2$, and $a_3$ have B-O, A-B, and A-AB patient-donor blood types, respectively. In transplant center $B$, pairs $b_1, b_2, b_3$, and $b_4$ have O-B, B-A, A-B, and AB-B patient-donor blood types, respectively. Since all incompatibilities between the patients and donors are only caused by blood-type incompatibility, the list of feasible exchanges are as follows: $(a_1, a_2)$, $(a_1, b_1)$, $(a_1, b_3)$, $(a_2, b_2)$, $(a_2, b_4)$, $(a_3, b_4)$, $(b_2, b_3)$, $(b_3, b_4)$. The following figure shows feasible exchanges among the pairs.




In all Pareto efficient matchings 6 pairs receive transplants (an example is $\{(a_1, b_1), (a_2, b_2), (b_3, b_4)\}$). Since there are 7 pairs, one of the pairs remains unmatched under any Pareto-efficient matching. Let $\phi$ be a Pareto-efficient mechanism. Since $\phi$ chooses a Pareto-efficient matching, there is a single pair that does not receive a transplant. This pair is either in Center $A$ or in Center $B$.

- The pair that does not receive a transplant is in Center $A$. In this case, if Center $A$ does not submit pairs $a_1$ and $a_2$ to the centralized match, and instead matches them internally to each other, then there is a single multi-center Pareto-efficient matching $\{(a_3, b_4), (b_2, b_4)\}$, and $\phi$ chooses this matching. As a result, Center $A$ succeeds in matching its all three pairs.

- The pair that does not receive a transplant is in Center $B$. In this case, if Center $B$ does not submit pairs $b_3$ and $b_4$ to the centralized match, and instead matches them internally to each other, then there is a single multi-center Pareto-efficient matching $\{(a_1, b_1), (a_2, b_2)\}$, and $\phi$ chooses this matching. As a result, Center $B$ succeeds in matching its all four pairs.

In either case we showed that there is a center that can successfully manipulate the Pareto-efficient multi-center matching mechanism $\phi$. $\Diamond$

# References


[1] F.T. Rapaport (1986). "The case for a living emotionally related international kidney donor exchange registry." *Transplantation Proceedings*, 18, 5-9.

[2] Alvin E. Roth, Tayfun Sönmez, and M. Utku Ünver (2004). "Kidney Exchange," *Quarterly Journal of Economics*, (May) 119: 457-488.

[3] Alvin E. Roth, Tayfun Sönmez, and M. Utku Ünver (2004). "Pairwise Kidney Exchange," NBER Working Paper.

[4] Alvin E. Roth, Tayfun Sönmez, and M. Utku Ünver (2005). "A Kidney Exchange Clearinghouse in New England," *American Economic Review Papers and Proceedings*, forthcoming.