Journal of Numerical Cognition
jnc.psychopen.eu | 2363-8761

PsychOpen GOLD

Empirical Research

Fraction Magnitude: Mapping Between Symbolic and Spatial Representations of Proportion

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Abstract

Fraction notation conveys both part-whole (3/4 is 3 out of 4) and magnitude (3/4 = 0.75) information, yet evidence suggests that both children and adults find accessing magnitude information from fractions particularly difficult. Recent research suggests that using number lines to teach children about fractions can help emphasize fraction magnitude. In three experiments with adults and 9-12-year-old children, we compare the benefits of number lines and pie charts for thinking about rational numbers. In Experiment 1, we first investigate how adults spontaneously visualize symbolic fractions. Then, in two further experiments, we explore whether priming children to use pie charts vs. number lines impacts performance on a subsequent symbolic magnitude task and whether children differentially rely on a partitioning strategy to map rational numbers to number lines vs. pie charts. Our data reveal that adults very infrequently spontaneously visualize fractions along a number line and, contrary to other findings, that practice mapping rational numbers to number lines did not improve performance on a subsequent symbolic magnitude comparison task relative to practice mapping the same magnitudes to pie charts. However, children were more likely to use overt partitioning strategies when working with pie charts compared to number lines, suggesting these representations did lend themselves to different working strategies. We discuss the interpretations and implications of these findings for future research and education. All materials and data are provided as Supplementary Materials.

Keywords: fractions, number lines, pie charts, decimals, area models, magnitude, rational numbers

Journal of Numerical Cognition, 2020, Vol. 6(2), 204–230, https://doi.org/10.5964/jnc.v6i2.285

Received: 2020-02-11. Accepted: 2020-07-28. Published (VoR): 2020-09-09.

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Learning and understanding fractions is extremely important both for everyday life and for later math skills. For example, over two-thirds of sampled workers in the USA report using fractions for their jobs (Handel, 2016). Furthermore, fractions are an important gatekeeper for learning algebra, an important topic in math education (Booth & Newton, 2012; Siegler et al., 2012). However, fractions pose a substantial challenge for many students who make both procedural and conceptual errors throughout fraction education and well into adulthood (e.g., Christou & Vosniadou, 2012; Lortie-Forgues, Tian, & Siegler, 2015; National Mathematics Advisory Panel, 2008; Ni & Zhou, 2005; Vamvakoussi & Vosniadou, 2010). Thus, it is essential to understand the difficulties children face when learning about fractions and investigate ways to help children overcome them.

A notable aspect of fraction notation is that it represents both a relation between two quantities (a part whole relation between the numerator and the denominator) and a single numerical magnitude on a continuum (e.g.,

¾ represents 3 of 4 parts and the magnitude 0.75). Children and adults have great difficulty with the latter aspect of fraction symbols; that is, interpreting the numerical magnitude associated with fractions (Bonato, Fabbri, Umiltà, & Zorzi, 2007; DeWolf, Grounds, Bassok, & Holyoak, 2014; Hurst & Cordes, 2016, 2018). As such, there has been a recent emphasis in both research (Siegler et al., 2013) and education (National Governors Association Center for Best Practices, 2010) on children's understanding of fraction *magnitudes* in particular. Furthermore, typical errors when trying to understand fraction magnitudes have also been shown to impact health and financial decision making (e.g., Reyna & Brainerd, 2007). In the current study, we focus on how children and adults visualize the magnitudes associated with symbolic fractions as spatial representations (i.e., pie charts, number lines) and whether priming specific spatial representations promotes better magnitude processing, as spatial visualizations play an important role in both learning fractions and using fractions for every day reasoning contexts (e.g., Cramer et al., 1997; Galesic et al., 2009; Keijzer & Terwel, 2003; Rau & Matthews, 2017; Shah & Hoeffner, 2002).

Symbolic Numerical Magnitude

The majority of work investigating how children and adults think about numerical magnitude has been in the context of whole numbers. This research suggests that children and adults represent whole numbers as approximate, ordered magnitudes. This evidence often comes from performance on symbolic numerical comparison tasks in which the participant is asked to decide which of two numbers is largest as accurately and quickly as possible. These studies reveal ratio-dependent responding, such that the speed and accuracy with which a person responds is dependent upon the ratio between the numbers (Moyer & Landauer, 1967, 1973; Sekuler & Mierkiewicz, 1977). Critically, this pattern of responding suggests that these representations are both noisy (i.e., the representation of 6 is not exact, but instead also bleeds into the representations of other numbers) and ordered (i.e., 6 overlaps more with 7 than with 10; Moyer & Landauer, 1967, 1973).

This finding of ratio-dependence is robust for whole number comparisons but has only recently been demonstrated in comparisons involving other kinds of rational numbers, including fractions and decimals. This research suggests that adults and children are able to represent approximate magnitudes when comparing two fractions, two decimals, and when comparing across notations (i.e., fraction vs. decimal, fraction vs. whole number, decimal vs. whole number; DeWolf et al., 2014; Faulkenberry & Pierce, 2011; Ganor-Stern, 2012, 2013; Hurst & Cordes, 2016, 2018; Meert, Grégoire, & Noël, 2010; Schneider & Siegler, 2010; Sprute & Temple, 2011; Varma & Karl, 2013; Wang & Siegler, 2013; although there are some contexts where accessing the magnitudes may be more difficult, e.g., Bonato et al., 2007; Kallai & Tzelgov, 2014). From these findings, researchers suggest that adults and children are able to represent the magnitudes of fractions, decimals, and whole numbers in an integrated way.

Spatial Representations of Magnitude

In line with research on whole numbers (Ramani & Siegler, 2008; Siegler & Ramani, 2009), recent evidence suggests that fraction understanding benefits from the use of number lines, resulting in better learning than using more traditional area models, including pie charts (Cramer et al., 2002; Gunderson et al., 2019; Hamdan & Gunderson, 2017; Keijzer & Terwel, 2003; Saxe et al., 2013; Wang & Siegler, 2013). For example, training studies have found that children taught to divide and color a number line to represent fraction magnitudes performed better on a symbolic magnitude comparison task than children who were taught to divide and color area models to represent fraction magnitudes (Gunderson et al., 2019; Hamdan & Gunderson, 2017). Moreover, another study found a number-line-focused experimental curriculum, designed to help students make connections between the numerical magnitudes associated with fractions and integers, resulted in benefits over business-as-usual schooling for a range of integer and fraction concepts (Saxe et al., 2013).

However, other work suggests that area models (e.g., pie charts) may be useful for representing part-whole information, a critical part of formal fraction education (e.g., Cramer et al., 1997; Cramer et al., 2002). Learning part-whole relations is important for learning the meaning of the fraction symbols (e.g., that 2/3 corresponds to 2 out of 3 and not 2 and 3) and to begin coordinating the proportional relation described by those symbols, serving as a building block for learning additional fraction concepts (Clark et al., 2003; Mix & Paik, 2008; Saxe et al., 1999). Relatedly, pie charts may be useful for communicating relative proportional information, rather than absolute information, when compared to other kinds of charts and graphs (Shah & Hoeffner, 2002).

When taken together, these studies suggest that number lines and area models may show distinct advantages and disadvantages, with number lines conveying magnitude information and pie charts conveying part-whole information. But do these distinct affordances make one spatial representation align better with one type of symbolic representation? Some evidence suggests that adults are more accurate when mapping between fraction symbols and pie charts than between fractions and number lines (Hurst, Relander, & Cordes, 2016). This may not be surprising, given research suggesting that both fractions and area models may be less transparent than decimals and number lines for communicating or learning magnitude, respectively (DeWolf et al., 2014, 2015; Rapp et al., 2015). On the other hand, decimals are generally preferred for representing continuous numerical magnitudes, which may make them more likely to align with number lines (DeWolf, Bassok, & Holyoak, 2015; DeWolf et al., 2014; Hurst & Cordes, 2016, 2018).

The Current Study

In the current study, we focused primarily (although not exclusively) on fraction notation. Given that fraction notation represents both magnitude and part-whole information, it can convey both *continuous* magnitude and *discrete* part-whole features. Yet, thinking about fraction magnitude is difficult, even for children actively learning fractions and decimals (Bonato et al., 2007; DeWolf et al., 2014; Hurst & Cordes, 2016, 2018). In this study, we explored how people report thinking about symbolic fraction magnitude, whether children can be prompted to think about continuous fraction magnitude by giving them a number line (versus a pie chart), and whether written partitioning strategies are spontaneously used to map between symbolic and spatial representations. We investigated these questions by first assessing how adults report thinking about fraction magnitudes and, in particular, if they spontaneously used linear or part-whole visualizations to think about fractions (Experiment 1). However, adults rarely reported using number lines, making it impossible to address this question using self-reports. Thus, in Experiments 2 and 3, we provided children with short mapping activities that explicitly involved either number lines or pie charts prior to completing a symbolic magnitude task. We then investigated how this experience with distinct spatial representations may impact children's performance on a subsequent symbolic task. Therefore, across three experiments we investigated the role of distinct spatial representations in conceptualizing fraction magnitude information.

