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Math Abilities in Deaf and Hard of Hearing Children: The Role of Language in Developing Number Concepts

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Deaf and hard of hearing (DHH) children who are not exposed to fluent sign language from birth generally fall behind their hearing peers in mathematics. These disparities are pervasive and emerge as young as 3 years old and continue throughout adulthood. While these limitations have been well-documented, there has been little attempt to empirically explain why one consequence of deafness seems to reflect difficulties with numbers and mathematics. The purpose of this review is to describe the math abilities of DHH children while providing an explanation as to *why* we see this disparity. In particular, we review evidence suggesting that limited/reduced language access, particularly in the first few months of life, may play a role in delaying the acquisition of early number concepts and its potential interference when solving math problems. We also consider the potential role executive functions, specifically working memory, play in mathematical learning and how lower working memory capacity seen in some DHH children may impact early numerical learning and task performance. Finally, we propose future research aimed to explain why deafness is often accompanied by difficulties in numerical cognition while informing our broader understanding of the relationship between language and numerical concepts.

Keywords: deaf, hard of hearing, hearing loss, math abilities, numerical cognition

Language plays an important role in acquiring numerical concepts. For example, individuals whose native language lacks words to denote specific numerosities above two or three tend to struggle when mentally representing *exact* quantities above those values (e.g., Gordon, 2004, Pica et al., 2004; Spaepen et al., 2011). Other work indicates that the linguistic structure of language may facilitate the development of both nonverbal (e.g., Barner et al., 2007) and verbal (e.g., Le Corre et al., 2016; Slusser & Sarnecka, 2011) number concepts, while greater exposure to "number language" (talk about number) is associated with superior number knowledge in preschool (e.g., Klibanoff et al., 2006; Mix et al., 2012). Such findings highlight an important link between numerical abilities and linguistic input. However, because language acquisition is confounded with age and the development of other cognitive abilities that may also impact the development of numerical abilities, it is difficult to understand the exact role language plays in this relationship.

Understanding how deaf and hard of hearing (DHH) children learn about numerical concepts provides a unique opportunity to gain insight into the role language may play in numerical abilities. Evidence suggests that DHH children born to parents who are not already fluent in sign language, and consequently are not exposed to a complete language from birth (denoted from hereafter as DHH-wo to signify that this population is *without* language access from birth), have a unique progression of language development compared to their hearing peers (Geers et al., 2009) These children also show significant delays in the development of numerical concepts compared to their hearing peers (e.g., Kritzer, 2009; Leybaert & Van Cutsem, 2002; Pagliaro & Kritzer, 2010; Titus, 1995) *and* when compared to other DHH children with access to fluent sign language from birth (e.g., Hrastinski & Wilbur, 2016; Kritzer, 2009; Mousley & Kurz, 2015). These challenges in math achievement have been consistently demonstrated over the last several decades (e.g., Bull, 2008; Hine, 1970; Wood et al., 1986; Wollman, 1965), and are thought to primarily lie in the acquisition of *symbolic* number concepts such as counting, arithmetic, and fractions (e.g., Kritzer, 2009; Leybaert & Van Cutsem, 2002; Pagliaro & Kritzer, 2010; Titus, 1995). This is reflected in their underperformance on different math assessments compared to their age-matched hearing peers (e.g., Pagliaro & Kritzer, 2013; Traxler, 2000). Most notably, these disparities in performance are observed primarily in DHH-wo children, DHH *without* access to fluent language from birth. Deaf children born to Deaf parents who are fluent signers, and thus have access to a fluent sign language from birth (denoted as DHH-w to signify that this group is *with* fluent language access from birth) do not appear to display the same challenges with mathematics as those with language deprivation early in development (e.g., Hrastinski & Wilbur, 2016; Kritzer, 2009; Mousley & Kurz, 2015), as signed languages are complete, natural languages that consist of their own unique grammar and syntax (Stokoe et al., 1965). This distinction in math abilities between DHH-wo (without language access from birth) and DHH-w, highlights an important relationship between language access and acquiring numerical concepts.

In this article, we explore the source of the disparity in math abilities between DHH-wo children and their hearing peers. First,

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we note that these math delays appear unique to DHH children *not* exposed to fluent sign language from birth; that is, there is sparse—but fairly consistent—evidence that DHH-w children do not demonstrate these same delays (Hrastinski & Wilbur, 2016; Kritzer, 2009; Mousley & Kurz, 2015). Then, we propose the theory that limited language access, from early in development, underlies the math difficulties generally observed in DHH-wo populations. We also explore the possibility that working memory limitations may work—brought on by, or in conjunction with, limited language access—to delay the acquisition of numerical concepts in DHH-wo children. We then propose future directions that can test this theory, while also shedding light on the dependence of numerical development on language.

Language Environments of DHH Children

Deafness is uncommon; out of every 1,000 children born in the United States, only 2–3 children are identified with a permanent, detectable deafness. Notably, over 90% of these children are born to hearing parents (Quick Statistics about Hearing, 2016), and less than 5% of them are born to at least one parent fluent in American Sign Language (ASL; Mitchel & Karchmer, 2004). The vast majority of DHH children are not born to fluent signers, and thus are not exposed to a fluent language from birth. Although parents may begin to learn ASL after their child's deafness is identified, as with learning any new language, fluency takes time. During this time these children do not get the same foundation for early language development as hearing children or deaf children born to fluent signers (DHH-w).

Regardless of whether parents choose to learn sign language, at least 85% of parents choose to provide hearing aids and/or cochlear implants in order to access spoken language (Brown, 2006). Yet hearing aids and/or cochlear implants are not immediately fit at birth, thus there is a prolonged period in early infancy during which DHH-wo children are not exposed to fluent language. Moreover, regardless of how long it takes to get fitted with hearing technology, this technology does not equate to 100% language access. For example, even with hearing aids/cochlear implants, access to speech sounds can be limited (Baer et al., 2002; Turner, 2006), resulting in periods of auditory (hence spoken language) deprivation throughout development (Moore & Linthicum, 2007).

