The Dynamics of Law Clerk Matching: An Experimental and Computational Investigation of Proposals for Reform of the Market

Online Appendices

Ernan Haruvy **Alvin E. Roth** **M. Utku Ünver**

Appendix: Genetic Algorithm and Representation of Strategy Strings

The outline of one simulation session can be stated as follows:

Algorithm: Generate the initial population of strategies for each pool (that has a population of s strategies at a time) that will cause an outcome in matching behavior similar to the experimental subjects in market 1.

For g=1…G, the total number of generations, run the following algorithm for the existing set of strategies for each pool.

- Make a **tournament** of T matching games by randomly choosing strategies i=1,…,s from each pool k=1,…,5. The existing 5 types are Judge 1, Judge 2, Judge 3, Judge 4 and workers. Determine the reinforcement (or fitness) of each strategy as the average payoff that it brought to the players who adopted it in the tournament.
- For i=1,..,h, **select** the i'th **highest fitness** strategy of each type k to the next generation offspring. Return these strategies to the population pool for crossover.
- For i=1,…,(s-h)/2, **crossover** 2 parents for the (2i-1)'th and (2i)'th spots in the offspring generation for each type k=1,…,5 using the following technique:
    - Use **tournament selection** to determine two parents P^k_{2i-1} and P^k_{2i}: Choose four parent candidates C_1, C_2, C_3, C_4 for type k randomly using the discrete uniform density. The higher fitness strategy of C_1, C_2 and C_3, C_4 become the two parents P^k_{2i-1}, P^k_{2i} for type k.

- With probability p, crossover the parents, with probability 1-p directly copy the parents as the offspring using single point **linear crossover**.
  - If crossover is adopted, randomly draw a crossover digit, c in $\{1,2,..,l^k-1\}$, in the strategy string of the size l^k.
  - Otherwise set c=0. Copy the digits 1,..,c of P^k_{2i-1} and c + 1 ,.., l^k digits of P^k_{2i} to form the child O^k_{2i-1}, copy the digits 1,..,c of P^k_{2i} and c+1,..,l^k digits of P^k_{2i-1} to form the child O^k_{2i}.

- For i=1,..,s, **mutate** each decision variable d=1,…,l^k in the offspring strategy O^k_i of each type t with probability $q = (1 - g / G)\, p^m_{max} + (g / G)\, p^m_{min}$ where g is the current generation number. Let $O^k_i(d)$ be the current decision variable.
  - If mutation is adopted, randomly draw an integer x in $\{r_1,\dots,r_2\}$, the range of the current decision variable $O^k_i(d)$, and replace it with x.
  - If mutation is not adopted, directly copy the existing digit.
- The strategies for generation g+1 are the offspring of generation g. ◊

The artificial adaptive agents are constructed to choose among strategies represented by strings of decision variables. The strategies are conditioned on the rank of players as well as the current information available in each year. The applicants are ex-ante identical, so they use the same pool of strategies. The judges have different ex-ante qualities; therefore judges of different types consider different pools of strategies. Therefore there are 5 pools of strategies. The strategies are coded using **integer coding**.[1] We use a bounded rational representation for the strategies.

In the law clerk market simulations, a judge strategy is represented as a string of 6 decision variables:

$$T\text{-}A^1\text{-}R^1\text{-}A^2\text{-}R^2\text{-}R^3$$

Variable T is an integer in {1,2,3}. This decision variable is the year when the judge is going to start accepting applications from applicants. This is automatically set to 1 in the treatments without announcements in all generations. In the treatments with

[1] Each decision variable is represented by an integer.

announcements, T is chosen from its full domain and evolves over time. A^t is in {0,1}. When A^t=1, the judge may hire an applicant in year t. When A^t=0, the judge will not hire an applicant in year t. A judge may announce admitting applications in a period, but this judge does not have to hire an applicant in the same period. This is the reason why we use two variables T and A^t (A^3=1 is automatically set at the beginning of the simulations, so it is not a decision variable.) R^t is in {1,2,3,4}. This decision variable is the threshold rank of the applicant that the judge is going to hire,[2] in the case A^t=1. When the applicants have lower ranks than R^t, simply the judge does not hire anybody in that period. Otherwise, it hires the best applicant. In treatments with announcements, depending on the values of T some values of A^t and R^t may not be used.

