Notes on Ricardian Continuum Model with Trade Costs

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Suppose there are trade costs of the iceberg melting type such that τ percent of a shipment melts away, $1+\tau = t$ units must be sent for 1 unit to arrive. The trade cost leads to a range of nontraded goods where absolute advantage is not large enough to overcome the trade cost.

Let goods be indexed in order of increasing home comparative advantage. Relative unit labor requirements for any good $z \in [0,1]$ are given by $A(z) \equiv a(z)/a^*(z)$ and $A' < 0$. The foreign country exports low z goods, for which $a^*(z)w^*t < a(z)w$, where t is the trade cost factor. Home exports a high z range for which $a^*(z)w^* > ta(z)w$.

The marginal export of the home country is defined by

$$a(\bar{z})wt = a^*(\bar{z})w^*$$

The marginal export of the foreign country is defined by

$$a(\bar{z}^*)w/t = a^*(\bar{z}^*)w^*$$

The cumulative expenditure on goods exported by the foreign country is given by

$$B^*(\bar{z}^*) = \int_0^{\bar{z}^*} b(x)dx$$

The cumulative expenditure on goods exported by the home country is given by

$$B(\bar{z}) = \int_{\bar{z}}^1 b(x)dx$$

The requirement that home income is equal to world expenditure on goods produced at home yields

$$B[wL + w^*L^*] + (1 - B - B^*)wL = wL$$

The requirement that foreign income is equal to world expenditure on goods produced by the foreign country yields

$$B^*[wL + w^*L^*] + (1 - B - B^*)w^*L^* = w^*L^*$$

Either of the last two equations simplify as a ratio:

$$\frac{w^*}{w} = \frac{B^*}{B}\frac{L}{L^*}$$

This equation combines with the two marginal export equations

$$A(\bar{z})t = w^*/w$$

and

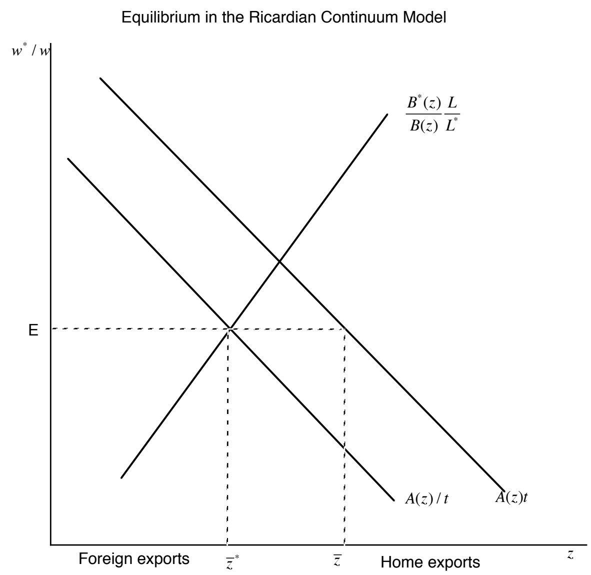
$$A(\bar{z}^*)/t = w^*/w$$

to solve for the three unknowns $w^*/w, \bar{z}, \bar{z}^*$.

For diagrammatic convenience, it is handy to use the two marginal export equations to solve for

$$\frac{A(\bar{z})}{A(\bar{z}^*)}t^2 = 1 \Rightarrow \bar{z}(\bar{z}^*, t),$$

where $\bar{z}$ is increasing in $\bar{z}^*$. Then plot in $(w^*/w, z)$ space the functions $A(z)/t$ and $LB^*(z)/L^*B[z^c(z, \tau)]$. The solution yields the equilibrium relative wage and $\bar{z}^*$.