Therefore, most DHH-wo children generally experience limited exposure to a fluent first language in infancy or childhood. This means fewer opportunities to learn new words, creating greater difficulty to develop age-appropriate vocabularies and language skills, a fact that may have significant consequences for numerical cognition. In fact, on average, spoken language development in DHH-wo children is different compared to hearing peers (e.g., Geers et al., 2009) including delayed singular–plural language acquisition (Stelmachowicz et al., 2002), verbal reasoning, grammar, vocabulary (e.g., Edwards et al., 2011), and reading comprehension (e.g., Traxler, 2000; Wake et al., 2004) as well as lower vocabulary skills (Carrigan & Coppola, 2020; Convertino et al., 2009; Lund, 2016; Schorr et al., 2008; Wake et al., 2004). This general reduced access to language, coupled with language delays, makes DHH-wo children a unique population that could provide additional insight into the role of language in forming numerical concepts. In this article, we first review the research on math abilities in DHH-w and DHH-wo children and then propose two theories to explain the observed lags in DHH-wo that could inform our understanding of the language-number dynamic while pointing the way for new research in this area.

The Importance of Access to Fluent Sign Language From Birth

Before delving into the literature on math abilities in DHH children, it must first be acknowledged that this is not a homogeneous population. The extent of a child's deafness, age of diagnosis, mode of communication, and age of exposure to fluent language, not to mention differences in schooling—all factors which may contribute to a child's math outcome—vary within this population. For the purposes of this review, it is important to acknowledge the distinction between research that includes DHH-w children and those that include only DHH-wo. As noted above, only a small proportion (~5%) of DHH children are born to a parent fluent in a sign language such as ASL. As a result, there are only a handful of studies characterizing the math abilities of DHH-w children born to fluent signers (but see Ansell and Pagliaro, 2006; Pagliaro & Ansell, 2012). Notably, consistent with our theory that language access may be an important contributor to numerical development, findings from these studies generally do not reveal any evidence of math delays in DHH-w children (Hrastinski & Wilbur, 2016; Kritzer, 2009; Mousley & Kurz, 2015). Moreover, other work finds positive correlation between ASL abilities and math performance (Ansell & Pagliaro, 2006; Pagliaro & Ansell, 2012), again suggesting that language may play an important role in numerical development.

However, the vast majority of DHH children are *not* born to fluent signers and consequently experience some degree of language deprivation early in development. Thus, most of the research in this field, and therefore this review, has included samples of exclusively DHH-wo children *or* may have collapsed data analyses across both populations (DHH-w and DHH-wo).[1] We do our best to note when native signers are included in the sample and report findings from such participants that are different from the sample as a whole.

Mathematics Abilities in DHH-wo Children

While a consensus exists that DHH-wo children, on average, underperform on formal math assessments, there is little work attempting to empirically explain *why* hearing loss is so strongly associated with difficulties in mathematics. Understanding the source of math difficulties in this population may shed light on the relation between language and mathematical abilities. To understand the scope of the numerical difficulties in DHH-wo children, it is important to distinguish between symbolic and nonsymbolic number knowledge. Symbolic number knowledge is more commonly referred to as any general math ability involving numerical symbols—whether they be number words or Arabic numerals (1, 2, 3, . . . , etc.)—such as counting, standardized tests, geometry, fractions, etc. Nonsymbolic number skills, on the other hand, involve our ability to mentally represent numerical quantities,

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[1] Throughout, when information about the sample in a particular study was not specified in the study, we will refer to the sample as DHH more generally. Notably, given that DHH-w children are less frequent in the population, studies including both populations in their sample or those not specifying the population are likely to include a majority of DHH-wo children.

Table 1
Prior Studies of Symbolic Number Abilities in DHH Children

Authors	Measures	*N* d/ Deaf	*N* hearing	Ages	At least one parent with hearing loss	Language	Language measure included	Findings[a]
Arfé et al. (2011)	Count to 20; digit comparison	10	99	4–6 years	0	Italian	No	No difference between groups
Hine (1970)	Schonnel's Essential Arithmetic Test	104	0	7–16 years	Nr	Nr	Nr	Deaf and Hard of Hearing (DHH) below hearing norms
Hrastinski and Wilbur (2016)	Northwest Evaluation Association Measures of Academic Progress	85	0	6th–11th grade	47	American Sign Language (ASL), English	Yes	Higher ASL proficiency correlates with higher math scores
Kritzer (2009)	Test of Early Mathematics Ability - 3	29	0	4–6 years	Nr	ASL, English	No	25 children scored below average compared to hearing norms
Leybaert and Van Cutsem (2002)	Count as high as you can, Set creation	21	28	3–6 years	3	Belgian French Sign Language/signed French/Signed Coded Complete French (19), Oral (2)	No	Same as control, but older
Pagliaro and Kritzer (2013)	Count as high as you can, set creation, geometry, puzzles, algebra	20	0	3–5 years	Nr	English	No	DHH struggled with most early number concepts
Pixner et al. (2014)	Subtraction, multiplication, math achievement	45	49	3rd–5th grade	Nr	Signed, Spoken	No	DHH showed general number impairments
Qi and Mitchell (2012)	Analysis of SAT data over 30 years	Nr	0	8–18 years	Nr	Nr	Yes	Trends show improvement in math scores, but still underperforming with oldest participants scoring at the 6–8 grade levels
Santos et al. (2021)	Give-N	14	45	3–6 years	Nr	English	Yes	DHH underperformed compared to hearing peers
Rodríguez-Santos et al. (2014)	Number comparison (1–9)	10	10	8–9 years	Nr	Spanish	Yes	Similar performance between groups, DHH slower to respond
Titus (1995)	Fractions	21	26	10–16 years	Nr	Nr	No	DHH underperformed compared to hearing peers
Wollman (1965)	Manchester Mechanical Arithmetic Test and additional math problems	Nr	162	14–16 years	Nr	Nr	No	DHH underperformed compared to hearing peers
Wood et al. (1984)	Vernon and Miller Arithmetic test	414	465	8–11 years	Nr	Nr	No	DHH 3–4 years below hearing children

Note. Nr = Not reported; DHH = Deaf and Hard of Hearing; ASL = American Sign Language; SAT = Stanford Achievement Test.
[a] Caution must be observed when interpreting the findings as some details about the participants' language experiences were not disclosed.

typically without language or symbols (Cordes et al., 2001; Feigenson et al., 2004). This is sometimes referred to as our "intuitive number sense" (e.g., Feigenson et al., 2004, 2013; Halberda et al., 2008; Szkudlarek & Brannon, 2017), and is typically assessed by our ability to rapidly indicate which of two arrays has the greater number of dots without counting. Much of the research on the numerical abilities of DHH-wo children focuses on verbal/symbolic math, revealing that these children largely struggle with formal mathematics (See Tables 1 and 2).