Experiment 1

In Experiment 1, we assessed adults' self-report of how they think fractions, after they completed a symbolic magnitude comparison task, to address two questions: (1) How do adults visualize fraction magnitudes? (2) Does the type of visualization relate to their abilities to process symbolic fraction magnitude? In particular, do individuals who visualize fractions as continuous magnitudes (e.g., falling along a number line) perform better on a fraction magnitude comparison task than those who visualize fractions as area models (e.g., pie chart)? As an additional research question, we were interested in replicating the effects of ratio-dependent responding seen in magnitude comparison tasks and investigating whether adults also rely on magnitude information during other fraction tasks. Thus, we also address (3) whether adults show ratio dependent responding when doing speeded and approximate fraction addition.

Method

Participants

Fifty adult college students from a university in the Northeastern USA (M_{age} = 19.2 years, range 18 to 24 years, 39 females) participated for partial course credit.

Measures

Adults completed all tasks in the same order on a 13-inch MacBook laptop using Xojo programming software (formerly named REALBasic): (1) magnitude comparison task, (2) speeded fraction arithmetic, and (3) fraction visualization questionnaire. Adults were tested one-on-one in a quiet room in our laboratory. The experimenter remained in the room for the magnitude comparison and speeded arithmetic tasks but left during the fraction visualization questionnaire. A list of trials and all the stimuli are available at the Open Science Framework (see Hurst, Massaro, & Cordes, 2020a).

Magnitude comparison task — Adults first participated in a magnitude comparison task (based on Hurst & Cordes, 2016, 2018) in which they were asked to rapidly judge which of two rational numbers was greater in magnitude. On each trial, two numbers were presented on the screen, one on the left and one on the right, and participants were asked to choose which of the two numbers represented the larger magnitude as quickly and accurately as they could by pressing the corresponding key on the keyboard (right or left arrow). The stimuli remained on the screen until the participant made a response and then a small fixation cross appeared in the middle of the screen for 1000ms until the next trial began. All adults received three blocks of trials that differed in the notation of the stimuli being compared: FvF trials involving two fractions (e.g. 3/5 vs 2/9), DvF trials involving one fraction and one decimal (e.g. 3/5 vs 0.22), and NvF trials involving one fraction and one whole number (e.g. 4/3 vs 2). At the beginning of each block, participants were given one notation-specific practice problem (the same problem across participants) with computerized feedback about their accuracy. After the practice problem, participants were invited to ask additional questions or clarify the task. On test trials, no feedback was given.

The order of trials within each block and the order of the blocks were randomized across participants. Stimulus pairs on each trial came from one of two ratio bins (ratio = larger numerical magnitude / smaller numerical magnitude): small ratio bin (ratios ranged from 1.35 to 1.51) and large ratio bin (ratios ranged from 2.2 to 2.9). For each Ratio (2) x Notation Block (3) combination there were four unique comparisons, shown twice (once with the largest number on the left and once with the largest on the right). Thus, there were a total of 48 trials (4 Unique Comparisons x 2 (shown twice) x 2 Ratios x 3 Notation Blocks).

The decimal stimuli (used in the DvF block) were presented to the hundredths digit, with a whole number before each decimal point (e.g. 0.15; 1.36). The fraction stimuli (used in all blocks: DvF, NvF, and FvF) had numerator and denominator values each less than or equal to 10. The two fractions for each comparison in the FvF block were made up of four different integers (i.e., a/b vs. c/d where a, b, c, d were all different positive integers) in order to prevent the use of whole number strategies (e.g., Schneider & Siegler, 2010). Furthermore, the larger fraction also had the larger numerator on 50% of the trials, the larger denominator on 25% of the trials, and the smaller “gap” (i.e., the difference between the numerator and the denominator) on about 38% of the trials, to avoid allowing participants to respond exclusively based on these heuristics (e.g., Kallai & Tzelgov, 2009, 2014; Meert, Grégoire, & Noël, 2010). In the NvF block, fractions ranged from 6/5 to 9/2 and whole numbers ranged from 1 to 6. In the FvF block, fractions ranged from 1/5 to 7/2. In the DvF block, fractions ranged from 1/5 to 5/3 and decimals ranged from 0.22 to 3.5. The DvF comparisons were the same as the FvF comparisons but with one value converted to an approximate decimal equivalent (e.g., 1/3 would be turned into 0.3). Decimals were approximately 2cm high x 5.5 cm wide, Fractions were approximately 5.5 cm high and 2.7 cm wide, and Whole Numbers were approximately 2 cm high and 1.2 cm wide. The fixation cross between trials was approximately 0.5 cm high x 0.5 cm wide, in the center of the screen.

Speeded arithmetic task — On each trial, two symbolic fractions were displayed on the screen with an addition sign (+) between them for 1500 ms (e.g., “1/7 + 2/9”, but with the fractions presented in their formal upright format), followed by a screen displaying the question “Less than 1 or More than 1?” Participants were asked to quickly judge whether the sum of the addition problem was more than one or less than one and press the corresponding key with stickers labeled as “more” (right arrow key) or “less” (left arrow key), respectively. Before beginning the task, participants were given one practice trial and were invited to ask any questions. Because the task was speeded (making it very difficult to compute an exact sum of two fractions with different denominators in such a short time), and because participants were not asked to provide an exact sum but only approximate whether the answer was above or below one, we considered this task to be an approximate magnitude addition task.

The task consisted of 16 unique trials, shown twice (with the order of the fractions reversed), resulting in 32 trials. The sum of the two fractions fell into two ratio bins surrounding the value one: close/small ratios of 3:2 and far/large ratios of 2:1. In each ratio bin, four trials summed to a value greater than one and the other four trials summed to a value less than one. In the small ratio bin, sums greater than one were approximately equal to 1.46 (range 1.41 to 1.53) and sums less than one were approximately 0.7 (range 0.65 to 0.78). In the large ratio bin, sums greater than one were approximately equal to 1.74 (range 1.69 to 1.80) and sums less than one were approximately 0.48 (range 0.37 to 0.59).

Fraction visualization questionnaire — Lastly, adults were presented with three questions: (1) “In what way do you think about fractions? In other words, when you think about a fraction (for example, 1/2 or 4/5) how do you visualize it?” (2) “If you were explaining fractions to someone, which visual references would be best to use?” and (3) “What kind of visual references do you remember learning fractions with most?” For questions 2 and 3, participants were provided with the options of a) Pie Chart, b) Number line, or c) No visual aid. For question 3, adults were provided with the additional option of “Other”, with a space to provide more details. Adults were asked to respond as honestly as possible.

Data Coding

Accuracy (proportion correct) and reaction time (RT) were measured on both the magnitude comparison and speeded arithmetic tasks. Only RTs from correct responses and those within three standard deviations of the individual’s average RT were included in the analyses. At the individual level, in order for average RT for each participant to accurately represent the speed in which they processed symbolic magnitudes on the magnitude comparison task, adults who performed at or below chance (4/8 or below) or who had fewer than three included trials (i.e., that were correct and within three *SD*s of their average RT) were excluded (similar criteria to those used in Hurst & Cordes, 2016). This resulted in a final sample of 38 (out of 50) participants having complete and useable RT data on the magnitude comparison task. Given the lower samples of useable RT data, we only report the analyses involving accuracy on the task (proportion of trials correct). However, RT performance showed a similar pattern – RT results are reported in the Supplementary Materials (see Hurst, Massaro, & Cordes, 2020b) and data is available on the OSF Project Page (see Hurst, Massaro, & Cordes, 2020a).

The responses from the first question on the fraction visualization questionnaire were coded based on three major themes: (1) area models, which included any description that involved visualizing an object or image involving parts of a whole shape or object, such as a pizza, a pie chart, or a rectangle with shaded in sections, (2) number lines, which included any method of visualizing a number line or continuum, and (3) symbolic methods, which included any responses that involved thinking about the magnitude using only symbols, for example estimating the proximity of the value to anchors like 1/2 or 1 and/or converting to another symbolic notation (e.g., decimal). Some participants gave responses that did not fit into any of these categories, for example “one number on top of another” or “as a ratio”. Participants’ responses could be given multiple codes if they fell into more than one of these categories. Two independent coders coded all responses and disagreed on 8/50 of the responses. Disagreements were discussed and settled by a third coder.

Data Analysis

All data analysis was done in R (version 3.5.1) (R Core Team, 2018) with RStudio (R Studio Team, 2016) using packages from *tidyverse* (Wickham, 2017), as well as *jmv* (version 0.9.5; Selker, Love, & Dropmann, 2018), *ez* (version 4.4-0; Lawrence, 2016), *psychReport* (version 0.4; Mackenzie, 2018), *effsize* (version 0.7.4; Torchiano, 2018), and *irr* (version 0.84.1; Gamer, Lemon, & Puspendra Singh, 2019). Bayesian analyses were conducted using JASP (Version 0.10; JASP Team, 2019). For the ANOVA models, we compare models that are matched except for a given effect and evidence for the inclusion of the effect is reported as BF_{incl} and evidence for the exclusion of the effect (i.e., $1/BF_{incl}$) is reported as BF_{excl}. For all other analyses, we use the more typical notation of B_{10} as the Bayes Factor for the alternate hypothesis (comparing the alternate model, 1, to the null model 0) and B_{01} as the Bayes Factor for the null hypothesis (comparing the null model, 0, with the alternate model, 1). For each test, we report the Bayes Factor (either BF_{incl}/BF_{10} or BF_{excl}/BF_{01}) that is larger than one, as these are easier to interpret, and they are always reciprocals of each other. Each Bayes Factor can be interpreted as how much more likely the data are under the referenced model (i.e., either inclusion or exclusion; alternate or null) relative to the other model. For example, if $BF_{incl} = 10$, then the data are 10 times more likely under the model that includes the effect than the model that does not include the effect. Bayes analyses were performed in JASP using default priors (JASP Team, 2019).