An applicant strategy is a string of 20 decision variables:

$$S^1_1\text{-}N^1_1\text{-}S^1_2\text{-}N^1_2\text{-}S^1_3\text{-}N^1_3\text{-}S^1_4\text{-}N^1_4\text{ - }S^2_1\text{-}N^2_1\text{-}S^2_2\text{-}N^2_2\text{-}S^2_3\text{-}N^2_3\text{-}S^2_4\text{-}N^2_4\text{- }N^3_1\text{-}N^3_2\text{-}N^3_3\text{-}N^3_4$$

Variable S^t_r is in {0,1}. When applicant is ranked r^{th} among the others, if S^t_r=1, she sends at least 1 application in year t; otherwise if S^t_r=0 she does not send any applications in year t. (S^3_r=1 is automatically set, so it is not a decision variable.) N^t_r is in {1,2,3,4} and denotes the number of judges that she will send an application in year t when she is ranked r at year t[3] and S^t_r=1. If none of these judges are available, she sends an application to the best available judge.[4]

The bounded rationality feature of the strategy representations comes from one source. We do not model all information sets using these representations. For example, subjects observe who already got matched and left the market before each period and they also observe actual grades of applicants not only their current rankings. As we find in our results, the current representations model subject learning pretty well in the experiment even though they are bounded rational.

[2] R^t is the rank of least acceptable applicant among the available ones.
[3] If none of these best N^t_r judges are available, she only sends an application to the best available judge.
[4] To keep the information sets simple, in the computational simulations ties are broken arbitrarily in every period, so there are never two students with the same rank.

Appendix: Sensitivity Analysis

Simulation Parameters

In this section we report results of three sets of sensitivity tests for the artificial adaptive agent simulations. These tests aim to see how much the results achieved through simulations depend on the choice of genetic algorithm parameters.

In the first set of tests, we conduct comparative static exercises by changing one parameter at a time. In three tests of this first set, we change number of simulation markets from 500 to 5000, we change number of simulations from 20 to 100 and we change number of tournament games from 1000 to 10000 one by one. In each of the tests we measure mean welfare of applicants in all treatments, welfare difference between centralized-idealized and decentralized treatments, welfare difference between centralized-idealized and centralized-coerced treatments, welfare difference between decentralized and centralized-coerced treatments, and welfare difference between announcement and no-announcement treatments in each of the three market designs. We take the average of these in last 50 markets and report in Table 7 as well as the results for the original simulations reported earlier. We observe that average welfare across all treatments is almost the same in every exercise.

We observe that welfare in decentralized markets is catching up with the welfare in centralized-idealized markets in the 5000 generation treatment, although the latter is always higher. Moreover decentralized markets continue to raise more welfare than the centralized-coerced markets. The differences between announcement and no-announcement sessions are usually robust. In all cases announcements increase welfare slightly except the centralized-coerced treatment. In the original experiments for this treatment, announcement causes less welfare but this is not significant.

In the second set of tests, we conduct active nonlinear tests (ANTs) to obtain a multivariate sensitivity analysis. This is a technique found by Miller (1998). An ANT is a hill climbing optimization procedure, which tries to maximize some objective in the simulation by randomly searching in the parameter space. We use 100 iterations for the optimization procedure. Using this technique, the worst case scenarios for simulation models can be easily determined. The results show how much the results obtained in the original simulations depend on the choice of parameters and how much at worst results