Symbolic Number Knowledge

Rote Counting

The evidence shows DHH-wo children fall behind hearing children in both general math abilities and more abstract concepts, as early as 3 years of age (e.g., Pagliaro & Kritzer, 2013). Counting abilities have been marked as an important predictor of math abilities later in life (Geary, 2011). Children with difficulties or delays in counting competence continue to display math learning difficulties later in life (Jordan & Levine, 2009). As such, one potential contributor to delayed numerical competence could be that the acquisition of verbal counting—the earliest indication of a child's understanding of symbolic number—is delayed in DHH-wo children.

Although counting may seem like a simple procedure, the protracted period of development during which children acquire verbal counting is evidence for its complexity. Much like learning the alphabet before learning to read, children can recite number words in a rote order long before they acquire a real understanding of the meaning of those words (e.g., Wynn, 1992). However, studies suggest DHH-wo children fall behind their hearing peers in rote counting, providing evidence for delays in the earliest demonstration of number abilities. Leybaert and van Cutsem (2002) found that 3–6-year-old signing deaf children (communicating nonverbally,

Table 2
Prior Studies of Nonsymbolic Number Abilities in DHH Children

Authors	Measures	*N* d/ Deaf	*N* hearing	Ages	At least one parent with hearing loss	Language	Language Measure Included	Findings[a]
Arfé et al. (2011)	Dot comparison	10	99	4–6 years	0	Italian	No	Deaf and Hard of Hearing (DHH) outperformed hearing controls
Bull et al. (2018)	Approximate Number System (ANS)	75	75	5–12 years	Nr	Signed, Spoken, Mix	No	DHH lower ANS acuity
Santos et al. (2021)	Approximate Number System	14	45	3–6 years	Nr	English	Yes	DHH lower ANS acuity
Rodríguez-Santos et al. (2014)	Dot and finger comparison (1–9)	10	10	8–9 years	Nr	Spanish	Yes	Similar performance between groups, DHH slower to respond
Zarfaty et al. (2004)	Nonsymbolic , small & Large	10	10	3–4 years	Nr	Spoken	No	DHH outperformed hearing controls in temporal conditions, same in spatial conditions

Note. Nr = Not reported; DHH = Deaf and Hard of Hearing; ANS = Approximate Number System.
[a] Caution must be observed when interpreting the findings as some details about the participants' language experiences were not disclosed.

with Belgian French Sign Language; BFSL) lag at least 2 years behind age-matched hearing children when demonstrating knowledge of a count list (count as high as you can). Notably, only 3 (out of 21) of the children had deaf parents, suggesting that most of these children were not exposed to fluent language from birth.

Pagliaro and Kritzer (2013) completed a comprehensive assessment of the numerical abilities of 3–5 year-old oral DHH-wo children (all but one communicated without a sign language). To assess proficiency in counting, geometry, measurement, problem-solving, reasoning and algebra, researchers compared performance of DHH-wo children to mathematical development standards compiled from the Principles and Standards for School Mathematics (National Council of Teachers of Mathematics [NCTM], 2000), PBS Child Development Tracker, and proposed math learning trajectories (e.g., Sarama & Clements, 2009). Similar to Leybaert and van Cutsem (2002), researchers found the count lists of DHH-wo children to be lower than standards developed from hearing peers—nine of the 20 children could not count to 5, with only two of the 20 children counting at or above age-level.

Number Knowledge

After learning to recite the count list, evidence suggests that hearing children begin to acquire the meaning of each number word in succession, beginning with understanding that "one" means one and only one, following in time by learning "two," "three," etc. Learning the meaning of the number words and understanding cardinality—that is, understanding that the last word recited in a count represents the cardinality of the set—however, are considered to be more critical factors in early counting. Typically, number word knowledge is assessed through the Give-a-Number procedure (Give-N; Le Corre & Carey, 2007; Wynn, 1990). In this process, children are asked to give N items to the experimenter, beginning with one. If they are successful, they are asked to give $N + 1$ items, and when they fail to give the correct number of items, they are asked to give $N - 1$ items. This titration procedure continues until the child is successful at N twice, and fail at $N + 1$ twice (and are then classified as an N-knower, e.g., 3-knower) or when the child correctly gives up to six items twice (and are then considered to be a *cardinal principle knower)*. Children who have mastered cardinality (CP-knowers) are expected to know the meaning of the number words in their count list and that the last word in the count list refers to the amount of objects in the group. Although children begin to recite counting words around age 2, it takes an additional 1–3 years for hearing children advance to the CP-knower stage (Wynn, 1990). This of course is variable, likely the result of differences in linguistic structure and number language experience (e.g., Sarnecka et al., 2007).

Being a cardinal principle knower helps children expand their understanding of quantity and develop more sophisticated knowledge of how numbers are related, fundamentally providing important groundwork for math learning (Baroody et al., 2006). The age at which children acquire cardinality is an important predictor of many of their later math abilities Nguyen et al., 2016. Although DHH-wo children seem to fall behind hearing peers in rote counting, the developmental time-course of the acquisition of cardinality in DHH-wo children is less clear. Two studies have found ambiguous evidence for whether DHH-wo children can create sets of a given size with the same competence as hearing peers (as required per the Give-N task e.g., Leybaert & Van Cutsem, 2002; Pagliaro & Kritzer, 2013).

Pagliaro and Kritzer (2013) used a different counting task to assess cardinal knowledge. They asked 4–5−year-old DHH-wo children ($n = 16$) to count a display of five objects, after which the researcher covered the items. Children were then asked to report how many objects were covered. Notably, only seven of 16 children were able to complete this simple counting task successfully. Critically, because most children did not perform well on the task and that this assessment greatly differs from the widely accepted standard assessment of cardinal understanding (the Give-N task), there is reason to question whether findings truly suggest cardinal competence in DHH-wo children.

Leybaert and Van Cutsem (2002), on the other hand, used a task more similar to the Give-N task. Across 13 trials, they asked

4–6−year-old DHH-wo children and 3–5−year-old hearing children to give a number of small objects to a frog puppet (ranging from 3 to 14 objects). All participants started with the small set and were given subsequent set sizes if they were successful on at least one of the trials in the previous set. At first glance, it appears that the deaf children performed comparably to the hearing group. However, researchers matched children by *grade* not age (hence leading to different average age ranges between the two groups), suggesting that while they performed at grade level, they were still at least a year behind in chronological age.

