

Ec871

Factor Proportions Model Exercise

Suppose production functions for two goods, food (F) and shelter (S) are functions of two factors, labor (L) and land (R), where:

$$\begin{aligned} F &= \min [L_F/2, R_F] \\ S &= \min [L_S, R_S/2] \\ 100 &= L_F + L_S \\ 100 &= R_F + R_S, \end{aligned}$$

and the subscripts denote the sector to which the factor is allocated.

Aggregate utility is $U = \min [X_F/2, X_S]$, where X denotes consumption of the subscripted good.

(i) Derive the autarky competitive equilibrium price vector and production bundle. Show that the factor income suffices to purchase the commodity bundle.

(ii) Suppose the country can trade all it wants at a relative price of unity. Derive the equilibrium consumption, production and trade bundles; and the price vector. Show that the factor income suffices to buy the consumption bundle.

(iii) Suppose the relative price of F rises to 1.1. Compute the new equilibrium price vector and production and trade bundle.

(iv) Suppose the labor supply rises to 110. Compute the new equilibrium bundle and price vector, and note any relation to results in (iii).

(v) Suppose another economy, differing from the first only in that $U^* = \min [F^*, S^*/2]$, enters trade with the first (with labor supply at 100 again). Compute the autarky equilibrium for the new economy, and the free trade equilibrium.

(vi) Now suppose the other economy has the same utility function as the first, but differs in that labor supply equals 150. Compute the foreign autarky equilibrium and the trade equilibrium. Show that the Heckscher-Ohlin prediction holds.

(vii) show that even when there is substitutability in production and consumption, much of the foregoing still holds. (How much?)

Production indeterminacy

(viii) Now suppose that a third good, Entertainment (E) is produced with technology

$$E = \min [L_E, R_E].$$

(a) If international prices are unity for food and shelter, what must be the value of P_E to permit all goods to be produced? Show that the production bundle is indeterminate with this price vector.

(b) Now suppose that P_E is equal to one. What will the new factor price and production bundle be?