will be distorted if different parameters are chosen. We form the search space of deviations from the original parameters as {-50%, -40%, -30%. -20%, -10%, 0%, +10%, +20%, +30%, +40%, +50%} for the parameters, which have real number values. These parameters are the ratio of selected best strategies for the next market under selection pressure, crossover probability, initial mutation probability, final mutation probability, strategy population. We define the search space of other parameters as selection pressure and no-selection pressure operator; random generation of initial strategies and forcing initial worker generation to submit applications in each period (i.e. using strategies similar to initial human subject strategies). We try to maximize and minimize the welfare measures used in the comparative static exercises. The results are displayed in Table 8.
We observe that the welfare differences between market types are very robust and the choice of parameters does not affect the fact that highest welfare is raised under centralized-idealized markets, followed by decentralized markets, followed by centralized-coerced markets. Mean welfare across all treatments is close to maximum under the original sessions. However the lack of selection under pressure accompanied by a combination of other parameters can decrease mean welfare substantially. This is due to substantial decrease in welfare for the centralized-coerced treatment with announcements.

In the third set of sets, we change the strategy representation used. Instead of using two digits to represent the action of an agent in each set (the first for deciding to do anything in the current period or not and the second one for deciding what action to take in case the agent decides to apply/hire in the current period) we use a single digit to represent agent actions: for a judge the range of actions is given by 0 to 4 where 0 denote not hiring anybody and k>0 denotes the threshold rank of an applicant to hire in the current period, for the applicant the range is analogously defined. As we see in Table 9, the ordering of welfare results are identical in the new simulations with that of the original simulations. However the magnitude of changes is different and as a big difference we observe that decentralized announcement treatment's relative well-being is slightly higher with respect to the no announcement treatment in the new set of simulations. Our strategy representation used in the original simulations has slightly better fit than the one considered here for the experimental data.

Sensitivity Analysis on Experimental Design Parameters

In this section, we conduct robustness analysis by changing experimental design parameters. With different experimental designs, we run additional simulations.

First, we impose two types of changes on payoff structure and on grade generation process for applicants. We choose new payoffs so that the maximum possible welfare of applicants is the same as the original experiments and they decrease the marginal utility of match quality for type j: in the original design this marginal utility is j, in subsequent comparative static exercises we have 3j/5 and 3j/7.[5] For different grade generation processes, we uniformly draw the grades of students from {0,1,…,5} and {0,1,…,10} instead of {0,1,2}. These decrease the probability of ties. The welfare measures discussed in the previous section are also calculated for these new exercises. The results are displayed in Table 10 for the last 50 markets of the simulations. We observe that with decreasing marginal utility of match quality, mean welfare of treatments increases, and particularly the welfare differences between treatments decrease. With decreasing probability of ties among student grades, the results are not substantially affected across different market designs. However announcements become slightly more effective and quicker in raising welfare.

Next, we impose a dramatic change in the design. In the original coerced-central treatment, the applicants can only apply in two periods to the available judges to get a right to be matched in the centralized period. In this new design, we create an application period preceding the centralized match period and succeeding the second period. In particular, before this new stage, all information about the applicant qualities becomes public information. The welfare implications of this design change in the coerced-central treatment are given in Table 11 and Figure 4. In Table 11, summary statistics about the average welfare in last 50 generations are given. We observe that the welfare in the new central-coerced treatment increases although still stays lower than the decentralized treatment. Moreover, the effect of announcements in the central-coerced treatment is reversed now: announcements no longer decrease welfare. Figure 4 shows average group welfare over time in all three treatments. One can trace the increase in the welfare of the

[5] In the original design, utility of quality j agent from a quality i match is ij, in the subsequent exercises it is 3(ij+5)/5 and 3(ij+10)/7 respectively.

central-coerced treatment in this new design by contrasting it with Figure 2b. We conclude that our findings about the signs of welfare differences within treatments are robust for this design change as well, although the magnitudes of the differences are not robust.