One recent study systematically compared Give-N performance (the gold standard of number knowledge) in age-matched DHH-wo children and hearing preschoolers. Contrary to previous work (e.g., Leybaert and Van Cutsem, 2002; Pagliaro & Kritzer, 2013), this study found 3–6-year-old DHH-wo children demonstrated significant delays in their number knowledge development compared to age-matched hearing peers. Results of this study suggest that early language deprivation may negatively impact the development of basic number knowledge in DHH-wo (Santos et al., 2021).

In sum, whether DHH children acquire number knowledge at the same rate (based upon their chronological age) as their hearing peers is currently ambiguous. A thorough exploration of the acquisition of number knowledge and the cardinal principle in DHH-wo children should be considered to provide a clear description of this process in children with limited access to language and to understand whether delays in cardinal knowledge may contribute to later math performance deficits.

Math Assessments

Significant research has focused on broad math abilities, as captured by standardized math assessments. Research reveals a clear gap in math performance between DHH-wo children and their hearing peers (e.g., Kritzer, 2009; Pagliaro & Kritzer, 2013; Traxler, 2000). Kritzer (2009) administered the Test of Early Math Ability-3 (TEMA-3) to 29 DHH children, ages 4–6 years, whose primary mode of communication was English or ASL. Most children (62%) had "good" or "fluent" exposure to ASL, and 60% of children had at least one deaf parent (suggesting majority may have been DHH-w). When compared to the norms of hearing children, DHH scores on the TEMA-3 showed evidence of delays in participants as young as 3 years old (Kritzer, 2009). Most notably, the DHH children with deaf parents scored higher than the DHH children born to hearing parents.

Qi and Mitchell (2012) demonstrated that these disparities in math achievement observed in childhood are pervasive and stretch into adulthood. In their analysis of over 30 years of math achievement data from the Stanford Achievement Test (SAT), researchers observed a trend of DHH children (not specified whether DHH-w or DHH-wo) consistently underperforming on the math achievement subscales between 8 and 18 years of age. The oldest participants earned scores that were equivalent to 4th—7th grade, far below what is expected at that age. These disparities have been apparent for almost 50 years (e.g., Hine, 1970; Traxler, 2000; Wood et al., 1984).

Even informal assessments of math skills reveal challenges for DHH-wo children. Pagliaro and Kritzer (2013) also explored preschool-aged (3–5 years) oral DHH-wo children's knowledge of number (e.g., symbol recognition and estimation), geometry (shapes and puzzles), measurement (time, length, and order), problem-solving, creating patterns/reasoning, and word problems. Consistent with previous reports, they found that participants were significantly challenged in every area of assessment and did not demonstrate age-appropriate number skills for most of the tasks.

Fractions

This limited early numerical foundation seen in preschool is only compounded in the learning of more advanced mathematics, such that DHH-wo children fall behind their hearing peers in more abstract mathematics, such as fraction understanding. While fractions are generally challenging for many children, they appear to be considerably more challenging for DHH children. Titus (1995) explored fraction understanding with 11–12-year-old and 13–16-year-old DHH adolescents and their hearing peers.[2] As expected, younger children performed worse than older children in the hearing group, revealing age-related changes in fraction knowledge in the hearing group. The DHH children, however, did not show this pattern: older DHH children performed comparably to their younger DHH peers, suggesting that DHH children were not acquiring more sophisticated fraction knowledge between the ages of 11–16 years. Moreover, even the youngest group of hearing children outperformed the oldest group of DHH children, again revealing that DHH children fall behind their hearing peers in fraction knowledge. Mousley and Kurz (2015) corroborated this finding, revealing difficulty with fraction knowledge in 14 deaf students[3] (8–16 years old), 11 of whom relied solely on ASL to communicate. Overall, participants showed difficulty with fraction magnitude, order, and equivalence with a range of 17%–83% correct on the written fraction assessment. These studies reflect challenges in the fraction domain, however, more research needs to be done to explore the root of these challenges in fraction learning to rule out issues in teaching as a possible explanation.

Nonsymbolic Number Abilities

Nonsymbolic number abilities refer to a person's ability to mentally represent quantities without the use of language (i.e., number words) or other symbols. It is widely acknowledged that humans and nonhuman animals have access to an approximate, nonverbal/nonsymbolic system for tracking and representing number, termed the Approximate Number System (ANS; Feigenson et al., 2004; Szkudlarek and Brannon, 2017). It has been suggested that ANS representations provide the foundation for the acquisition of the meanings of symbolic number (i.e., count words, Arabic numerals; Gallistel & Gelman, 1992), and as such, much work has focused on exploring the link between nonsymbolic numerical acuity (ANS acuity) and math achievement. Research with hearing children and adults has established a strong relationship with ANS acuity and formal math abilities. Halberda et al. (2008) found that ANS acuity—as determined by the ease (speed and accuracy) with which an individual determines which of two arrays of dots is more numerous—in ninth grade was positively correlated to previous scores on standardized math achievement tests as early as Kindergarten. This relationship appears in preschoolers as well (Libertus et al., 2011). Even ANS abilities during infancy correlate with math

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[2] The home environment and primary communication mode of the DHH sample was not described in the article.

[3] It is unclear how many of this sample was DHH-wo, however, at least 5 of the 16 participants relied on ASL at home.

achievement (TEMA-3) 3 years later (Starr et al., 2013). Furthermore, research indicates that mathematical disabilities may stem from impaired ANS acuity (Mazzocco et al., 2011). If so, then understanding whether nonsymbolic numerical abilities are affected in DHH children is important for understanding the potential source of these mathematical difficulties. That is, do DHH children have less precise abilities to track number nonsymbolically? Do these acuity differences emerge prior to the acquisition of number words/cardinality?