A specific example illustrates. Suppose that $B^* = \bar{z}^*$, a uniform distribution of tastes. Then $B = 1 - \bar{z}$. Let $A(z) = A/z, A > 0$. The two cutoff equations imply $\bar{z} = t^2\bar{z}^*$. Then the equilibrium $\bar{z}^*$ is solved from

$$\frac{\bar{z}^*}{1 - \bar{z}^* t^2}\frac{L}{L^*} = \frac{A}{\bar{z}^* t} \tag{1}$$

(1) implies a quadratic equation $t(L/L^*)(\bar{z}^*)^2 + At^2\bar{z}^* - A = 0$. The positive root is the solution. After some algebraic manipulation this yields

$$\bar{z}^* = \frac{1}{2}\frac{L^*}{L}At[-1 + \sqrt{1 + 4/(t^3 AL^*/L)}], \tag{2}$$

and

$$\frac{w}{w^*} = \frac{1}{2}\frac{L^*}{L}t^2[-1 + \sqrt{1 + 4/(t^3AL^*/L)}]$$

where the second equation uses the home export cutoff equation for the solution for the factoral terms of trade.

For $A = 1/2$ and $L^*/L = 1$, symmetric equilibrium obtains. The foreign range of exports $\bar{z}^*$ is increasing in AL^*/L, the effective size of the foreign country.

It can be shown that $d\bar{z}^*/dt < 0$. It is convenient to work with the elasticity.

$$\frac{t\partial\bar{z}^*}{\bar{z}^*\partial t} = 1 - \frac{3}{\frac{1}{2}\frac{L^*}{L}At^3[-1 + \sqrt{1 + 4/(At^3L^*/L}]\sqrt{1 + 4/(At^3L^*/L}}. \quad (3)$$

The right hand side simplifies to $1 - \frac{3}{1+2-\sqrt{1+4/(At^3L^*/L}} < 0$.

The factoral terms of trade effect of a change in t is ambiguous. It is negative as L^*/L is large (smaller countries suffer terms of trade deterioration as trade costs rise) and as A is small (the home country is relatively more efficient, effectively increasing its relative size).

Welfare rises with a fall in t directly, helping to offset the possibly negative factoral terms of trade effect. Welfare per capita is given by the real wage $\ln w - \ln P$ where the log price index is given by $\ln P = \int_0^1 b(z)\ln p(z)dz$. With full use of the properties of the model, the condition for welfare loss can be shown to be $d\ln w/d\ln t > 1$,[1] where the foreign wage is the numeraire so w is the factoral terms of trade. The condition for welfare loss looks extreme, but it cannot be ruled out.

1 Real Wages

$$\ln P = (\ln w^* + \ln t)\int_0^{\bar{z}^*} b(x)dx + \ln w\int_{\bar{z}^*}^1 b(x)dx + \int_0^{\bar{z}^*} b(x)\ln a^*(x)dx + \int_{\bar{z}^*}^1 b(x)a(x)dx.$$

The comparative static derivative of real income (real wages) with respect to exogenous variables follows readily. For trade costs, differentiating $\ln P$

[1]See next section. Essentially, both t and the factoral terms of trade affect welfare through the goods imported, and the condition means world prices fall by less than the home relative wage.

with respect to $\ln t$, the effects through the changes in endpoints cancel out, so

$$\frac{d\ln P}{d\ln t} = B^* + (1 - B^*)\frac{d\ln w}{d\ln t}.$$

The equation uses the numeraire property of $w^* \Rightarrow d\ln w^* = 0$. The change in real income is given by

$$d\ln w - d\ln P = B^*(-1 + d\ln w/d\ln t).$$

Other comparative static derivatives have similar structure.

A perverse case where home benefits from a rise in trade cost is illustrated below.

Ricardian Continuum Example

This numerical example illustrates factoral terms of trade in the Ricardian continuum example as trade costs change. L is the home relative endowment of labor. w is the home relative wage. The formulas are taken from the preceding notes. For the case drawn, the home country is much larger than the foreign country so its relative wage is always below one, given the position of the relative technology function : A (z) = 1 / z.

Home benefits from the rise in t over the range below about t = 1.4.

```
Clear[L, A, t, w];
A = 1;
L = 10;

w := (t^2 / (2*L)) * F;
F := -1 + (1 + 4 L / (A*t^3)) ^0.5;

Plot[w, {t, 1, 2}]
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Out[55]=