A Strong-Scatter Theory of Ionospheric Scintillations for Two-Component Power Law Irregularity Spectra

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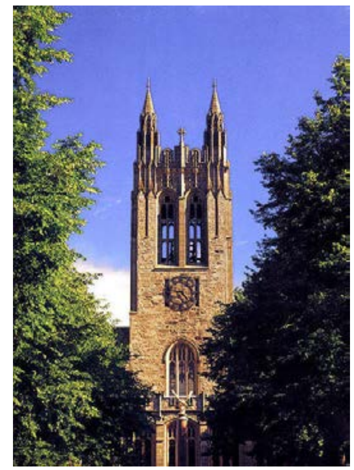

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Introduction

- We extend the phase screen theory for ionospheric scintillation to the case where the irregularities follow a two-component power law spectrum.
- The two-component model includes, as special cases, an unmodified power law and a modified power law with an outer scale or inner scale. We use it to explore the effects of a spectral break on the scintillation statistics.
- We solve the 4th moment equation for propagation through two-dimensional field-aligned ionospheric irregularities. A specific normalization is invoked to exploit self-similar properties of the problem and achieve a universal scaling.
- The numerical algorithm employs specialized quadrature algorithms and a Python library for arbitrary-precision floating-point arithmetic (mpmath).
- These advancements enable simulation of stronger scattering conditions than previously possible, enabling us to validate new theoretical predictions for the behavior of the scintillation index and intensity correlation length.

Solution to 4th Moment Equation

- Spectrum of intensity fluctuations: $$\Phi_I(q) = \int \exp\left\{-g\left(r, q\rho_F^2\right)\right\}\exp(-iqr)dr$$

Fresnel scale: $\rho_F = (z/k)^{1/2}$

- Structure interaction term: $$g(r_1, r_2) = 8\int \Phi_I(q)\sin^2(r_1 q/2)\sin^2(r_2 q/2)\,\mathrm{d}q/2\pi$$

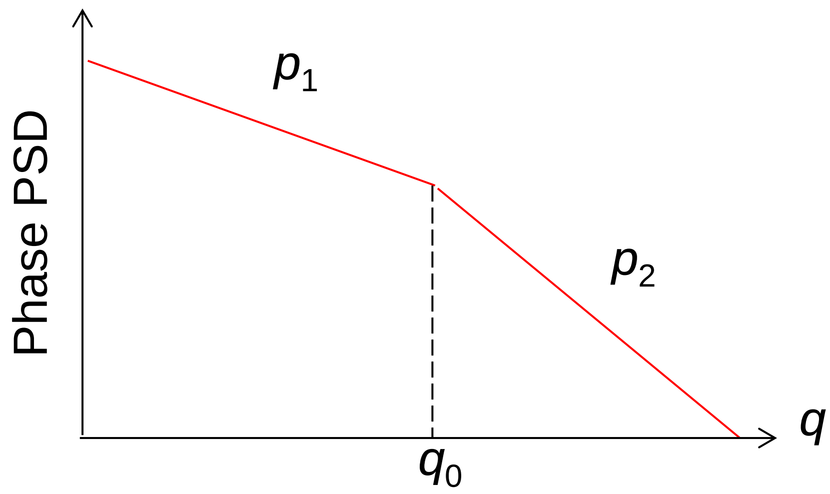
Scintillation Statistics

- Scintillation index: $$S_4^2 = \tfrac{1}{2\pi}\int \Phi_I(q)dq - 1$$

- Intensity correlation: $$R(r) = \tfrac{1}{2\pi}\int \Phi_I(q)\cos(qr)dq$$

Two-Component Structure Model (Rino)

Phase spectrum: $$\Phi_{\delta\phi}(q) = C_p \begin{cases} |q|^{-p_1}, & q < q_0 \\ q_0^{p_2 - p_1} |q|^{-p_2}, & q > q_0 \end{cases}$$



- p_1=0 gives an outer scale
- p_1=p_2 gives an unmodified power law
- p_2>p_1 gives a two-component spectrum
- Can emulate an inner scale if p_1<3, p_2>3

With this piecewise model we can produce analytic results valid for general p_1<5, p_2>1.

Compact Strong Scatter Model (Rino)

- We use the Fresnel scale to normalize the phase and intensity spectra:

$$P(\mu) = \Phi_{\delta\phi}\left(\mu;\rho_F\right)/\rho_F = \begin{cases} U_1\mu^{-p_1}, & \mu \le \mu_0 \\ U_2\mu_0^{p_2-p_1}\mu^{-p_2}, & \mu > \mu_0 \end{cases}$$

$$I(\mu) = \Phi_I\left(\mu;\rho_F\right)/\rho_F = \int \exp\left\{-\gamma(\eta,\mu)\right\}\exp(-i\eta\mu)d\eta$$

Normalized spatial wavenumber: $\mu = q\rho_F$

Normalized turbulent strength parameters: $U_1 = C_p\rho_F^{p_1-1}, \quad U_2 = C_p q_0^{p_2-p_1}\rho_F^{p_2-1}$

- Define *U** as the normalized phase spectral power *at the Fresnel scale*

$$U^* = \begin{cases} U_1, & \mu_0 \ge 1 \\ U_2, & \mu_0 < 1 \end{cases}$$

- Parameters p_1, p_2, μ_0, and U^* specify all solutions for 2-component spectra (i.e. different combinations of perturbation strength, propagation distance, and frequency produce the same results).

New Theoretical Results

- **Limiting behavior of S_4**
  - If $1<p_1<3$: $S_4 \rightarrow 1$ (although a local maximum exceeding unity may be achieved for intermediate values of the scattering strength).
  - If $3<p_1<5$: $$S_4 \rightarrow \sqrt{\frac{p_1 - 1}{5 - p_1}}, \quad 3 < p_1 < 5$$

    Quasi-saturation with $S_4>1$ dictated by spectral slope of large scale irregularities

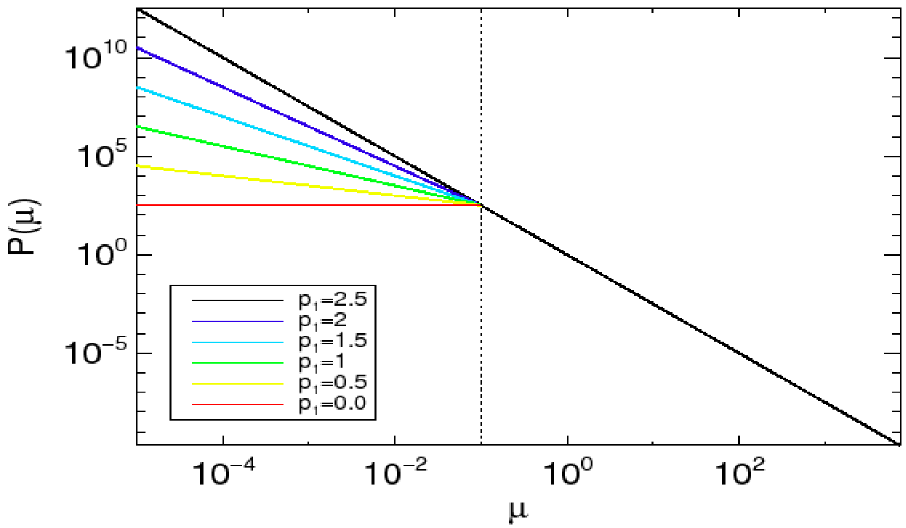
- **Limiting behavior of correlation length ξ_c**
  - If $p_1<p_2<3$: $$\xi_c \rightarrow C_1(p_1, p_2, \mu_0)(U^*)^{-1/(p_2-1)}$$