While there have been relatively few studies examining nonsymbolic number abilities in DHH children, these studies have provided conflicting evidence. Zarfaty et al. (2004) examined the ability to mentally represent small sets in 2.5–4.5-year-old children ($n = 20$; 10 DHH, 10 hearing). The DHH children used spoken language at home and in school (and thus were likely DHH-wo). Participants witnessed a puppet place 2, 3, or 4 bricks into a box and were asked to place the *same* number of bricks into another box. When the blocks were placed sequentially (i.e., one at a time) into the box, there was no difference in performance between the two groups in reproducing the same set size. However, when the blocks were presented simultaneously (i.e., all at once) into the box, the DHH group reproduced the correct number of bricks more often than the hearing children suggesting that DHH children may have actually had a better ability to track number nonsymbolically, at least when presented simultaneously.

Other studies have had similar findings. Arfé et al. (2011) asked 4–6-year-old Italian-speaking DHH-wo children with cochlear implants ($n = 10$) and age-matched hearing peers ($n = 99$) to identify which card with different numbers of dots (1–9) had the greater amount. Again, DHH-wo children outperformed hearing children in this nonsymbolic number comparison. Moreover, in another study, 8–9-year-old Spanish speaking DHH-wo children ($n = 10$) showed comparable performance to Spanish speaking hearing children ($n = 10$) when asked to compare and identify the larger pairs of nonsymbolic arrays of dots or fingers (set sizes of 1–9) that were briefly displayed (Rodríguez-Santos et al., 2014). Together, these studies suggest that nonsymbolic number abilities, at least for small sets (1–9 items), may be unaffected by reduced language input.

However, other works involving larger set sizes (i.e., not including sets with 4 or fewer items) have revealed significant delays in nonsymbolic numerical processing in DHH children (Bull et al., 2018; Santos et al., 2021). Compared to age-matched hearing peers, 5–12-year-old English-speaking DHH children[4] ($n = 75$) displayed worse performance when asked to discriminate between pairs of arrays containing anywhere from 5 to 35 items briefly displayed on a computer screen. These differences persisted after controlling for inhibition (Bull et al., 2018). Moreover, another work (Santos et al., 2021) has found delays in nonsymbolic number abilities in 3–5-year-old English-speaking DHH-wo children ($n = 14$) compared to age-matched hearing peers ($n = 45$) when children were asked to judge which of two arrays had a larger number of dots (Panamath task; Halberda & Feigenson, 2008). Notably, they found that differences in the amount of time (in months) the DHH-wo children had access to fluent language through cochlear implants or hearing aids (their "hearing age") fully accounted for group differences, providing strong support that language access plays an important role in nonsymbolic number development. Together, results suggest that nonsymbolic numerical abilities (at least for sets larger than 4) may be delayed in DHH children due to limited language access early in development.

In sum, there is evidence that the tracking of small sets (1–4 items) may not be affected in DHH-wo children, however, nonsymbolic number abilities for sets over 5 appear to be delayed in this population. However, evidence is clear that DHH-wo children generally demonstrate significant delays in nearly all aspects of symbolic mathematics. These delays appear pervasive, ranging from delays in rote counting (Leybaert and Van Cutsem, 2002), numeral identification (Kritzer, 2009; Pagliaro & Kritzer, 2013), problem-solving, and pattern recognition (Pagliaro & Kritzer, 2013) to more advanced fraction understanding (Mousley & Kurz, 2015; Titus, 1995). Differences between DHH-wo children and their age-matched hearing peers have been demonstrated as early as 3 years of age (Pagliaro & Kritzer, 2013), and have been found to continue into adulthood (Traxler, 2000). Given the importance of mathematics for academic, health, and financial outcomes in life (e.g., Gerardi et al., 2013; Eyler et al., 2017), it is important to understand the source of these deficits in order to provide a means for targeting these delays in this population.

So *Why* Do DHH-wo Children Struggle With Math?

It is unlikely that deafness itself is accompanied by innate difficulties in learning numerical concepts. As others have put it, deafness is merely a risk factor for difficulties with mathematics (Nunes & Moreno, 1998, 2004). Are these mathematical delays driven by language, or another factor, in early childhood? Uncovering the source of these delays will help to inform our understanding of the relation between language and numerical development across both DHH and hearing populations. Here we explore two potential explanations of the observed difficulties in mathematical abilities focusing on *reduced language access* and possible *differences in domain-general cognitive processing* in DHH-wo children.

Reduced Language Access and Abilities

It is hard to dispute that language is an important part of learning and processing numbers. It has been argued that the language we speak determines when we acquire basic numerical concepts. For example, Le Corre et al. (2016) found that native English-speaking infants, who acquire a singular/plural distinction in language, demonstrate an understanding of the number "one" *before* infants whose native language does not seamlessly differentiate between one and more than one object in the language (Mandarin), suggesting that grammatical markers may scaffold numerical acquisition. Barner et al. (2007) found that young children's abilities to track as many as four objects (and reliably discriminate it from a single object) emerge at the same time parents report that their child begins to use plural markers in language ("-s"), again suggesting that grammatical markers in language may support numerical development. In other work, it has been reported that both receptive and expressive vocabulary knowledge are related to early number word knowledge (Negen & Sarnecka, 2012) and hearing children diagnosed with Specific Language Impairment have reported difficulties with numerical concepts (Durkin et al., 2013). Thus, significant evidence ties language abilities to numerical concept acquisition.

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[4] No reference to early language experience.

DHH-wo children who experience limited language access may be at a disadvantage when it comes to learning numerical concepts as a result of generally lower language abilities overall and/or likely reduced access to *numerical* language, in particular.

The limited language access DHH-wo children endure as a product of their environment likely also results in less exposure to *numerical* language. Number can show up in everyday language in a variety of ways, for example, through number words (one, two, three, . . .), counting, comparison of quantities (“Sam has more toys than Joey”), references to cardinality (“there are 3 dogs”), time (“in 5 min”), or individuation (“you can have one cookie”). Work with hearing children has found that the more number language toddlers are exposed to, the more advanced their number knowledge is in preschool (Gunderson & Levine, 2011; Klibanoff et al., 2006; Levine et al., 2010; Mix et al., 2012). Levine et al. (2010) observed families over a period of 16 months, from the time children were between 14 and 30 months of age, and then assessed children’s numerical abilities later in preschool. Results revealed that the amount of number talk children heard when they were 1.5–2.5 years old, predicted a child’s number knowledge 1.5 years later. In fact, the *type* of number language the parent employed was actually important, specifically, parental counting and labeling of *large sets* (4–10 items), but not of small sets (1–3 items), of objects was predictive of the toddlers’ later number knowledge (Gunderson & Levine, 2011). In a similar study performed with preschool teachers, researchers found that the amount of math talk in the classroom was significantly correlated with gains in number knowledge preschoolers demonstrated across the school year (Klibanoff et al., 2006). Moreover, Mix and colleagues found the relation between exposure to number talk and number knowledge to be causal, such that children learn the meaning of the count words more quickly when the cardinality of sets are consistently labeled before counting, than if the set is not labeled (Mix et al., 2012).