Results and Discussion

Magnitude Comparison Task

Accuracy on the magnitude comparison task was analyzed using a repeated measure ANOVA with notation (3: FvF, NvF, and DvF) and ratio (2: small, larger) as within subject factors (see Figure 1). Bayes factor (either BF_{incl}/BF_{10} or BF_{excl}/BF_{01}, whichever is greater than one) is reported for each effect as well.

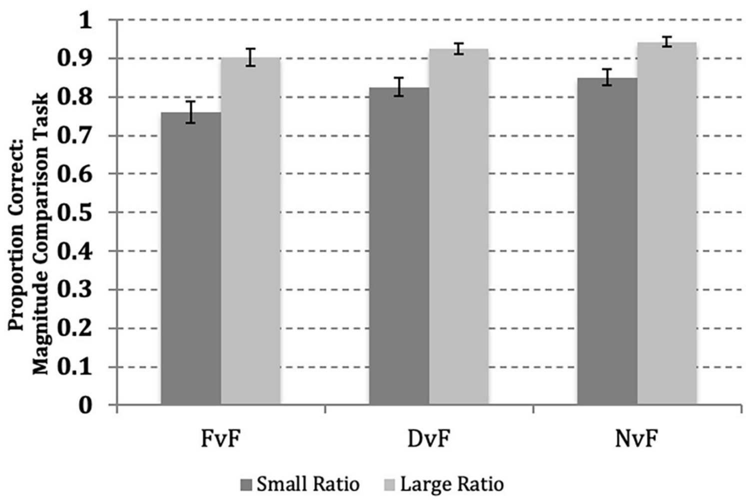

Figure 1. Performance on the magnitude comparison task of Experiment 1, by ratio and notation.

Note. Error bars represent standard error (*SE*) of the mean.

There was a main effect of notation, $F(2, 98) = 6.7$, $p = .002$, $\eta_p^2 = .12$, $BF_{incl} = 19.6$, such that performance on FvF trials, $M = 0.83$, was significantly lower than NvF trials, $M = 0.89$, $t(49) = 3.2$, $p = .002$, Cohen's $d = 0.5$, $BF_{10} = 14.5$, and DvF trials, $M = 0.88$, $t(49) = 2.5$, $p = .016$, Cohen's $d = 0.3$, $BF_{10} = 2.5$, which were not significantly different from each other, $t(49) = 1.3$, $p = .210$, Cohen's $d = 0.2$, $BF_{01} = 3.0$. There was also a main effect of ratio, $F(1, 49) = 50.4$, $p < .001$, $\eta_p^2 = .50$, $BF_{incl} = 1.6 \times 10^{10}$, with higher accuracy on the large ratio, $M = 0.92$, than the small ratio, $M = 0.81$. There was not a significant interaction between notation and ratio, $F(2, 98) = 1.4$, $p = .253$, $\eta_p^2 = .03$, $BF_{exclu} = 5.8$.

Overall, these findings replicate previous work showing ratio-dependent responding when adults compared fractions and whole numbers, fractions and decimals, and two fractions (e.g., Faulkenberry & Pierce, 2011; Ganor-Stern, 2013; Hurst & Cordes, 2016; Schneider & Siegler, 2010). In addition, performance when comparing two fractions was significantly less accurate and slower than when comparing one fraction with a whole number or decimal, replicating patterns in other work (Hurst & Cordes, 2016). This finding suggests that the difficulties encountered when thinking about rational number magnitudes may be particularly tied to the fraction notation itself. That is, comparing two fractions within the same notation was more difficult than comparing a fraction to a value in a different notation. Together, these findings are consistent with claims that adults consider fractions as falling along an integrated continuum with whole numbers and decimals, but also highlight the difficulty in processing the numerical magnitudes associated with fraction notation in particular.

Speeded Arithmetic Task

We were primarily interested in whether adults would display significant ratio effects on a fraction addition task that only required an approximate sum. Thus, we used paired *t*-tests to compare performance on the small and large ratio bins in terms of both accuracy and RT. On the large ratio trials, M_{acc} = 0.92, M_{RT} = 571 ms, adults performed significantly more accurately, $t(49) = 4.46$, $p < .001$, Cohen's $d = 0.5$, $BF_{10} = 450$, and faster, $t(49) = 6.59$, $p < .001$, Cohen's $d = 0.36$, $BF_{10} = 4.7 \times 10^5$, than on the small ratio trials, M_{acc} = 0.85, M_{RT} = 742 ms. This suggests that adults do access approximate representations of fraction magnitude information in other contexts beyond magnitude comparisons.

Fraction Visualization Reports

Only *one* participant (2%) reported using a number line in response to our first question about how they think about or visualize fractions. In contrast, 64% of individuals ($n = 32$) reported using a visual area model (e.g., imagining a pie chart) and 46% ($n = 23$) reported using symbolic methods (e.g., estimating decimal form; note, however that 10 adults are included in both groups as they reported examples from both categories). In addition, 8% ($n = 4$) reported using a method that did not fit into these categories (e.g., "one number on top of another", "as a ratio").

Responses to the second question aligned with this pattern of results. That is, when asked to pick (of three options) which would be the *best* visual reference to use to explain fractions to someone else, almost all participants reported a pie chart: 84% ($n = 42$), with fewer reporting a number line (14%, $n = 7$) or using no visual reference (2%, $n = 1$). Similarly, when asked which visual references they remember learning with (note that adults could select more than one option for this question), 82% ($n = 41$) said they remember using a pie chart and only 22% ($n = 11$) reported using a number line (note, again, that $n = 7$ of these adults reported both and so are included in both categories). In addition, 2% ($n = 1$) reported not remembering any visual reference and 14% ($n = 7$) selected "other". The "other" responses included: real world examples, shaded objects, sets of objects, money, and base-10 blocks (sets of stackable blocks that can be organized into sets of 10 to easily communicate place-value).

Overall, these self-reports reveal that many of the adults spontaneously thought about fractions via area models – with most also reporting pie charts as being the best way to teach someone about fractions and the way they remember learning about fractions. It may not be surprising that pie charts were chosen so frequently when it was provided as an option in our multiple-choice questions, especially since pie charts may be the canonical fraction representation. More striking, however, is adults' responses on the open-ended question about the way in which they "think about fractions" (which came before the multiple-choice questions). Even without prompting, most adults reported using a visual strategy involving an area model or image, like a pie chart or a shaded object, highlighting the overall preference for this type of representation amongst our adult sample. Notably, in contrast, only *one* person spontaneously reported using a number line. Even when number lines were provided as a multiple-choice option, markedly few participants selected them, suggesting that number lines are not perceived as being a particularly useful representation for thinking about fraction magnitudes, at least by the adult college students tested. Of note, however, is that this pattern may be attributable to the age of our sample, as the emphasis on number lines in the classroom has been relatively recent (National Mathematics Advisory Panel, 2008) and may not have been part of the curriculum when these adults were learning fractions.

Performance Differences Across Fraction Visualization Methods

Our primary interest was to investigate whether there were individual differences in magnitude understanding between people who opted to use area models versus number lines to think about fractions. However, almost none of the participants reported using number lines and instead area models were the primary way adults reported visualizing fractions, making it impossible to make any meaningful inferences from the data between these two choices. Yet, many adults did report using a symbolic method for thinking about fractions. These symbolic methods required adults to think about the magnitudes associated with the values relative to other symbols, either within the same notation (e.g., ¼ is less than ½) or across notations (converting 3/5 to a decimal or percentage), despite not involving a visual figure aid. Given this, we hypothesized, post-hoc, that this feature of symbolic reasoning may be theoretically similar to the hypothesized benefits of the number line.

To explore this hypothesis, we compared performance on the magnitude comparison task between those who spontaneously reported using only area model visualizations and those who spontaneously reported using only symbolic methods. Notably, given that 20% of adults (n = 10) reported using both of these methods, we isolated this analysis to only those adults who reported *only one* of them. In line with our predictions, accuracy on the magnitude comparison task was higher for those who reported using symbolic methods (n = 13), M = 0.90, compared to those who reported an area model visualization (n = 22), M = 0.82, $t(23.3) = 2.49$, p = .020, BF_{10} = 3.6. There was not a significant difference in performance on the speeded arithmetic task between adults who reported a symbolic method, M = 0.88, versus area model visualization, M = 0.87, $t(22.1) = 0.29$, p = .775, BF_{01} = 2.9.