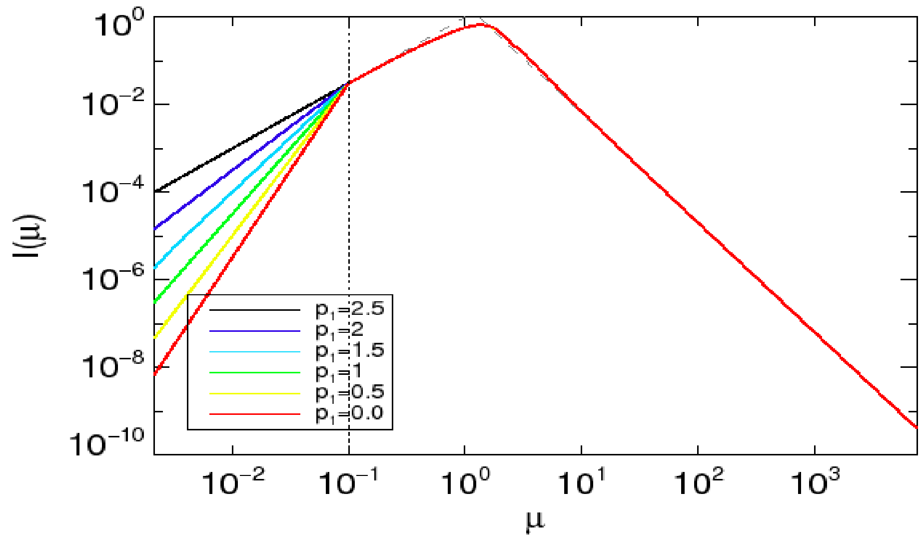
    Rate of decrease with increasing U^* dictated by slope of small scale irregularities
  - If $p_1<3$, $p_2>3$: $$\xi_c \rightarrow C_2(p_1, p_2, \mu_0)(U^*)^{-1/2}$$

    Rate of decrease independent of irregularity slopes. Applies to shallow spectra with inner scale or steep spectra with outer scale (expected to occur in nature).

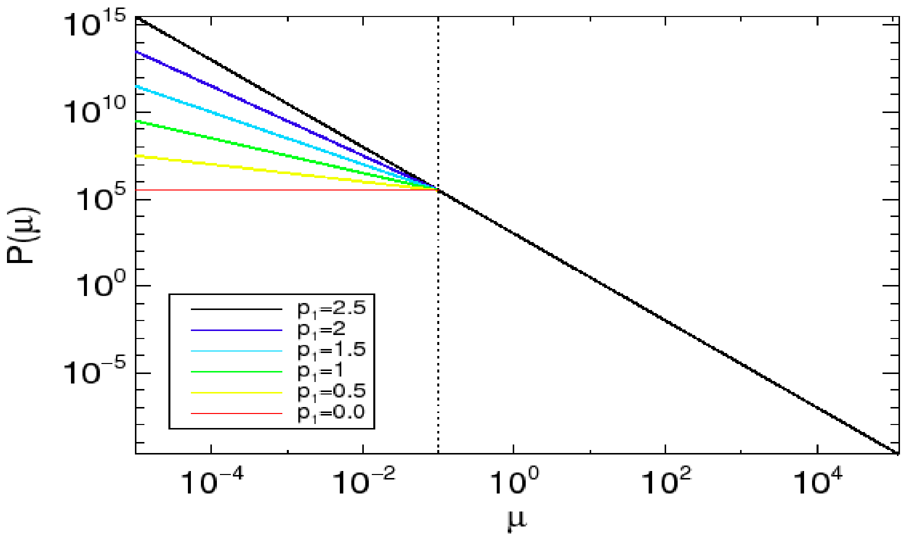
Effect of an Outer Scale for Shallow Spectra, p_2=2.5