Purpura et al. (2017) provide direct evidence for the importance of math-specific language in the development of early number abilities of preschool children. Researchers performed a math-specific language intervention with 3–5−year-old children in Head Start classrooms. This intervention resulted in significant improvements in not only math-specific language abilities, but in performance on math tasks as well. This evidence for a causal relationship between number language input and math abilities supports the contention that access to number language plays a key role in the development of numerical concepts.

In sum, evidence from hearing populations suggests that both general language abilities, and access to *numerical* language, in particular, are critical components to early number learning and more sophisticated numerical concepts. Given that DHH-wo children experience reduced language access early in development, and thus likely have reduced numerical input, then it is possible that this would impede the acquisition of basic numerical concepts. Research should explore whether differences in access to numerical language exist between children who are, and are not, deaf.

Language and Math in DHH-wo

Compared to hearing peers, DHH-wo children experience reduced access to language that likely explains reported delays in general language abilities in this population (Tomblin et al., 2015). If linguistic structures promote the acquisition of basic numerical concepts in hearing children (e.g., Barner et al., 2007; Le Corre et al., 2016), then DHH-wo children who receive limited linguistic input may be at a disadvantage. Early in development, DHH-wo children have delayed general language abilities compared to both DHH-w children and their hearing peers, which may be the root cause of their math difficulties.

There is the potential for these language difficulties to transfer to difficulties working with math problems. For example, it is understood that some mathematical operations rely more on language—such as subtraction with borrowing (LeFevre et al., 2010) or number line estimation—than other types of math, such as subtraction without borrowing. These problems require processing place-value information, which typically invokes language-dependent strategies during problem-solving (LeFevre et al., 2010). In fact, when comparing performance on subtraction problems with and without borrowing, DHH-wo children do not do as well as hearing peers when borrowing was required (i.e., when the solution is dependent upon language). On the other hand, subtraction problems where the borrowing strategy is *not* needed are not as challenging for DHH-wo children, evident by similar performance to hearing peers on these problems (Pixner et al., 2014). As such, general language delays may continue to interfere with math performance well into middle childhood as children acquire these more sophisticated arithmetic procedures. Moreover, in line with this hypothesis, DHH children with lower reading scores on the American College Test (ACT) and the Stanford Achievement Test (SAT; Holt et al., 1997; Traxler, 2000) present lower scores on math problem-solving. More simply put, if a child cannot understand the math problem, they cannot solve it (Serrano Pau, 1995).

These challenges may also reflect differences in mathematics instruction. Other work suggests DHH children may successfully solve word problems when presented in ASL, though they tend to solve them using different strategies than those of hearing children (Pagliaro & Ansell, 2012). However, notably, DHH-wo children show delays in numerical concepts well before children are expected to read (e.g., Pagliaro & Kritzer, 2013), making reading difficulties or instructional differences only one possible piece to the puzzle, but likely not the sole source of DHH-wo children’s math difficulties.

While there is the possibility that general language delays may drive the reported math delays in DHH-wo children, we still do not know how much general math learning is language dependent. Despite the reported links between general vocabulary and number knowledge in preschoolers (e.g., Negen & Sarnecka, 2012) and examples of hearing children with language impairments underperforming on math assessments (Durkin et al., 2013), it may be more complicated than this. However, a recent study (Slusser et al., 2019) showed that general vocabulary is linked to number word knowledge which in turn predicted math abilities in hearing children. This supports our contention that the lower language abilities experienced by DHH-wo children early in development may impede the acquisition of early numerical concepts and thus, future achievement in mathematics.

Alternatively, reduced language access, and consequently reduced number language access, may impede opportunities for incidental learning of numerical concepts. The importance of number language experience on numerical development in hearing children has been well established (e.g., Klibanoff et al., 2006; Purpura et al., 2017). It is possible that the mere lack of exposure to number language early in development and informal mathematical thinking may slow the acquisition of foundational number concepts such as counting and cardinality, leading to continual delays in math achievement. Many researchers have suggested that this may be one possible explanation for the lags we see in math achievement in DHH-wo children (e.g., Gregory, 1998; Kritzer, 2009; Nunes, 2004).

Nonsymbolic Representational Limitations

The potential for reduced numerical input leads to another potential source of difficulty: Limited numerical language input may impede the development of nonsymbolic numerical abilities (i.e., ANS). There is some evidence to think that ANS abilities are shaped by numerical input and/or the acquisition of formal math abilities. For example, research indicates that numerical language, in particular, learning to count, coincides with a marked refinement of the acuity of this system (Shusterman et al., 2016). In this longitudinal study, preschool children's performance on ANS tasks improved considerably at the same time that they first demonstrated knowledge of the cardinal principle (i.e., mastered the counting routine). It is thought that cardinality is related to changes in the way number is conceptualized which may be reflected in performance on nonsymbolic tasks (Shusterman et al., 2016). More convincingly, Mussolin et al. (2014) specifically explored the direction of this relationship by examining large number discrimination (ANS acuity) and symbolic number knowledge (cardinality) with 3–4-year-old children. Children were tested at two time points, 7 months apart. Their analysis exposed a positive predictive relationship for their symbolic number knowledge at time one and their ANS acuity at time two. Other work reveals that adults with at least 1 year or more of formal schooling do better on ANS tasks than adults in the same culture that do not have formal education (Pica et al., 2004), supporting the view that number language experience is related to nonsymbolic number abilities.

Thus, if exposure to numerical language shapes *nonverbal, nonsymbolic* numerical abilities, then children with limited language access may not develop ANS acuity similar to their hearing peers. This, in turn, could result in a feedback loop where delays in symbolic number knowledge lead to slower development of ANS acuity, which in turn, would make it more difficult for children to learn numerical symbols. Consequently, creating an inefficient foundation from which to build formal numerical knowledge.