The significant difference on the magnitude comparison task may suggest that using symbols to reason about fractions reflects more advanced knowledge of fractions and/or that this reasoning requires an increased attention to magnitude, allowing for more accurate estimates of magnitude than part-whole visualizations. Thus, it may be that those individuals who thought about fraction magnitudes relative to other symbols had a better understanding of the relations among magnitudes in fraction, decimal, and whole number notation, and less of a focus on the part-whole components of fractions. However, given that this was a post-hoc hypothesis and analysis and that we did not find the same difference in the speeded arithmetic task, a different fraction task that also appears to rely on magnitude knowledge, additional research is needed to more fully investigate the differences in the way people spontaneously think about fraction values.

Overall, results of Experiment 1 reveal that adults generally do not spontaneously visualize fractions as falling along a number line, however the particular way they report reasoning about fractions was related to their performance on the magnitude comparison task. Therefore, although some clear and striking patterns emerged in the data, we were unable to investigate our central question about the utility of the number line for thinking about fractions because almost no participants reported spontaneously engaging in this kind of thinking. Thus, in Experiment 2 we investigated the relation between visual and symbolic representations of fractions by explicitly providing children with different visualizations of fractions before completing the symbolic comparison task. We focused on children in Experiments 2 and 3 because we thought they may be more able to readily adopt a number line visualization strategy for fractions, for two reasons: first, they are more likely to have experienced number lines in their classroom due to the more recent adoption of number lines in mainstream curricula (National Mathematics Advisory Panel, 2008) and second, they are still actively learning fractions, which may make them more flexible and less rigid in their preferred strategies. However, we did collect data from separate samples of adults for Experiments 2 and 3 and although the results are complementary to the results from the child data, we are only reporting the adult samples in the Supplementary Materials (see Hurst, Massaro, & Cordes, 2020b). All data are also available on the OSF Project Page (see Hurst, Massaro, & Cordes 2020a).

Experiment 2

In Experiment 2, 9- to 12-year-old children were assigned to one of two conditions in which they were asked to map fractions to either pie charts or to number lines. Following this task, participants engaged in the magnitude comparison task (as in Experiment 1). By allowing children to engage in a visual mapping task directly, we investigated whether priming children to think about fractions using either number lines or pie charts would impact subsequent performance on a symbolic magnitude task. Furthermore, by testing children we can more thoroughly investigate the impact of these spatial representations on fraction magnitude understanding with expectations that our findings may have implications for education. In particular, given that adults (based on Experiment 1) mostly reported learning fractions with area models and not number lines, they may be particularly resistant to number line priming, as these might be highly unfamiliar representations. Children who are in the process of learning about fraction and decimal magnitudes and who are more likely to have encountered fractions on number lines, however, may have more malleable visual representations of fractions and thus may be more open to adapting the way they think about fractions. Notably, this age group is older than children in other recent studies investigating the efficacy of number lines (e.g., Gunderson et al., 2019; Hamdan & Gunderson, 2017) so that we could investigate whether the way children approach symbolic fractions could be impacted using a brief practice rather than teaching an entirely new approach. Thus, we used children who were already familiar with fractions and had begun instruction on these topics.

In Experiment 2, we investigated two specific research questions: (1) Does practice mapping fractions to number lines result in better fraction magnitude performance than practice mapping fractions to pie charts? (2) Do children use overt partitioning strategies when dealing with number lines and pie charts? Research with adults suggests they are more error-prone when translating between fractions and number lines than pie charts (Hurst et al., 2016). However, whether this is due to a difference in the types of overt strategies people use when encountering a number line compared to a pie chart is unclear.

Method

Participants

Seventy 9-12-year-old children were included in the analyses and were assigned to one of two between-subject conditions: Number Line Condition (N = 35, M_{age} = 10.5 years, range: 9.0 to 12.8 years, 20 females, 14 males, 1 unreported), or Pie Chart Condition (N = 35, M_{age} = 10.5 years, range: 9.2 to 12.8 years, 16 females, 19 males). Children's grade was not consistently collected, but based on the education system in the Northeastern United States, they were in approximately 3rd to 5th grades, which is around the time that fractions are typically introduced. Children were tested at local after school programs, summer camps, and public parks, as well as in our laboratory or in their homes. Written consent was obtained from parents or legal guardians of all children and children provided both oral and written assent for their own participation. Children received a small prize, sticker, or $10 for their participation, depending on the regulations of the specific testing facility.

Design and Measures

All participants completed the visual representation activities and the magnitude comparison task. The visual representation activities involved either Number Lines or Pie Charts, depending on the participant's condition. The magnitude comparison task was identical to the task used in Experiment 1. The experimenter remained quietly in the room for the duration of the study and the entire experiment took approximately 25 minutes.

Visual representation activities — All participants received two separate 21.5 cm by 14 cm paper booklets: one booklet for Number-to-Position trials and one for Position-to-Number trials (adapted from Siegler & Opfer, 2003)[i], presented in that order (see Figure 2).

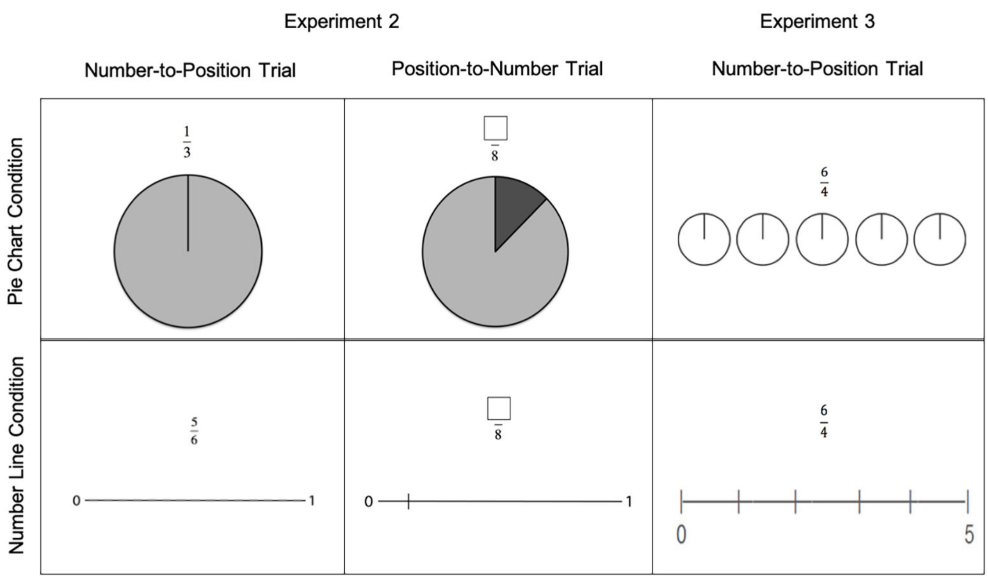

Figure 2. Stimuli from Experiment 2 (left, middle) and Experiment 3 (right) across the two conditions.

In the Number-to-Position task, participants were presented a symbolic fraction above an empty pie chart (4 cm radius) or 0-to-1 number line (13.8 cm long) and asked to place a mark on the number line or fill in the part of the circle that went with the number at the top. The number line had labeled end points of 0 and 1. On the pie charts, a vertical radial line extended from the center of the circle to the top. This was intended to be a reference line for participants; however, many chose not to use it and drew two of their own boundary lines.

On each trial, after the participant made their mark or filled in the circle, the experimenter showed the participant the correct response by making a mark on the line or on the pie chart with a yellow highlighter, using a premeasured cut out. The experimenter then gave the participant feedback by commenting on the participant's response relative to the correct response (e.g., "You were pretty close!" or "Not quite, it's actually smaller"). The participant completed seven of these problems, each presented on a separate page and in the same order, but with a different target magnitude: 1/2, 3/4, 1/3, 7/8, 3/5, 5/6, and 1/5.

In the Position-to-Number task, participants in the Number line condition were given a number line with endpoints of 0 and 1 with a 1 cm hatch mark somewhere along the line representing a magnitude. Participants in the Pie Chart condition were given a pie chart with a shaded portion. In both conditions, the number line and pie chart were accompanied by a symbolic fraction above the spatial representation with the correct denominator but an empty box in the numerator[ii]. Participants were asked to write down their best estimate of the correct numerator of the fraction, using the denominator provided. The experimenter corrected responses on each trial by showing the participant a card with a number line or pie chart divided up into the relevant units and indicating the correct answer. Participants completed seven trials with each problem presented on a separate page and in the same order, but with a different target magnitude: 1/2, 3/4, 1/4, 2/3, 7/9, 3/7, and 1/8.

Data Coding and Analysis

For the Number-to-Position task, percent absolute error (PAE) was used as a measure of performance accuracy (as in Siegler & Booth, 2004). PAE was measured as the absolute difference between the magnitude that was estimated and the target magnitude divided by the range of the line/pie chart and multiplied by 100:

PAE = 100*[abs(estimate – target)/range]

For example, if on the 0 to 1 representation a participant was trying to estimate ½ and put their mark at the location corresponding to 0.6, their PAE would be 100*[abs(0.5-0.6)/1] = 10%. Two independent coders measured responses on each booklet and reliability was measured using the intraclass correlation (ICC; modeled using consistency with a two-way model using R package *irr* by Gamer et al., 2019). Reliability was excellent for all magnitudes in each condition (ICCs's > .75) and the average value given by the two coders was used in the analyses.