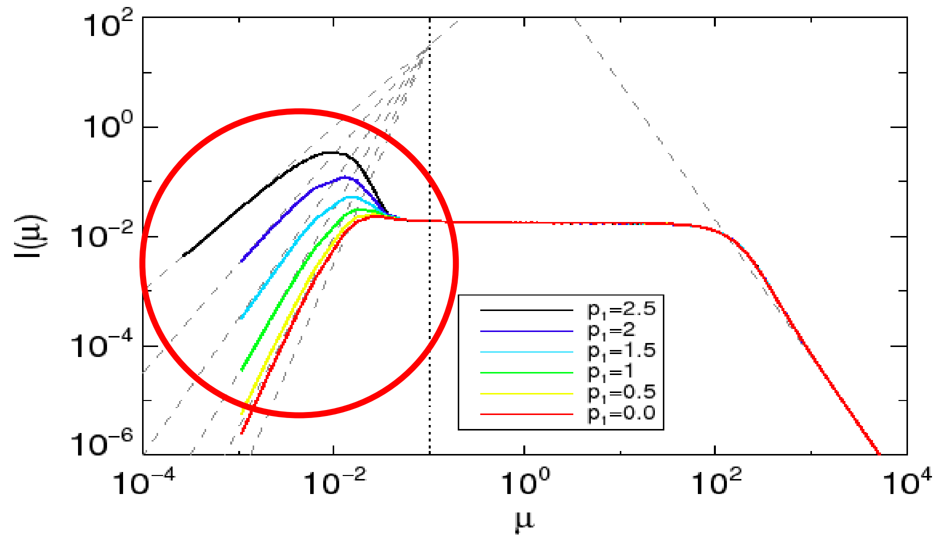
Moderate Scatter *U=1**





Strong Scatter *U=1000**

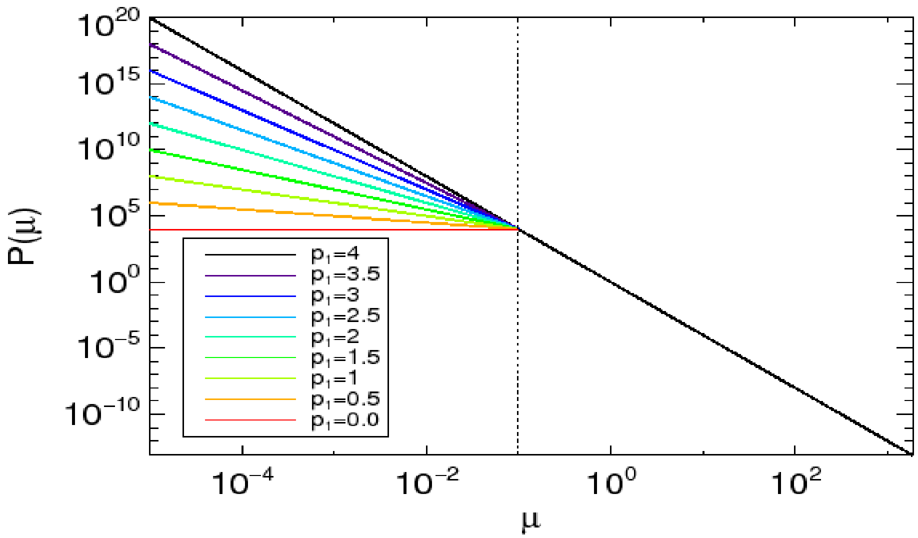


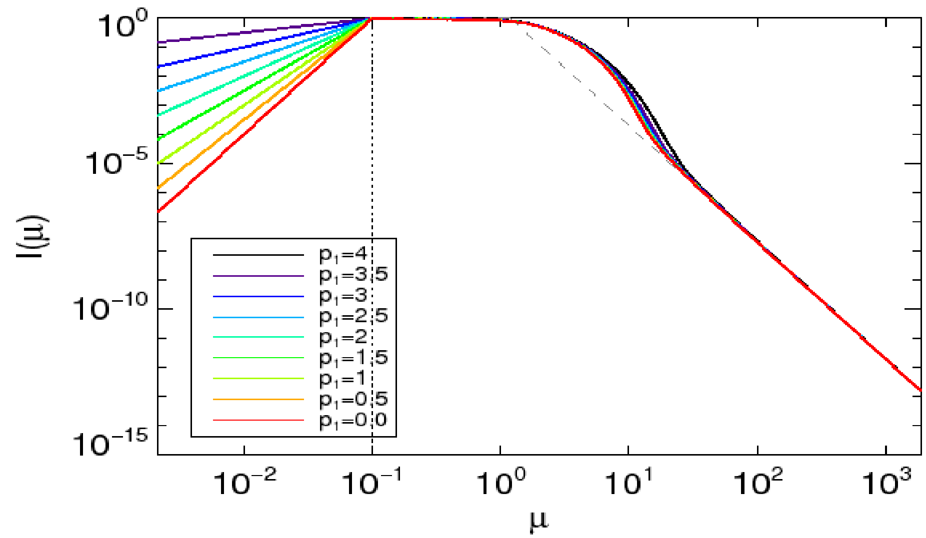


- Eroding large scale irregularity structure suppresses low frequency enhancement
- Outer scale does not effect high frequency content (correlation length unaffected)

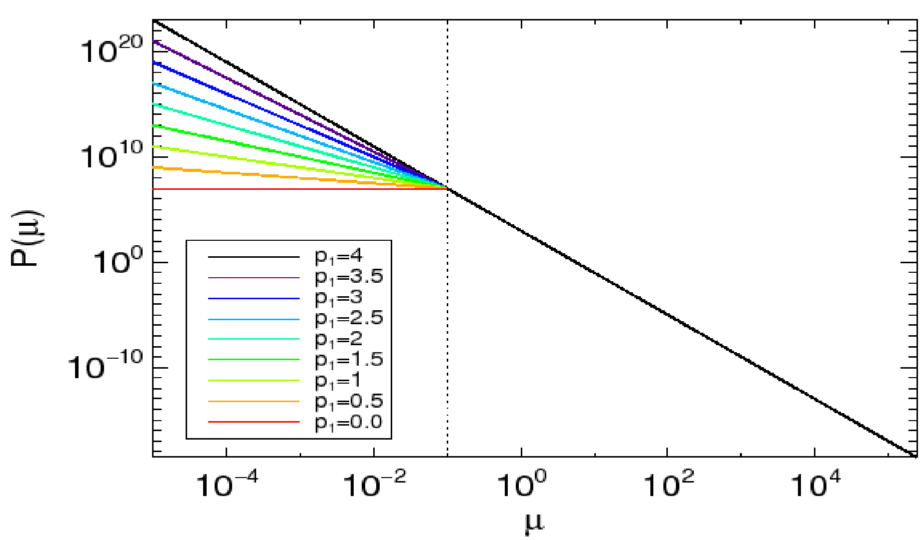
Effect of an Outer Scale for Steep Spectra, p_2=4.0