Nonsymbolic Numeric Processing in DHH Children

Unfortunately, not much is known about the development of the ANS in DHH children. In the only study of its kind, Bull et al. (2018) explored the relationship between ANS acuity, math, and domain-general abilities such as working memory, short-term memory, and inhibition in 5–12- year-old DHH children. Researchers found lower ANS acuity in DHH children, even after controlling for working memory and inhibition. Further, differences in math abilities disappeared after controlling for ANS acuity supporting the idea that poorer math abilities in DHH children may be connected to limitations with nonsymbolic representation.

Whether or not the ANS contributes to math abilities is still up for debate and the role of number language on the efficiency of the ANS has not yet been established. However, recent research has shown that the relationship between ANS and math ability is mediated by number knowledge (Slusser et al., 2019). So, poorer ANS acuity may be a part of the explanation for differences between math abilities in DHH-wo children and their hearing peers. It is difficult to know whether this theory could explain delays in math abilities in DHH-wo children, however, the potential for limited number language experience impeding the development of both symbolic and nonsymbolic number concepts is high. More research must be done to determine whether or not DHH-wo (and even DHH-w!) children experience less number language in order to consider this possibility.

Differences in Domain-General Cognitive Processing

Domain-general cognitive processing, specifically executive functions (EF), have been linked to math learning (e.g., Geary, 1995, 2007) and performance on math assessments (Cragg & Gilmore, 2014). These "general purpose control mechanisms" (Miyake et al., 2000, p. 50) govern our cognitive processing through three central mental processes: (a) *inhibition*, the ability to inhibit the dominant or previously learned response; (b) *flexibility*, switching to and from different tasks; and (c) *working memory*, our cognitive system that permits attention to and processing of multiple sources of relevant information in the environment (Baddeley & Hitch, 1974). While inhibition and cognitive flexibility both play roles in math abilities, working memory seems to be a key component within EF that is implicated in mathematical reasoning (see DeStefano & LeFevre, 2004 for a review).

Working memory allows us to briefly keep information in mind while attending to other sources of information in the environment. Two short-term storage systems, the phonological loop and the visuospatial sketchpad, are responsible for the short-term store of auditory/verbal and visual/spatial information, respectively. The "working" component, the central executive, is the control center that manages information within the two storage systems. Given the amount of mental processing of arithmetic involved in solving math problems, like following procedural order and keeping relevant numerical information in mind, it is not surprising that working memory is important for math, particularly during early math learning. Geary (1995) posits that the ability to limit external distractions from the environment and organize information is critical for *learning* mathematics, particularly the meaning of number words and their relationship to each other. As such, learning number word meanings requires substantial mental effort and requires attentional control to map the word onto its corresponding quantity.

There is extensive literature linking children with weaker working memory abilities to lower math skills (e.g., Bull, 2008), particularly those with a learning disability in mathematics (Geary et al., 2004). This research seems to identify the central executive as a key working memory component where deficiencies may be responsible for limited learning. Given the evidence that poorer working memory accompanies lower math abilities, it is essential that we explore the possibility that limited working memory skills, particularly within the central executive, may be responsible for delayed math learning in some DHH children.

Executive Functioning in DHH Children

There is research demonstrating weaker working memory abilities in DHH children (e.g., Bull et al. (2008); Monroy et al., 2019) compared to hearing peers. This delay may emerge as early as infancy, with DHH-wo 7–22 -month-old infants (in nonsigning households) displaying slower visual habituation rates compared to age-matched hearing controls and moreover, that visual habituation rates were positively correlated with spoken language abilities in DHH-wo infants (Monroy et al., 2019). This is important because visual habituation in hearing infants has been linked to later cognitive abilities, specifically executive functioning (Cuevas & Bell, 2014), and implies that limited language input very early in development may influence the foundation of basic domain-general cognition. If so, it is possible that weaker executive functioning abilities in DHH-wo children, potentially brought on by limited language access and/or abilities, interferes with mathematical learning and performance on numerical tasks. This is supported by research showing general cognitive abilities can explain math achievement in DHH children[5] (Chen & Wang, 2020). However, it is important to note that while this may be one piece of the puzzle, this is not likely to fully explain the disparity in math abilities between DHH-wo children and their hearing peers. Previous research with DHH children has controlled for working memory and still found delays on performance on nonsymbolic number tasks (Bull et al., 2018), but there are no other known studies that attempt to isolate working memory influence on math abilities in DHH children. Because of this, the role EF plays in learning and performance on formal math assessments and math learning in DHH children must still be considered.

What's Next?

Research consistently reveals that DHH-wo children underperform compared to their hearing peers on math assessments. These delays begin as early as 3 years of age and extend into adulthood. Unfortunately, the existing literature makes it difficult to determine exactly *why* DHH-wo children underperform in math, but we can speculate reasons and formulate theories to explore. We are left to wonder: Does limited language access contribute to lower language abilities, less access to numerical language, and/or lower nonsymbolic capacities? Is this the catalyst to the systematic lag in numerical understanding we see in DHH-wo children? And/or, perhaps inefficient working memory abilities limit the capacity for learning early number concepts and hinder the development of subsequent mathematical reasoning? By testing the theories offered here, we can answer these questions and improve our understanding of the development of numerical concepts in DHH children, while clarifying our understanding of the role of language in numerical cognition and where it is most critical in numerical development.

To really peel apart the influence of language on the development of numerical knowledge, there are several areas that need further research. To begin, a thorough *reexamination* of numerical development with DHH children should be done to establish and/or confirm what we know about the acquisition of their symbolic and nonsymbolic number concepts. Our understanding of the acquisition of basic math concepts in hearing children, as well as the specific tasks designed to assess these foundational abilities, have changed dramatically over the last decade. For example, our understanding of the role of nonsymbolic numerical abilities in mathematical achievement in hearing children was not known until recently (e.g., Halberda & Feigenson, 2008). It is important to explore these relationships in DHH children to confirm these children follow the same developmental trajectory.

More importantly, over the last 15 years, newborn hearing screenings and early interventions have become more prevalent in the United States. These changes have likely led to earlier language access for this population, which could potentially have decreased the gap in math abilities between DHH-wo children and their hearing peers. This is important to know because *if* early language access is key to numerical development, we may see DHH-wo children demonstrating greater competence in early math assessments relative to previous research of DHH-wo math abilities. Due to advances in both our understanding of math cognition and hearing assessments, it is critical to provide a broader assessment of math abilities in DHH populations in order to truly understand the source of their math difficulties.