Booklets were also coded for evidence of overt partitioning strategies. Two independent coders determined whether, for each magnitude, there was evidence of overt partitioning (additional lines on the number line or pie chart beyond the response) or not. Each participant was then categorized as either consistently using an overt partitioning strategy on every trial, consistently not using an overt partitioning strategy, or inconsistently applying strategies (i.e., partitioning on some trials, but not others). Inter-rater reliability on these overall categorizations (measured using Cohen's Kappa with the R package *irr*; Gamer et al., 2019) was excellent (Cohen's Kappa = .84) and the codes from the first coder were used in the analyses.

For both the PAE analyses and the partitioning categorization, performance on the magnitude of ½ was not included for three reasons: (1) overall performance on ½ was very accurate, (2) we are not able to disambiguate a partitioning strategy from only placing the answer because there is only one hatch mark needed to fully partition the representation, and (3) the visual aspects of our pie chart display may have made ½ easier for pie charts than for number lines.

For both conditions, accuracy on the Position-to-Number task was computed as the proportion of trials in which a correct numerator response was provided. On the magnitude comparison task, accuracy was used as the primary dependent variable.

All analyses were done using the software and packages as described in Experiment 1.

Results and Discussion

Magnitude Comparison Task

In order to investigate our primary question of whether the number line versus pie chart mapping tasks impacted subsequent performance on the magnitude task, we analyzed the magnitude comparison task using an ANOVA on proportion correct with notation (3: FvF, DvF, and NvF) and ratio (2: small and large) as within-subject factors and condition (2: Number Line and Pie Chart) as a between subject factor. Bayes factors are also reported for each main and interaction effect in the ANOVA based on comparing models that contain the effect to equivalent matched models that do not contain the effect. As in Experiment 1, for each test we report the Bayes Factor (either BF_{incl}/BF_{10} or BF_{excl}/BF_{01}) that is larger than one.

Data (see Figure 3) did not reveal a main effect of notation, $F(2, 136) = 0.5$, $p = .580$, $\eta_p^2 < .01$, $BF_{excl} = 23.1$. However, there was a significant main effect of ratio, $F(1, 68) = 49.9$, $p < .001$, $\eta_p^2 = .40$, $BF_{incl} = 1.25 \times 10^{10}$, and a ratio by notation interaction (reporting Huynh-Feldt correction for a violation of sphericity), $F(1.77, 120.02) = 5.3$, $p = .009$, $\eta_p^2 = .07$, $BF_{incl} = 5.5$. Paired *t*-tests indicated that there was a significant ratio effect, with lower accuracy on the smaller ratio than the larger ratio, on all three trial types: FvF trials: $M_{small} = 0.59$, $M_{large} = 0.81$, $t(69) = 4.7$, $p < .001$, Cohen's $d = 0.72$, $BF_{10} = 1563$; DvF trials: $M_{small} = 0.62$, $M_{large} = 0.76$, $t(69) = 5.3$, $p < .001$, Cohen's $d = 0.67$, $BF_{10} = 12821$; and NvF trials: $M_{small} = 0.68$, $M_{large} = 0.74$, $t(69) = 2.8$, $p = .006$, Cohen's $d = 0.29$, $BF_{10} = 5.2$. However, the size of the ratio effect (i.e., the difference in performance between the small and large ratio trials) was significantly smaller in the NvF trials than in the FvF trials ($p = .003$) and the DvF trials ($p = .040$), which were not significantly different from each other ($p = .162$). The presence of ratio effects replicates prior work using a similar task with similarly aged children (Hurst & Cordes, 2018) and adults (Hurst & Cordes, 2016), suggesting that children are able to access magnitude information in an integrated way from fractions, decimals, and whole numbers.

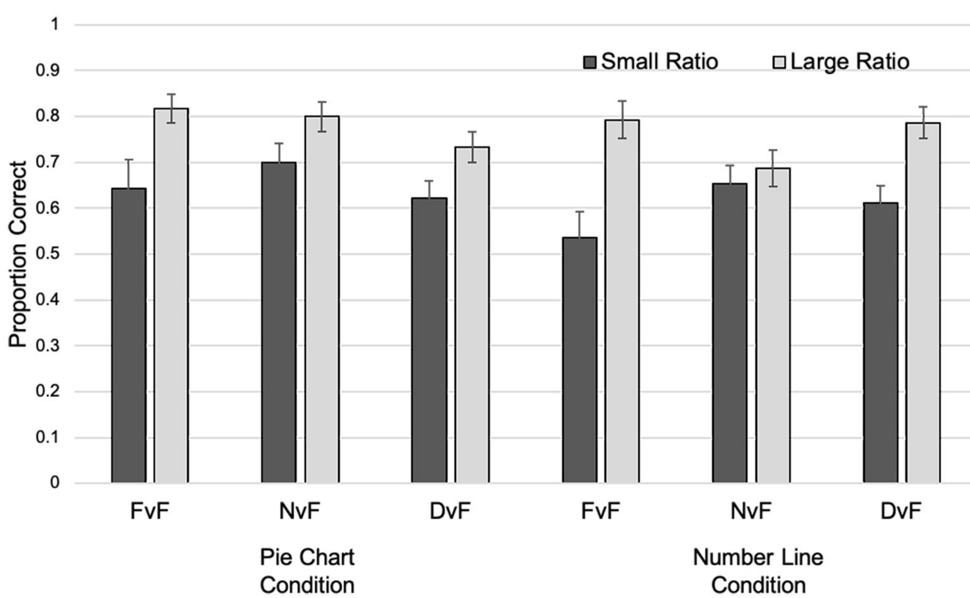

Figure 3. Children's performance on Experiment 2 separated by condition and presented by notation block and ratio.

Note. Error bars are standard error of the mean.

There was not a main effect of condition, $M_{PC} = 0.72$, $M_{NL} = 0.68$, $F(1, 68) = 0.98$, $p = .325$, $\eta_p^2 = .01$, $BF_{excl} = 2.9$. However, there was a small interaction between notation and condition, $F(2, 136) = 3.3$, $p = .039$, $\eta_p^2 = .05$, $BF_{excl} = 1.5$ (all other interactions with condition: Condition x Ratio $p = .516$, $BF_{excl} = 5.3$, Condition x Ratio x Notation, $p = .211$, $BF_{excl} = 3.16$). In particular, performance across the three notations was very similar in the Number Line condition, $M_{FvF} = 0.66$ $M_{NvF} = 0.67$, $M_{DvF} = 0.70$, (all pairwise *p*s > .100, $2.4 < BF_{01} < 5.5$), with only slight variation revealing numerically highest performance in the DvF trials and lowest in the FvF trials. On the other hand, in the Pie Chart condition, there was slightly more variation in performance across the three notations, M_{FvF} = 0.73 M_{NvF} = 0.75, M_{DvF} = 0.68, and the ordinal pattern of performance differed, with highest performance on the NvF trials and lowest performance on the DvF trials (NvF vs. DvF: p = .044, BF_{10} = 1.2; FvF v. DvF: p = .080, BF_{01} = 1.3; NvF and FvF: p = .593, BF_{01} = 4.8). Thus, this small interaction is likely caused by subtle changes in the pattern of performance across the three notations, with performance in the Pie Chart condition showing slightly more variability and slightly higher performance, particularly in the NvF and FvF trials. However, the simple tests comparing performance on each notation between the Number Line and Pie Chart conditions were not statistically significant (all ps > .050; 1.2 < all BF_{01} < 3.7), making it difficult to interpret this finding.

Visual Representation Task Performance

In order to further understand children's use of number lines and pie charts, we analyzed performance on the visual representation activities. On the Number-to-Position task, there was not a significant difference in PAE (excluding trial 1/2) between the Pie Chart condition, M = 6.6%, and the Number Line condition, M = 8.9%, $t(68)$ = 1.5, p = .133, Cohen's d = 0.36, BF_{01} = 1.5. On the Position-to-Number trials, children did very well overall and there was also not a significant difference between the two conditions on proportion correct, M_{NL} = 0.84, M_{PC} = 0.87, $t(61.6)$ = 0.61, p = .542, Cohen's d = 0.15, BF_{01} = 3.5.

Partitioning strategies — Next, we looked at differences in the use of overt partitioning across the two representations on the Number-to-Position task, as this is the task in which they were asked to draw on the spatial representation.

In general, pie charts were associated with partitioning, whereas number lines did not show much evidence of written partitioning strategies (see Table 1). When looking at only those children who were consistent in their strategy use (either never partitioning or always partitioning), we find a significant difference between the Pie Chart and Number Line conditions, χ^2 = 26.2, p < .001, BF_{10} = 2.6 x 10^5, with children in the Pie Chart condition much more likely to use a partitioning strategy and children in the Number Line condition much less likely to use written partitioning strategies. Children who were inconsistent in their strategy use were excluded from the prior analysis because it is unclear if they should be categorized as "partitioners", given that sometimes they did not rely on that strategy. However, when these children were grouped with the children who always partitioned, making the comparison children who partitioned on at least one trial vs. children who never partitioned, the difference between the Pie Chart and Number Line conditions is even larger, χ^2 = 30.3, p < .001, BF_{10} = 2.5 x 10^6.