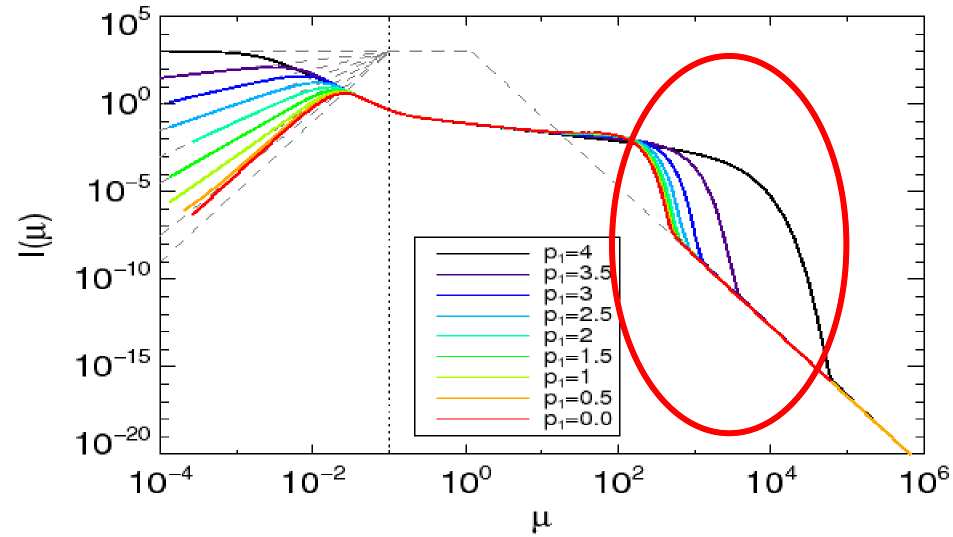
Moderate Scatter *U=1**





Strong Scatter *U=1000**

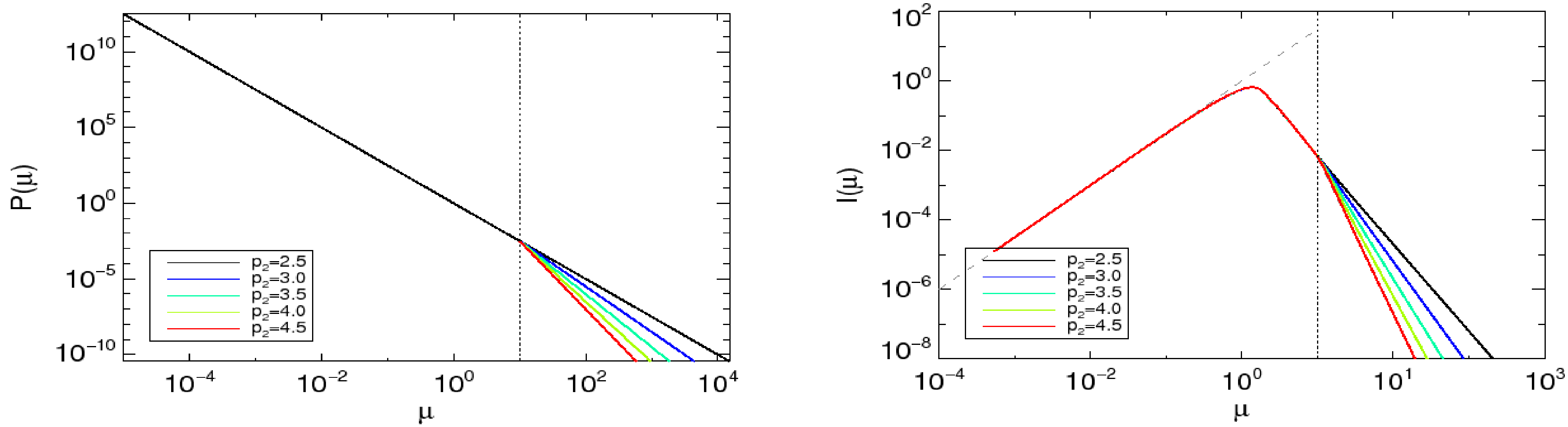




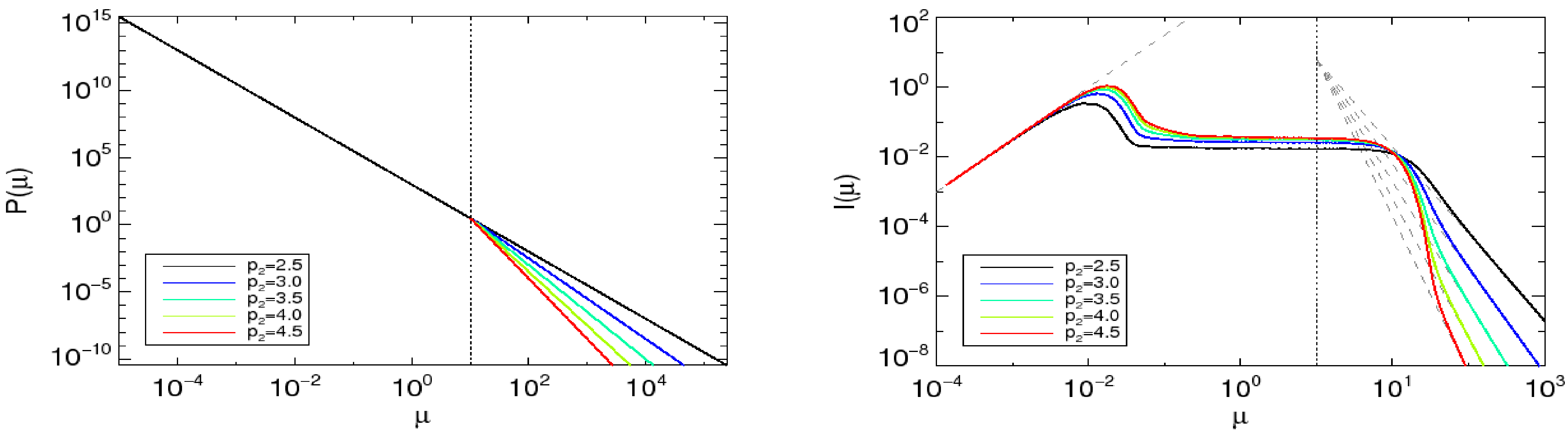
- Eroding large scale irregularity structure suppresses high frequency spectral broadening
- This mitigates strong focusing by large scales which commonly produces S_4>1

Effect of an Inner Scale for Shallow Spectra, p_1=2.5

Moderate Scatter *U**=1



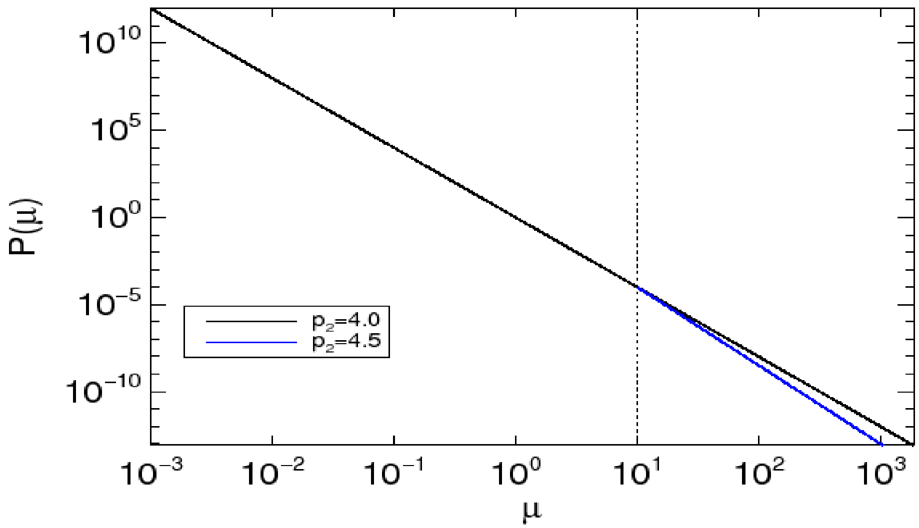
Strong Scatter *U**=1000

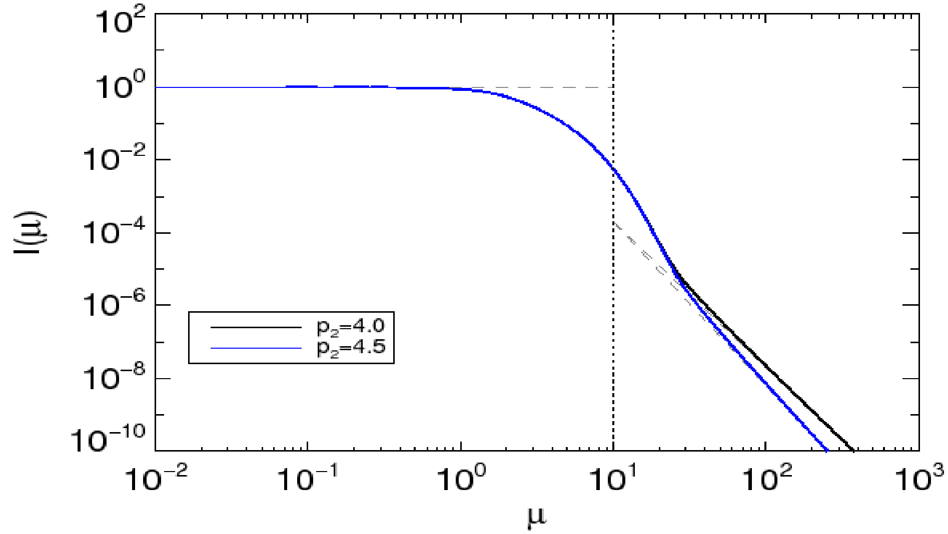


- Erosion of small scale structures promotes focusing by those which remain.
- Removed power is redistributed to scales up to the break to maintain saturation.