Table 7. Robustness Analysis for Simulations: Comparative Statics on Simulation Length, Simulation Number, and Tournament Length

Comparative Statics	Original Simulations	With 5000 generations	With 100 simulations	With 10000 games per tournament
Mean welfare of applicants in all treatments	**28.35 (0.0628)**[6]	28.77	28.38	28.56
Welfare diff. between central-idealized and decentral.	**0.87 (0.0495)**	0.26	0.78	0.69
Welfare diff. between central-idealized and central-coerced	**2.48 (0.0646)**	2.25	2.45	2.48
Welfare diff. between decentral. and central-coerced	**1.61 (0.0912)**	1.99	1.67	1.79
Welfare diff. between central-idealized A and NA	**0.16 (0.0541)**	0.07	0.17	0.11
Welfare diff. between decentral. A and NA	**0.25 (0.0969)**	0.14	0.27	0.28
Welfare diff. between central-coerced A and NA	**-0.12 (0.0603)**	0.17	0.01	0.07

Table 8. Robustness Analysis for Simulations: Multivariate Active Nonlinear Tests (ANTs) on other GA parameters

ANTs	Max.	Min.
Mean welfare of applicants in all treatments	28.81	23.23
Welfare diff. between central-idealized and decentral.	2.77	0.25
Welfare diff. between central-idealized and central-coerced	10.70	1.99
Welfare diff. between decentral. and central-coerced	7.93	0.98
Welfare diff. between central-idealized A and NA	0.37	-0.17
Welfare diff. between decentral. A and NA	0.25	-0.03
Welfare diff. between central-coerced A and NA	0.42	-6.39

[6] The numbers in parentheses are standard errors of the benchmark simulations.

Table 9. Robustness Analysis for Simulations: Comparative Statics on Strategy Representation

Comparative Statics	Original Simulations	Alternative strategy representation
Mean welfare of applicants in all treatments	**28.35 (0.0628)**	28.43
Welfare diff. between central-idealized and decentral.	**0.87 (0.0495)**	1.46
Welfare diff. between central-idealized and central-coerced	**2.48 (0.0646)**	2.20
Welfare diff. between decentral. and central-coerced	**1.61 (0.0912)**	0.73
Welfare diff. between central-idealized A and NA	**0.16 (0.0541)**	0.25
Welfare diff. between decentral. A and NA	**0.25 (0.0969)**	0.91
Welfare diff. between central-coerced A and NA	**-0.12 (0.0603)**	0.12

Table 10. Robustness Analysis for Experimental Design Through Simulations: Comparative Statics on Marginal Utility of Match Quality and Grade Generation of Students

Comparative Statics	Marginal Utility of Productivity for type j=3/5j	Marginal Utility of Productivity for type j=3/7j	Grades of applicants are uniformly drawn from {0,1,…,5}	Grades of applicants are uniformly drawn from {0,1,…,10}
Mean welfare of applicants in all treatments	29.01	29.29	28.41	28.40
Welfare diff. between central-idealized and decentral.	0.52	0.37	0.86	0.75
Welfare diff. between central-idealized and central-coerced	1.49	1.06	2.52	2.31
Welfare diff. between decentral. and central-coerced	0.97	0.69	1.66	1.55
Welfare diff. between central-idealized A and NA	0.09	0.07	0.12	0.17
Welfare diff. between decentral. A and NA	0.15	0.11	0.17	0.45
Welfare diff. between central-coerced A and NA	-0.07	-0.05	-0.07	0.09

Table 11. Robustness Analysis for Experimental Design Through Simulations: Effect of increasing application periods in the centralized-coerced treatment

Comparative Statics	Original Simulations	With 4 periods in coerced-central treatment
Mean welfare of applicants in all treatments	**28.35 (0.0628)**	28.80
Welfare diff. between central-idealized and decentral.	**0.87 (0.0495)**	0.87
Welfare diff. between central-idealized and central-coerced	**2.48 (0.0646)**	1.11
Welfare diff. between decentral. and central-coerced	**1.61 (0.0912)**	0.24
Welfare diff. between central-idealized A and NA	**0.16 (0.0541)**	0.16
Welfare diff. between decentral. A and NA	**0.25 (0.0969)**	0.25
Welfare diff. between central-coerced A and NA	**-0.12 (0.0603)**	0.19

Figure 4: Robustness Analysis for Experimental Design Through Simulations: Effect of increasing application periods in the centralized-coerced treatment on Average Group Welfare. Dashed lines represent announcement conditions.