Table 1

Number of Participants in Each Strategy Category

	Experiment 2		Experiment 3			
	NL	PC	Ext NL		Ext PC	
Strategy Category	Fractions	Fractions	Fractions	Decimals	Fractions	Decimals
Never Partitioning	28	5	32	33	9	8
Sometimes Partitioning	3	10	3	2	20	19
Always Partitioning	4	20	2	2	8	10

Thus, although we did not see significant performance differences when mapping fractions to number lines versus pie charts, there was a notable difference in the use of overt partitioning across our representation types. When working with number lines, children did not often use a written partitioning strategy. Although we cannot be certain what strategy these children did employ, the lack of written strategy might indicate that they used a more approximate approach, such as directly estimating the location of the fraction on the number line. On the other hand, when working with pie charts, children tended to partition the pie charts into pieces. Thus, it may be that number lines are associated with continuous magnitude in a way that encourages approximate estimation, or at least, does not encourage the use of written partitioning strategies. Conversely, individuals may be more likely to associate pie charts with discrete part-whole information in a way that does encourage the use of partitioning strategies. Notably, this was the case despite the fact that participants only mapped fractions, which may be difficult to process as a continuous magnitude (although not impossible; Hurst & Cordes, 2016, 2018; current manuscript Experiment 1). Thus, it is not that fractions always lead people to use partitioning strategies. Rather, children may attend less to the exact part-whole fraction information in the case of number line representations (by not using an overt partitioning strategy, but instead potentially engaging a magnitude estimation strategy) and to treat fractions like a part-whole structure with specific components in the case of pie charts (by using a partitioning strategy). Notably, however, given that we could only rely on overt, written strategies, it is possible that children simply instantiated these strategies in different ways (e.g., through their gestures on number lines and by drawing on pie charts). This is an important question for future work.

Overall, in Experiment 2 we did not find that a relatively short activity mapping fractions with number lines led to substantially better performance on the symbolic magnitude comparison task than practice mapping fractions to pie charts and the Bayes Factors generally suggest small evidence in favor of the null or inconclusive evidence in either direction. However, there was some evidence of differences between the two conditions, such that children in the Pie Chart condition showed more notation-dependent responding, with a slight increase in performance in the NvF trials in particular relative to the DvF trials. Based on previous work suggesting that teaching with or providing short trainings with number lines lead to better fraction understanding than area model representations, specifically pie charts (Cramer et al., 2002; Hamdan & Gunderson, 2017; Keijzer & Terwel, 2003; Saxe et al., 2013; Wang & Siegler, 2013), our lack of finding (and if anything, a pattern in the opposite direction) may be surprising. It is possible that using a number line representation is not beneficial for symbolic magnitude comparisons. However, there are several other, non-mutually exclusive, possibilities for why these activities did not lead to differences in the subsequent symbolic magnitude comparison task.

One possibility is that the primary advantage offered by the number line model is that it provides a visual representation that emphasizes the relation between different symbolic representations, including decimals, fractions, and integers – relations which are not as readily represented via pie charts (e.g., Saxe et al., 2013). In line with this hypothesis, those adults in Experiment 1 who reported using symbolic methods that integrated different magnitudes and/or different types of numbers performed better on the magnitude comparison task. Importantly, although some previous studies that showed a benefit for number lines included instruction with both fractions and decimals (e.g., Wang & Siegler, 2013), children in Experiment 2 only received number lines and pie charts with proper fractions (i.e., fractions between 0 and 1). Thus, it may be that children are unable to reap the benefits of the number line model under these circumstances, where a pie chart may be just as effective. This idea was explored in Experiment 3.

Experiment 3

In Experiment 3, we used a very similar paradigm to Experiment 2, but extended our visual mapping task to include symbolic magnitudes between 0 and 5 in fraction, decimal, and whole number notation. In so doing, we addressed two specific research questions, similar to those in Experiment 2: (1) Does practice mapping fractions, decimals, and whole numbers to number lines result in better symbolic magnitude performance than practice mapping the same values to pie charts? (2) Do children use overt partitioning strategies with number lines and pie charts and do these strategies differ for fractions and decimals? In particular, are the strategies that people engage during the mapping task more dependent on the notation they are given (decimals vs. fractions) or on the visual-spatial representation (number lines vs. pie charts)? On the one hand, in Experiment 2 children used distinct written strategies when dealing with number lines and pie charts even though they were given the same fraction magnitudes. Thus, we might expect that the same patterns would be true for decimals. On the other hand, decimals are more easily used to think about magnitudes (e.g., Hurst & Cordes, 2016, 2018) and do not directly represent information about the components. Thus, we might expect that people will be unlikely to engage in written partitioning strategies with decimals, even when given a pie chart. In Experiment 3, we included trials with fraction and decimal notation in order to directly test these distinct predictions.

Method

Participants

Seventy-four 9-12-year-old children participated in the study, separated across two conditions: Extended Number Line (N = 37, M_{age} = 10.4 years, range: 9 to 12.5 years, 17 females) and Extended Pie Chart (N = 37, M_{age} = 10.5 years, range: 9.3 to 12.3 years, 12 females). General aspects of our recruitment, consenting, and testing procedures were identical to Experiment 2.

Design and Measures

Procedures and tasks in Experiment 3 (see Figure 2 for stimuli) were identical to Experiment 2 except for the following differences in the visual representation activities: (1) participants were given 14 Number-to-Position trials, and no Position-to-Number trials (given the overall high accuracy on the Position-to-Number trials in Experiment 2) using the following rational numbers (fractions, decimals, and whole numbers): 4, 6/4, 1/5, 1.4, 3.8, 3/4, 2.3, 0.2, 8/3, 9/2, 2, 10/3, 4.1, and 2.7; (2) in the Extended Number Line Condition, all number lines were labeled with endpoints of 0 to 5, with 0.7 cm vertical hatch marks located at the whole number units on the line (i.e., 1, 2, 3, and 4); (3) in the Extended Pie Chart Condition, all pie charts were represented as five circles (radius = 1.5 cm) aligned horizontally across the center of the page (representing all values from 0 to 5) with 0.3 mm between each circle and each circle containing a radius line extending from the center of circle to the top; and (4) after the participant completed their response, the experimenter used a premade cut out of the correct answer to put a yellow hatch-mark in the correct spot on the line or fill in the correct amount in the circles (as in Experiment 2), however participants were not given evaluative verbal feedback but instead the experimenter simply said: “This shows the number”, pointing to their correct yellow answer.

Data Coding and Analysis

Coding of responses and strategies on the visual representation activities mimicked that of Experiment 2. For analyses involving performance on the spatial representations, we did not include the whole number values as those trials were trivially easy (the number line included markings at these locations and the pie charts are whole objects) and were only included to help children think about the integration across notations. Additionally, as in Experiment 2, we did not look at strategies or accuracy on the fractions involving 2 in the denominator (9/2), since we cannot disambiguate between a partitioning strategy and the correct answer. Again, two independent coders coded for strategy use (overt partitioning strategy or not, for decimals and fractions separately; all Cohen's Kappas > .85) and codes for the first coder are used in the analysis. Two independent coders also measured the accuracy of responses for average error on fractions and decimals separately (using the same model and methods as Experiment 2; all IRRs > .90) and the average value between the two coders was used in the analyses. Eleven children in the Extended Pie Chart condition responded to the booklet in an atypical fashion, making it impossible to score the accuracy of these participants in a way that is comparable to the others (e.g., coloring in the pie charts as you might a rectangle). Thus, their data were not included in analyses involving performance on the spatial mapping tasks, but they were included in analyses involving the other tasks and strategies on the spatial mapping task. All analyses were done as described in Experiment 2.