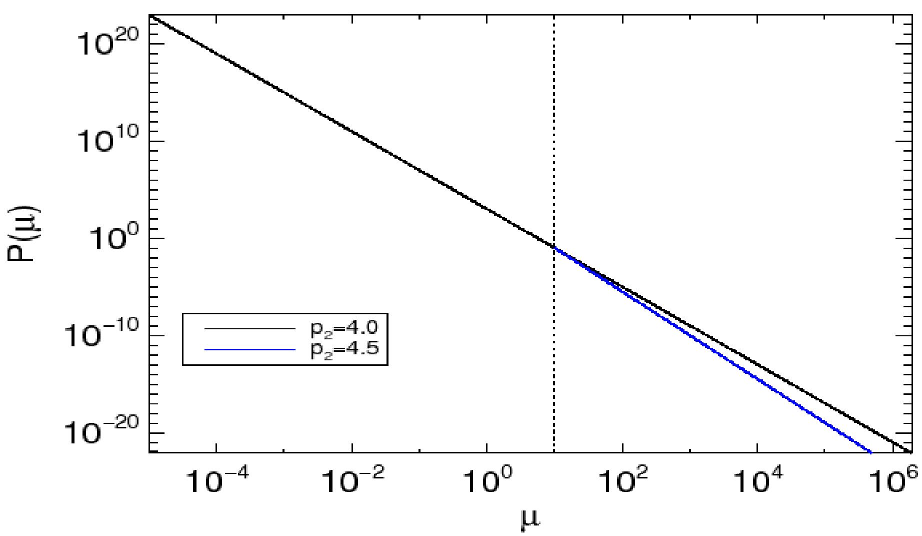
Effect of an Inner Scale for Steep Spectra, p_1=4.0

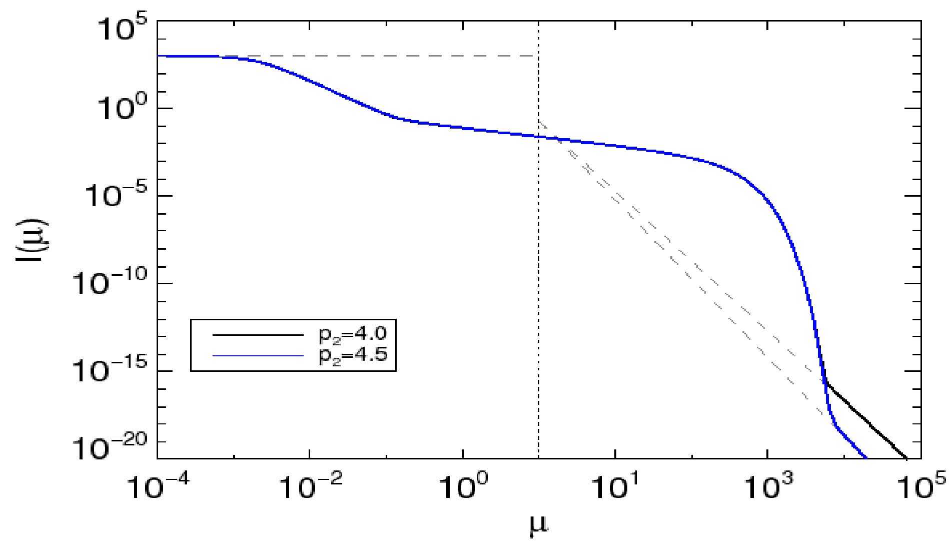
Moderate Scatter *U=1**





Strong Scatter *U=1000**

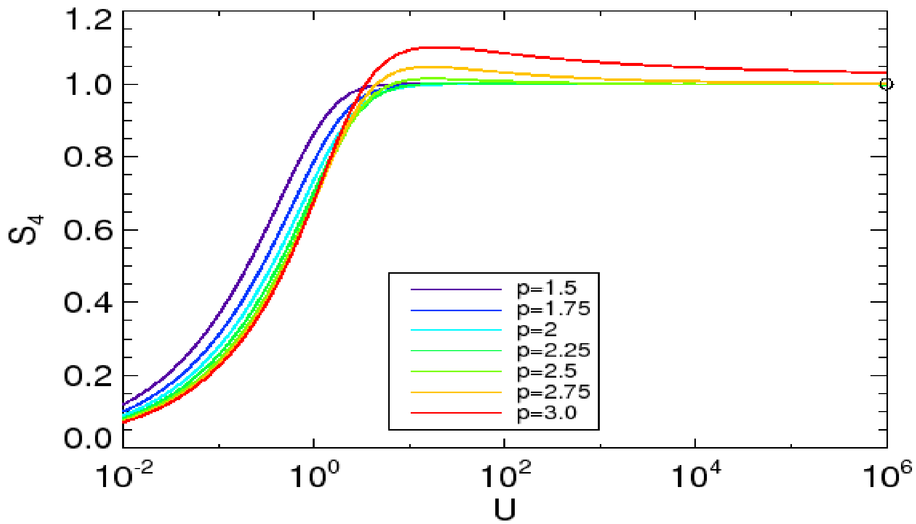


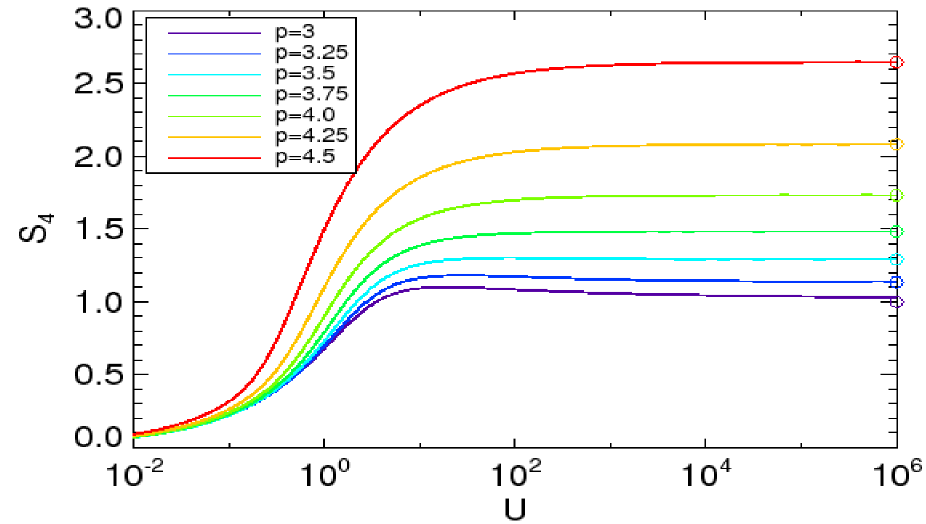


- Eroding small scale structure has little effect when the spectrum is already steep
- Imposing inner scale does not alter the correlation length