Next, given often reduced language access early in development, it seems likely that numerical language access may similarly be limited in DHH-wo populations. Yet, very little is known about the language environments of DHH children, particularly number language experience with DHH-wo and DHH-w. Aragon and Yoshinaga-Itano (2012) showed similar numbers of conversational turns and exposure to adult words between 14- and 36-month-old DHH-wo Spanish-speaking toddlers and their English-speaking hearing peers—however, whether or not there is *in fact* a clear difference between the amount of *number language* experienced by DHH-wo children, reflecting differences in linguistic content, is unknown. Thus, it is critical to perform naturalistic observations in the homes of parents and their young DHH and hearing children. Subsequent analyses of (a) the amount of language the child is exposed to and (b) the breadth of which the language includes numerical language should be performed to understand whether limited numerical input plays a role in the acquisition of numerical concepts.

Once we know how much number language DHH children experience compared to hearing children, it is important to identify factors mediating their underperformance in formal math assessments. Is it general vocabulary? Overall number word knowledge? Parent input? Working memory? Age at which children access fluent language? Future studies should always include measures of language ability, working memory, and age of access to a fluent language, whether it be auditory or manual. Given the strong role working memory plays in mathematical learning and performance coupled with lower working memory skills in DHH children, future research should be careful to control for working memory. If differences in math performance remain after controlling for working memory abilities, then working memory is not likely the key component to these developmental differences. If we find that delays in DHH-wo children are fully explained by limited language and/or in limited number talk they experience from their parents (controlling for working memory), we will understand that is the specific mechanism responsible for these math delays.

To date, it is difficult to speculate whether DHH children begin with a similar capacity to represent number in infancy because of the paucity of research exploring ANS acuity in DHH children. There is still a dearth of research exploring the ANS in DHH children younger than 3 years old. As a result, we do not know when

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[5] No discussion of early language experience.

DHH children begin to show disparities in their nonsymbolic number abilities. Because of its proposed importance in formal math abilities, the ANS in DHH children needs to be examined from infancy, before the onset of language. If it does differ, this would suggest that differences in early language exposure prior to the onset of language are already changing the trajectory of numerical development in DHH children. If it does not differ, then it is important to pursue the timing of the divergence in representational abilities to determine when deficits in ANS acuity and/or differences in symbolic number first emerge in order to isolate the source of these deficits. Given recent findings that show differences in habituation rates among DHH-wo and hearing infants (Monroy et al., 2019), we may find that language input is influential in the processing of numerical information earlier than expected. If DHH infants do demonstrate comparable nonsymbolic numerical processes to hearing infants, yet older DHH children do not, then this would strongly point to a feedback loop in ANS acuity such that learning symbols sharpens our ANS acuity.

It is also important to consider the role of the learning environment, specifically DHH children's exposure to math curriculum. The disparities in math performance we see between DHH children and their hearing peers could be the result of differences in curriculum and a limited mathematical focus in the classroom. It is widely known that Deaf education programs focus on language and literacy at the detriment to adequate attention to mathematics (e.g., Swanwick et al., 2005; Wood et al., 1984, 1986). Perhaps language impacts numerical abilities early on in development, but other factors, such as educational foci, may exacerbate these mathematical difficulties as DHH children get older.

Finally, future research must strive to include DHH-w children that are native signers with access to a fluent signed language from birth and avoid treating them as one group. *If* early language access is critical to developing early number concepts, then native signers should show no lag in math achievement. On the other hand, there is the potential that, like other cross-linguistic comparisons, differences in linguistic structure between signed and spoken languages, as well as potential differences in the amount of numerical language and incidental learning experienced by native signers (all variables that need to be explored) may influence the development of numerical cognition. However, recent research offers a clue regarding the importance of early fluent language access. Hrastinski and Wilbur (2016) showed 6th–11th-grade DHH-w students receiving fluent ASL exposure from birth (native signers) achieved significantly higher math scores (17th–64th percentile) on the Northwest Evaluation Association (NWEA) Measures of Academic Progress (MAP) than nonnative (less proficient) DHH-wo signers (4th–23rd percentile). Additionally, both Kritzer (2009) and Mousley and Kurz (2015) showed that DHH-w children with *Deaf* parents (thus exposed to fluent sign from birth) outperform DHH-wo children with *hearing* parents. These studies point to the importance of fluent language access very early in development for the acquisition of formal math abilities. Further, it is currently unknown whether early fluent language access promotes nonsymbolic number abilities. As such, future research should strive to include separate samples of DHH-w children with access to fluent and native sign language from birth to address these open questions regarding the role that fluent sign language access from birth may play in the development of numerical abilities in DHH children.

Lastly, and perhaps most importantly, future research involving numerical development in DHH children must acknowledge the heterogeneity in language experience among this population. It seems to have become commonplace to settle the language experiences of all DHH children together which inspires an overgeneralization of the results and ultimately a misrepresentation of cognitive abilities of children with hearing loss. *If,* as we argue, the reduced language experience endured by DHH-wo children is responsible for their lag in numerical development, then researchers must quantify and isolate language experience to pull apart the potential influence this may have on math achievement. Isolating language experience in cognitive development will help paint a clearer picture of the role of language in numerical development without diminishing the important findings that are necessary for interventions to close the gap in math achievement in DHH children.

Conclusions

DHH-wo children demonstrate various delays in mathematical processing, though it is not clear what the source of these difficulties are—it could be general language demands, reduced linguistic input, or other domain-general processing delays. A thorough investigation of any and all of these possibilities is important to gain a complete picture of the role of language and domain-general processing in developing numerical concepts. Future work may be able to isolate how each of these factors may contribute to acquisition of mathematical concepts. While we have presented two theories to explain the source of mathematical delays in DHH-wo children, we argue that the most likely explanation for lower math abilities in DHH-wo children is limited and reduced language experience. Specifically, while they likely begin with the same capacity to learn about number in infancy (i.e., similar ANS abilities early in development), due to reduced linguistic input, DHH-wo children fall behind their hearing peers. If this theory is supported, it would corroborate previous work emphasizing the role language plays in the conceptual development of number and stress that early access to language, especially number language, is paramount for typical development of numerical concepts.

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