Results and Discussion

Magnitude Comparison Task

As in Experiment 2, we analyzed accuracy data from the magnitude comparison task using an ANOVA with notation (3: FvF, DvF, and NvF) and ratio (2: small and large) as within-subject factors and condition (2: Ext-NL and Ext-PC) as a between subject factor (see Figure 4). Bayesian analyses were conducted and are reported as for Experiments 1 and 2. We replicated the main effects of Experiment 2: there was a main effect of ratio, $F(1, 72) = 46.5$, $p < .001$, $\eta_p^2 = .39$, $BF_{incl} = 3.8 \times 10^8$, and a ratio by notation interaction, $F(2, 144) = 5.06$, $p = .008$, $\eta_p^2 = .07$, $BF_{incl} = 3.2$, but not a significant main effect of notation, $F(2, 144) = 2.3$, $p = .106$, $\eta_p^2 = .03$, $BF_{excl} = 4.5$. Within each notation, performance was better on the large ratio than the small ratio, FvF: $M_{small} = 0.61$, $M_{large} = 0.79$, $t(73) = 4.8$, $p < .001$, Cohen's $d = 0.61$, $BF_{10} = 1791$, DvF: $M_{small} = 0.64$, $M_{large} = 0.76$, $t(73) = 5.2$, $p < .001$, Cohen's $d = 0.60$, $BF_{10} = 1.0 \times 10^4$, and NvF: $M_{small} = 0.71$, $M_{large} = 0.76$, $t(73) = 2.2$, $p = .030$, Cohen's $d = 0.24$, $BF_{10} = 1.3$. Furthermore, NvF showed significantly smaller ratio effects than both FvF ($p = .003$) and DvF ($p = .028$), which were not significantly different from each other ($p = .264$). However, there were no significant main or interaction effects involving condition (ps > .100; Condition $BF_{excl} = 3.8$, Condition x Ratio $BF_{excl} = 5.9$, Condition x Notation $BF_{excl} = 7.9$, Condition x Ratio x Notation $BF_{excl} = 8.4$). Therefore, as in Experiment 2, a brief practice with number lines or pie charts did not impact children's subsequent performance on a symbolic comparison task, even when the mapping activities involved different kinds of numbers. Furthermore, the Bayes Factors for the effects involving condition provide small-strong evidence for the null hypothesis that there is no difference in performance on the symbolic comparison task between the two conditions.

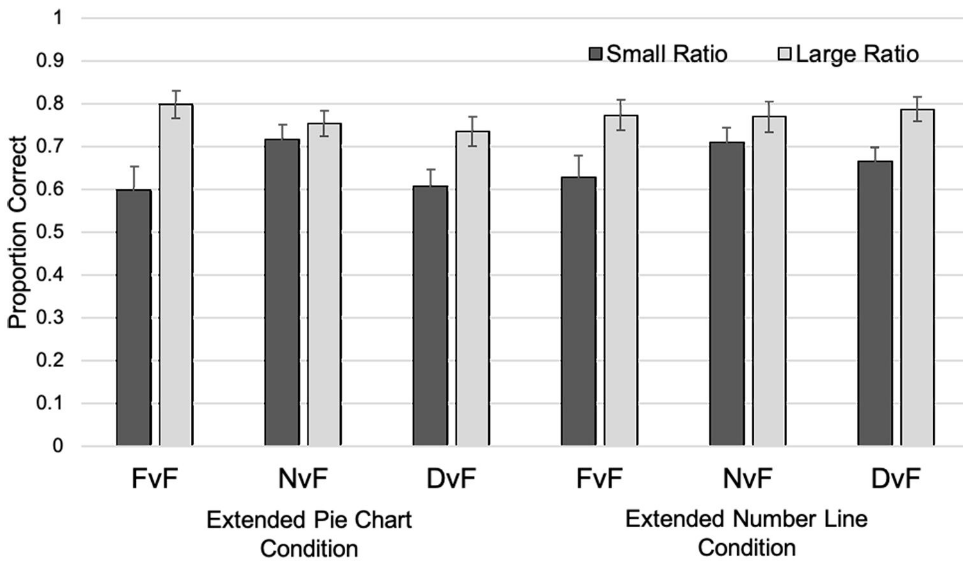

Figure 4. Children's performance on Experiment 3 separated by condition and presented by notation block and ratio.

Note. Error bars are standard error of the mean.

Visual Representation Task

Next, we analyzed performance on the spatial representation activities using a repeated measures ANOVA with Notation (2: Fractions, Decimals) as a within-subject factor and Condition (2: number lines vs. pie charts) as a between-subject factor on PAE. No main effects or interactions were significant; main effect of notation: $F(1, 61) = 2.47$, $p = .121$, $\eta_p^2 = .04$, $BF_{exclu} = 1.2$; main effect of condition: $F(1, 61) < 0.01$, $p = .982$, $\eta_p^2 < .001$, $BF_{exclu} = 3.8$; interaction: $F(1, 61) = 1.08$, $p = .302$, $\eta_p^2 = .02$, $BF_{exclu} = 2.5$: $M_{frac\text{ - }NL} = 11.3$, $M_{frac – PC} = 9.6$, $M_{dec – NL} = 7.0$, $M_{dec – PC} = 8.7$.

Additionally, we explored written partitioning strategies across the two conditions for fractions (although, as in Experiment 2, excluding 9/2 given that almost every participant provided the answer without overt partitioning strategies) and decimals separately. The full counts broken down by condition and age group are presented in Table 1.

Most children in the Extended Number Line condition consistently provided the answer without using a written partitioning strategy for both fractions and decimals. In the Extended Pie Chart condition, however, children showed much less consistency in their strategies, with many children using partitioning strategies on some trials but not all of them. Furthermore, when looking at just those children who consistently used or did not use a partitioning strategy the pattern of responses between Extended Number Line and Extended Pie Chart conditions were significantly different for both fractions, $\chi^2 = 12.2$, $p < .001$, $BF_{10} = 77.3$, and decimals, $\chi^2 = 16.9$, $p < .001$, $BF_{10} = 809$. As in Experiment 2, children who inconsistently partitioned the visual reference were excluded from these analyses. However, when inconsistent partitioners are grouped with the consistent partitioners (i.e., comparing the number of children who partition at least one trial versus children who never partition), the same difference is found between the Extended Number Line and Extended Pie Chart conditions for both fractions, $\chi^2 = 28.9$, $p < .001$, $BF_{10} = 1.1 \times 10^6$, and decimals, $\chi^2 = 34.2$, $p < .001$, $BF_{10} = 2.3 \times 10^7$.

Thus, although children did not show differences in their error between number lines and pie charts, they did show significant differences in their strategies. Children were more inconsistent in their approach to the pie charts, sometimes dividing them and sometimes not, but children were very unlikely to divide up the number lines and tended to simply put their answer on the line without indicating additional markings. In line with Experiment 2, this suggests that children may be inclined to adapt their strategy based on the visual-spatial representation, using a part-whole partitioning strategy for pie charts and using a different strategy for number lines. As in Experiment 2, we cannot know for certain what children who did not partition were doing, but it may be that these children were using more approximate estimation strategies that did not require providing the exact part-whole information. Notably, the number of children using each type of strategy is nearly identical for the fractions and decimals within each condition (both *p*s > .500, comparing fraction vs. decimal, within the NL and PC conditions separately), suggesting that the notation did not substantially impact strategy selection and instead the spatial representations impacted written strategy use for both fractions and decimals similarly.

Comparing Experiments 2 and 3

We also directly compared performance on the visual mapping tasks across the two experiments to investigate how performance and strategy use when mapping between symbolic and spatial representations may differ for proper fractions between 0 and 1 (Experiment 2) and improper fractions between 0 and 5 (Experiment 3). First, we compared PAE on the fraction trials only for 0-to-1 fractions (Experiment 2) and 0-to-5 fractions (Experiment 3), using a 2 (Experiment) x 2 (Spatial Model: number line or pie chart) between-subject ANOVA. There was not a main effect of Experiment, $F(1, 129) = 2.3$, $p = .134$, $\eta_p^2 = .02$, $BF_{exclu} = 1.7$, Spatial Model, $F(1, 129) = 1.4$, $p = .238$, $\eta_p^2 = .01$, $BF_{exclu} = 2.6$, or an interaction, $F(1, 129) = 0.03$, $p = .870$, $\eta_p^2 < .001$, $BF_{exclu} = 3.9$.

Next, we compared the strategies children engaged in across the two experiments (see Table 1 for number of children in each category across experiments). There was not a significant difference between the Experiment 2 and Experiment 3 number line activities, when comparing the number of children who consistently used partitioning strategies or never used partitioning strategies on fraction trials, $\chi^2 = 0.87$, $p = .350$, $BF_{01} = 3.8$, and this is still true when the children who were inconsistent are grouped with those who always partitioned, to compare children who ever partitioned versus those who never partitioned, $\chi^2 = 0.55$, $p = .460$, $BF_{01} = 3.6$. However, there was a significant difference between Experiments 2 and 3 on the conditions using pie charts, when comparing the number of children who always partitioned vs. never partitioned, $\chi^2 = 4.9$, $p = .026$, $BF_{10} = 3.8$. Notably, this difference is not significant when the children who were inconsistent are grouped with those who always partition so that we compare children who partition on any trial vs. those that never partition, $\chi^2 = 1.2$, $p = .282$, $BF_{01} = 2.5$. Although children in Experiment 2 were likely to consistently partition the pie chart, children in Experiment 3 were less consistent, with some children always partitioning, others never partitioning, and many inconsistently using partitioning from trial to trial. Thus, when these “inconsistent” children are removed, the two experiments look very different – but when the “inconsistent” children are grouped with children who always partitioned, then the experiments look fairly similar (although, there is only weak evidence for the null in this case).

However, it should be noted that children were not randomly assigned to experiment and data from the two experiments were collected at different times. Thus, the cross-experiment comparisons should be interpreted with caution. However, they do suggest that further work directly manipulating this difference (values beyond 0 to 1 and from multiple notations) may provide insight into children’s approach to these distinct tasks.