Development of the Scintillation Index

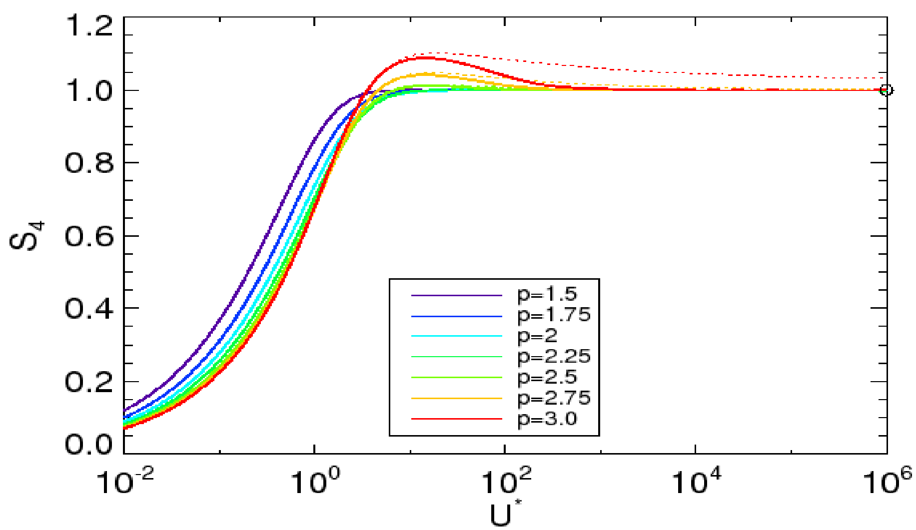
Unmodified power law

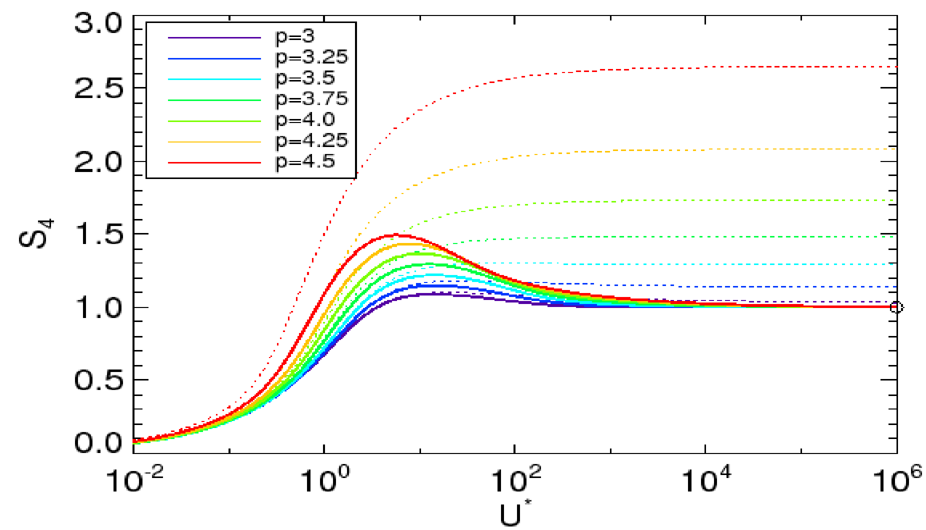




Unmodified power law spectra with *p*>3 admit sustained quasi-saturation states with S_4>1

Outer scale μ_0=0.1

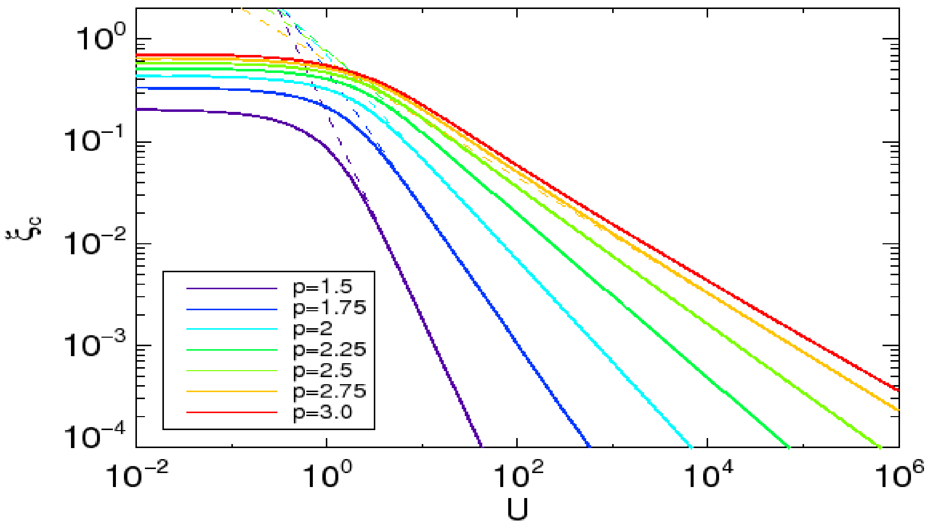


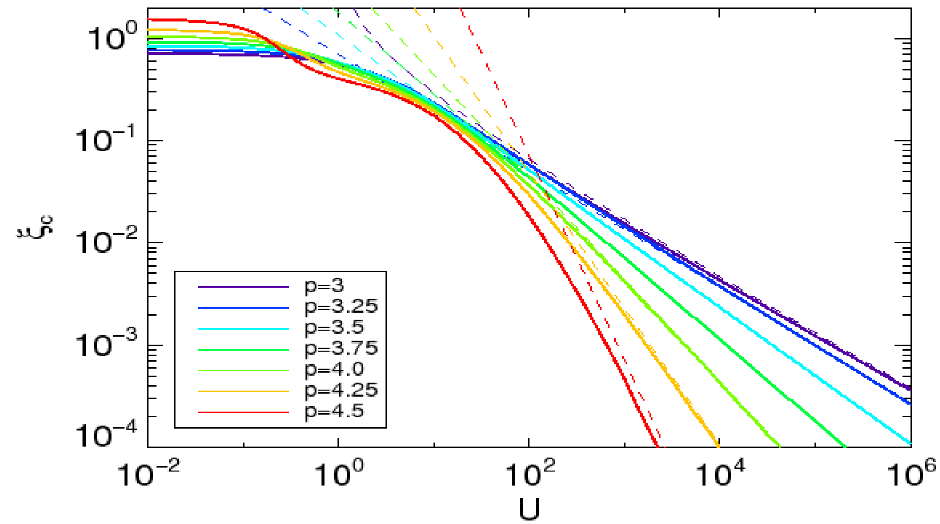


An outer scale suppresses strong focusing, driving S_4 to unity (from above if *p*>2)

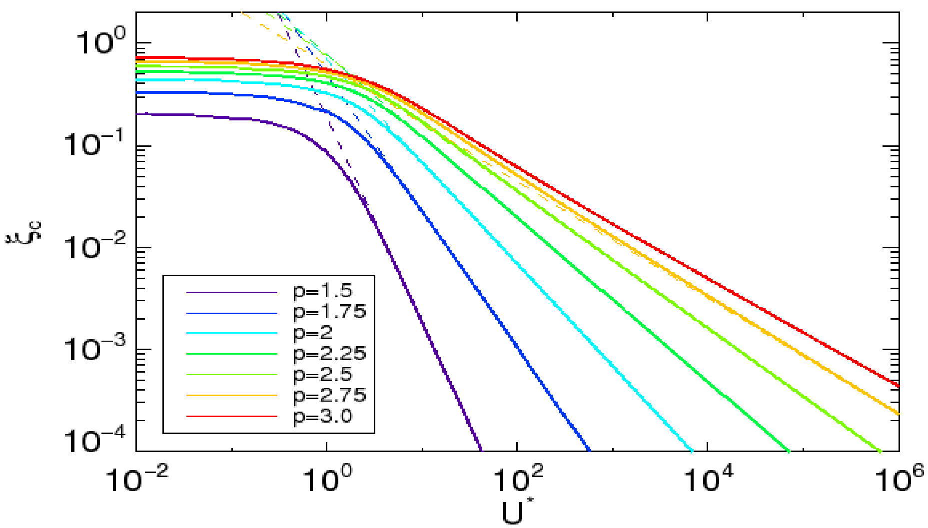
Development of the Correlation Length

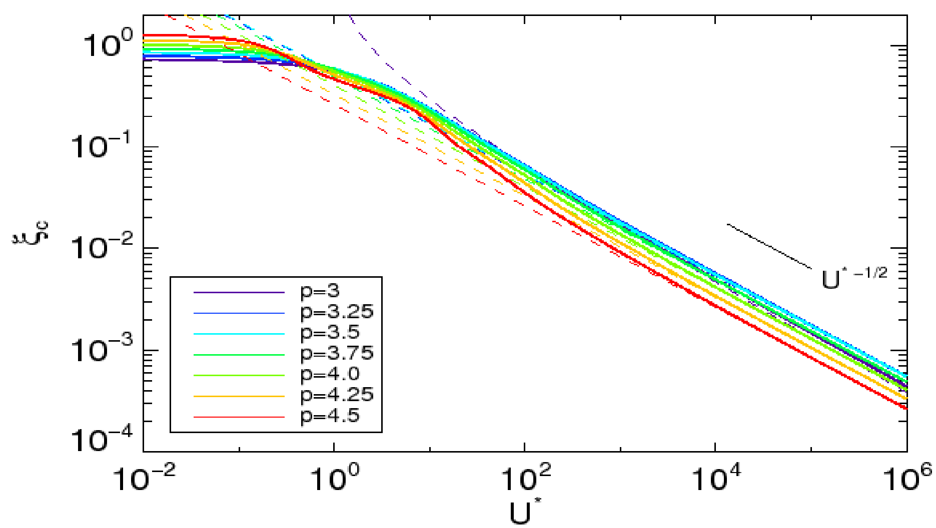
Unmodified power law





Outer scale μ_0=0.1

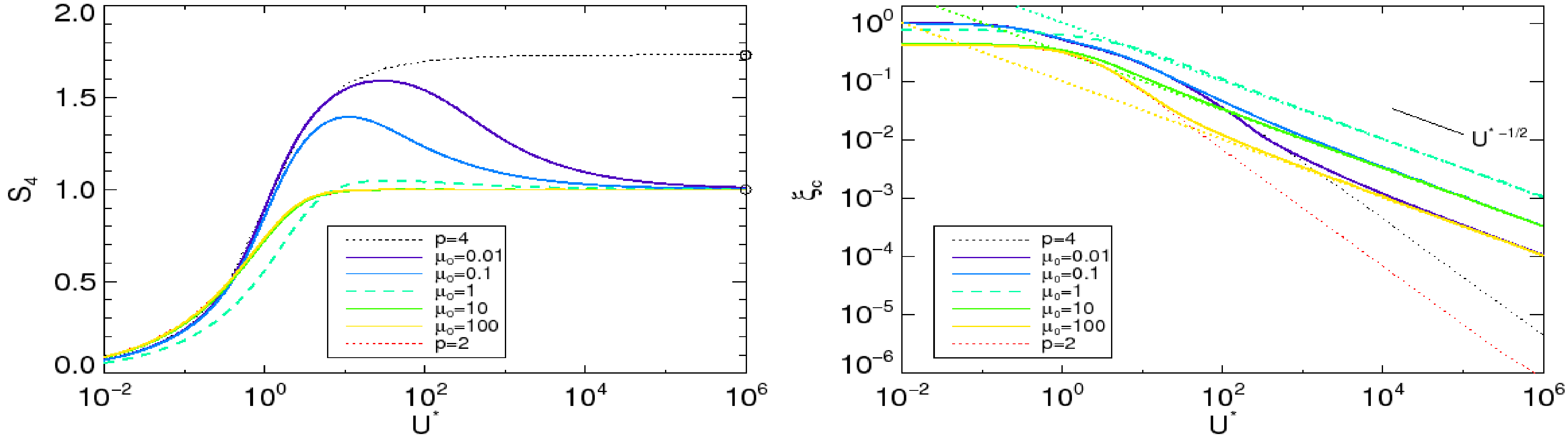




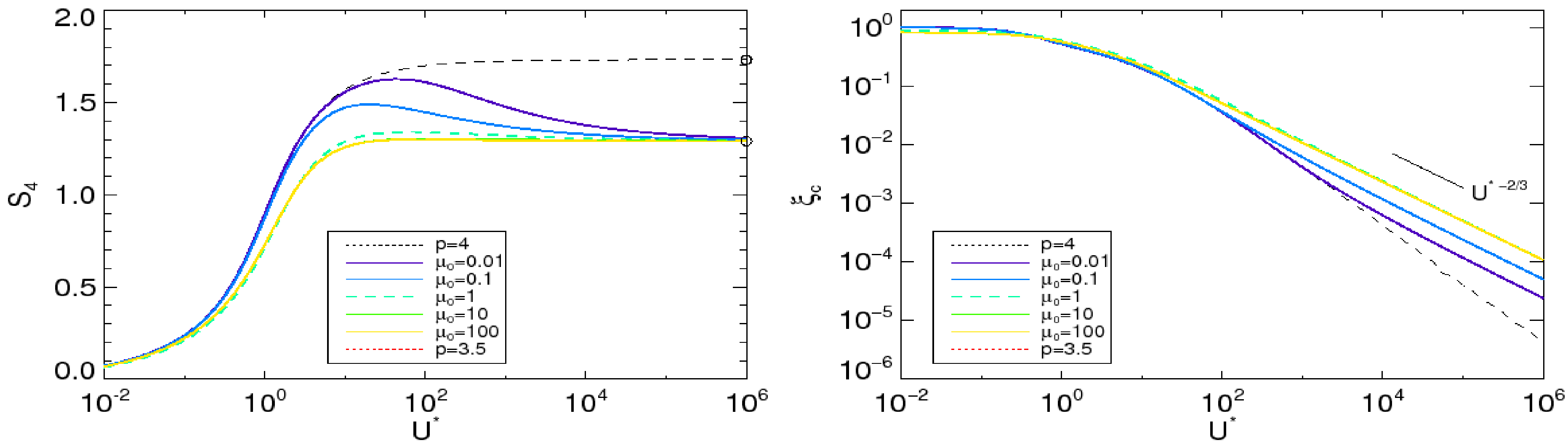
For shallow spectra, correlation length is unaffected by presence of an outer scale.
For steep spectra, suppression of refractive focusing greatly increases correlation length.

Effect of an Intermediate Break Scale

p_1=2, p_2=4, and μ_0 ranging from 0.01 to 100



p_1=3.5, p_2=4, and μ_0 ranging from 0.01 to 100



- Location of the spectral break relative to the Fresnel scale dictates the development

Conclusions

We make the following observations valid in the asymptotic strong scatter limit:

- For steeply sloped spectra with $p_2 > p_1 > 3$ the S_4 index approaches a quasi-saturation state exceeding unity as the scattering strength increases. Introducing a spectral break (with $p_1 < 3$) causes S_4 to retreat from its maximum to ultimately saturate at unity.