General Discussion

Across three experiments, we investigated the relation between symbolic magnitude understanding and the use of number lines and pie charts for thinking about the *magnitudes* of symbolic rational numbers. In general, and in contrast to our predictions, we did not find that our number line mapping tasks primed children to better compare symbolic fraction magnitudes, regardless of whether the mapping task involved proper fractions only or a mix of notations and magnitudes. However, we did find consistent evidence that children were more likely to apply explicit part-whole partitioning strategies with pie charts compared to number lines regardless of the specific magnitude or notation (Experiments 2 and 3) and that symbolic methods for reasoning about fraction may be associated with better fraction magnitude comparison performance than visual part-whole methods, at least in adults (Experiment 1). Furthermore, despite these differences in written strategy use, children did not show differences in error when using number lines vs. pie charts. Lastly, as evidenced by the significant ratio effects in all experiments, we replicated previous work with children and adults suggesting that they can represent the magnitudes of fractions, decimals, and whole numbers in an integrated fashion (Ganor-Stern, 2012, 2013; Hurst & Cordes, 2016, 2018) and further extend these findings to suggest that adults process fraction magnitude information in approximate arithmetic contexts as well.

These findings have implications for the way we think about fraction concepts, for fraction education, and for better understanding how visual representations may differentially evoke specific strategies more generally. In particular, these findings highlight that the magnitude component and the part-whole component of fraction symbols may be differentially accessed or used, depending on the specific spatial representations. Although adults may not spontaneously visualize fractions using number lines (Experiment 1), when children were forced to map between fractions and number lines they were less likely to engage in part-whole partitioning strategies, compared to when they mapped fractions to pie charts. Importantly, however, we cannot be certain what strategy children were using. One possibility may be that children attempted to map the fraction to the number line with a holistic magnitude approach that did not require them to partition or divide the line at all. The finding that partitioning strategies differed as a function of visual representation (number lines versus pie charts) is consistent with the idea that pie charts may be more likely to promote a focus on discrete parts, whereas number lines promote attention to continuous numerical magnitude. In addition to having implications for education and how these different conceptual fraction features (magnitude and part-whole components) are taught, this work also has broader implications for how best to convey fractional information in health, marketing, and/or financial scenarios. For example, when presenting fractional or proportional information, we may want to consider whether the fraction magnitude or the specific fractional components are most relevant and select different representational strategies accordingly.

In addition, although children in Experiment 3 tended to not use partitioning strategies for number lines (similarly to children in Experiment 2), they were less consistent in their strategy use with pie charts, both across and within children. This pattern with pie charts in Experiment 3 is distinct from the pattern in Experiment 2, in which children relied almost exclusively on part-whole partitioning strategies for pie charts. This may suggest that the inclusion of different notations on the same representation or the fact that the target magnitudes extended beyond one prompted some children to not bother partitioning the pie charts, maybe because they engaged in estimation strategies rather than partitioning strategies. Adults (reported in the Supplementary Materials, see Hurst, Massaro, & Cordes, 2020b) show a similar pattern such that when adults did partition, it tended to be on the 0 to 1 pie chart, however we generally saw lower levels of partitioning across the board in the adult data. This may be because adults were generally more likely to engage estimation strategies regardless of the representation. This is consistent with other work showing that adults do show ratio effects with both pie charts and number lines, suggesting that they are able think about magnitude without the presence of discretized units (Hurst, Relander, & Cordes, 2016). Alternatively, this qualitative difference between children and adults may reflect more general differences about their tendency to provide written information as part of their answer. It may be that the adults, who have not been recently instructed on how to use pie charts or number lines, were simply less likely to think including the partitioning lines was a necessary component of their response. Importantly, however, these patterns may suggest important educational and developmental differences in how people use visual presentations of fractional information.

When taken together, these findings provide some evidence suggesting that in order to teach fraction *magnitudes* it may be important to emphasize the relations between different magnitudes, in different notations, for values beyond just zero-to-one using a number line spatial representation. Given that children and adults may use number lines with an approximate, estimation-based approach, using this representation for magnitude may more closely align with the way number lines are used for whole numbers as well. On the other hand, children tended to use pie charts with a much more part-whole approach, especially for proper fractions. Thus, it may be that pie charts provide a benefit for conceptualizing the part-whole components of fractions. This is something that should continue to be investigated further.

Given these findings suggesting that the pie charts, particularly the 0-1 pie chart, may have been approached in part-whole way with overt partitioning, and the number lines were not, why did these strategy differences not result in performance differences, either in terms of error on the actual spatial mapping task or on the subsequent symbolic magnitude comparison task? In terms of the number line and pie charts tasks specifically, it may be that the different strategies were well-aligned with the needs of the particular representation, allowing children to maximize their performance on both representations. Alternatively, the differences in overt-written strategies may not provide a complete picture. For example, it may be that children partitioned both number lines and pie charts, but instantiated them differently, such as using *mental* partitioning strategies or *physical* strategies involving their fingers or hands, both of which we were not able to capture within the current data. In either case, this is informative for how people approach number line and pie chart spatial models, but is one possible explanation for why these differences in approaches did not culminate in differences in error patterns or in performance on the subsequent symbolic magnitude comparison task.

Furthermore, there are several other related possibilities as to why these differences in written strategy use on the priming task did not lead to differences in the subsequent symbolic magnitude comparison performance. One possibility is that the priming task did not involve explicit instruction and/or may have not been intensive enough to alter participant's approach to comparing symbolic magnitudes. Although participants did receive feedback on the task, they were not instructed as to how to use the number lines or pie charts. However, many of the recent studies showing an impact of number lines were with younger children who did not have much fraction understanding already, and as such, the primary manipulation involved instruction (e.g., Gunderson et al., 2019; Hamdan & Gunderson, 2017). It may be that the older children in the current study (who already had some knowledge of fractions) may have been more likely to default to their learned or practiced way of thinking about fractions, such that our visualization activities may have not altered their approach to the tasks. Moreover, although we tested children who are actively in the process of learning fractions and decimals and adults who have already completed instruction on these topics, we did not systematically collect information about the educational experience of these participants. In particular, it may be that some participants already had substantial experience with number lines, used number lines for whole numbers but not fractions, or had not used number lines in any capacity. Thus, the older age of these children, and therefore the more fraction experience they already have, may have limited the impact of our priming manipulation. Future studies may consider increasing the number of trials or difficulty of the priming tasks and/or explicitly drawing participant's attention to the use of mental visualization strategies, in order to promote greater transfer from the priming task to the magnitude comparison task. Additionally, future research should explicitly compare the malleability of fraction visualization across middle childhood to determine whether younger children may have more malleable fraction visualization strategies. If so, it may be that introducing the magnitudes of non-whole number quantities onto number lines before young children are introduced to the formal part-whole aspect of fraction notation can improve children's learning of rational number concepts more generally.

Conclusions

In summary, the current study reports three experiments providing convergent evidence that adults and children are able to mentally represent the magnitudes of fractions, decimals, and whole numbers in an integrated, approximate, and ordered way. In addition, despite not seeing substantial differences in accuracy or in priming between the pie chart and number line activities, our findings do provide some evidence that pie charts may be more aligned with part-whole based *strategies*, whereas number lines may be more aligned with magnitude-based strategies. Together, our results provide some insight into the use of visual-spatial representations for fraction education and indicate that both number lines and pie charts may convey distinct, but important, fraction information, providing a strong case that both visual representations be used to promote fraction understanding.

Notes

i) Although these terms (Number-to-Position and Position-to-Number) have typically been used for number line mappings only, for ease in communication we will be using them for mapping with both number lines and pie charts.

ii) We provided the denominator because we were concerned that without this cue the task would be too confusing or difficult for the children, and children might not provide a fraction at all but might provide a whole number or a decimal instead. Based on our results, however, this decision may have inadvertently made the task too easy.

Funding

The authors have no funding to report.

Competing Interests

The authors have declared that no competing interests exist.

Acknowledgments

The authors have no support to report.

Author Note

A version of this manuscript with a subset of the data was reported in M.A.H.'s PhD dissertation in August, 2017.

Data Availability

For this study, a dataset is freely available (Hurst, Massaro, & Cordes, 2020a).

Supplementary Materials

The Supplementary Materials published on PsychArchives include a written methods and results section for the smaller adult samples collected as a comparison sample to the children in Experiments 2 and Experiments 3. These data were largely consistent with the child data and were much smaller samples, and thus are included as supplements for completeness.

The OSF Project page includes (1) all of the data from all three experiments reported here and the adult samples reported in the Supplementary Materials, (2) all the analysis code in R used to analyze all the data, and (3) all the materials, including image files, administration programs, and PDFs of the paper-pencil priming materials. The wiki on the OSF page includes additional details about each of the files provided (for unrestricted access, see Index of Supplementary Materials below).

Index of Supplementary Materials

Hurst, M. A., Massaro, M., & Cordes, S. (2020a). *Spatial mapping of fraction magnitudes* [Research data and materials]. OSF. https://osf.io/eycdk

Hurst, M. A., Massaro, M., & Cordes, S. (2020b). *Supplementary materials to "Fraction magnitude: Mapping between symbolic and spatial representations of proportion"* [Additional analyses]. PsychOpen. https://doi.org/10.23668/psycharchives.3164

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