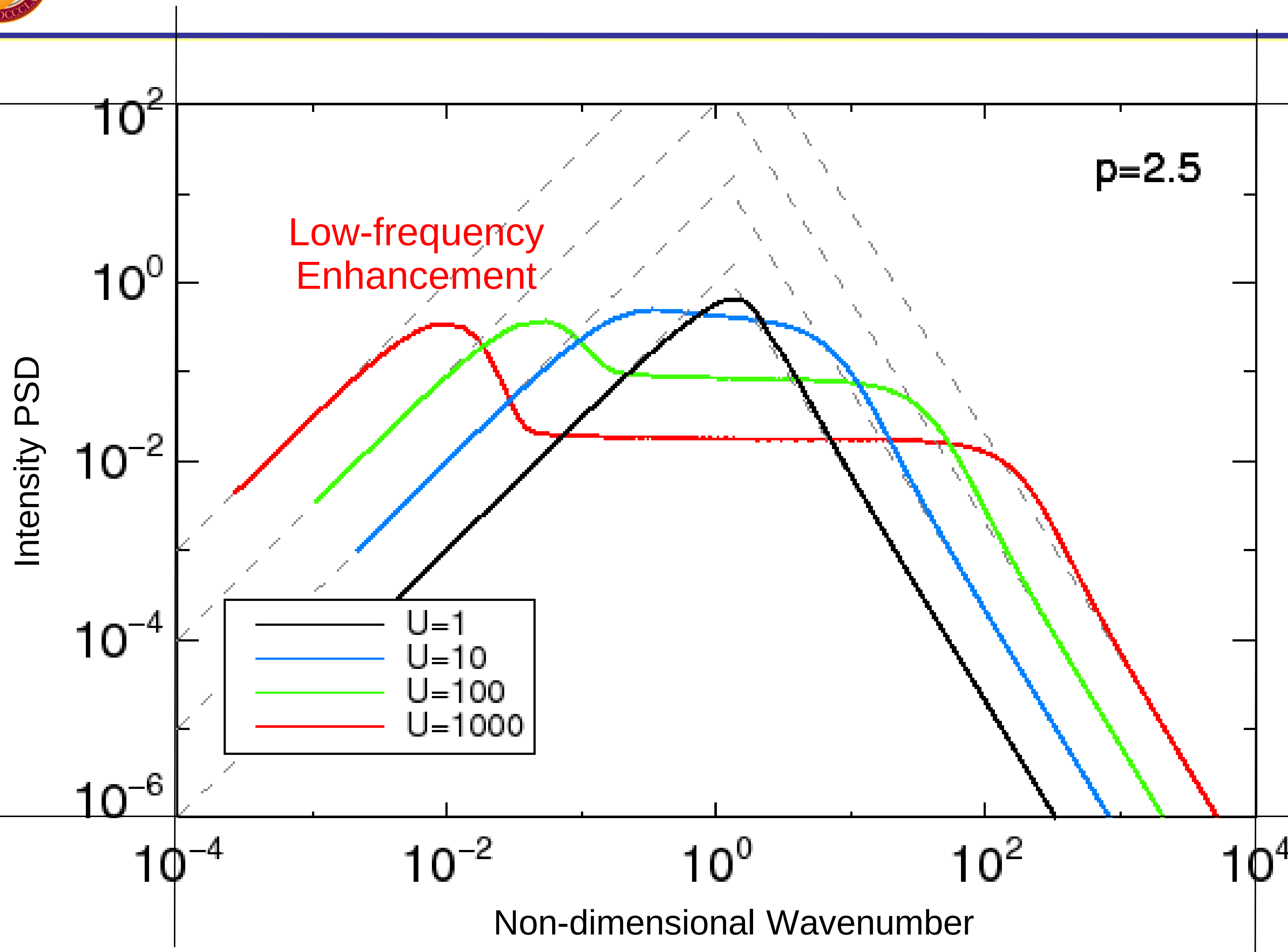
- When outer and inner scales are present the rate of decrease in correlation length with increasing scattering strength is slower than for an unmodified power law with any slope. We expect this slower rate of decrease to be observed in nature.

- Shallow spectra ($p < 3$) are insensitive to an outer scale but are sensitive to an inner scale. Steep spectra ($p > 3$) are insensitive to an inner scale but are sensitive to an outer scale. Spectra with $p \approx 3$ are relatively insensitive to both outer and inner scales.

- For the general case with $p_1 < 3$, $p_2 > 3$, irregularity scales sizes both smaller and larger than the Fresnel scale can contribute to the intensity statistics. The location of the spectral break relative to the Fresnel scale dictates the development of intensity.

Extra Slides

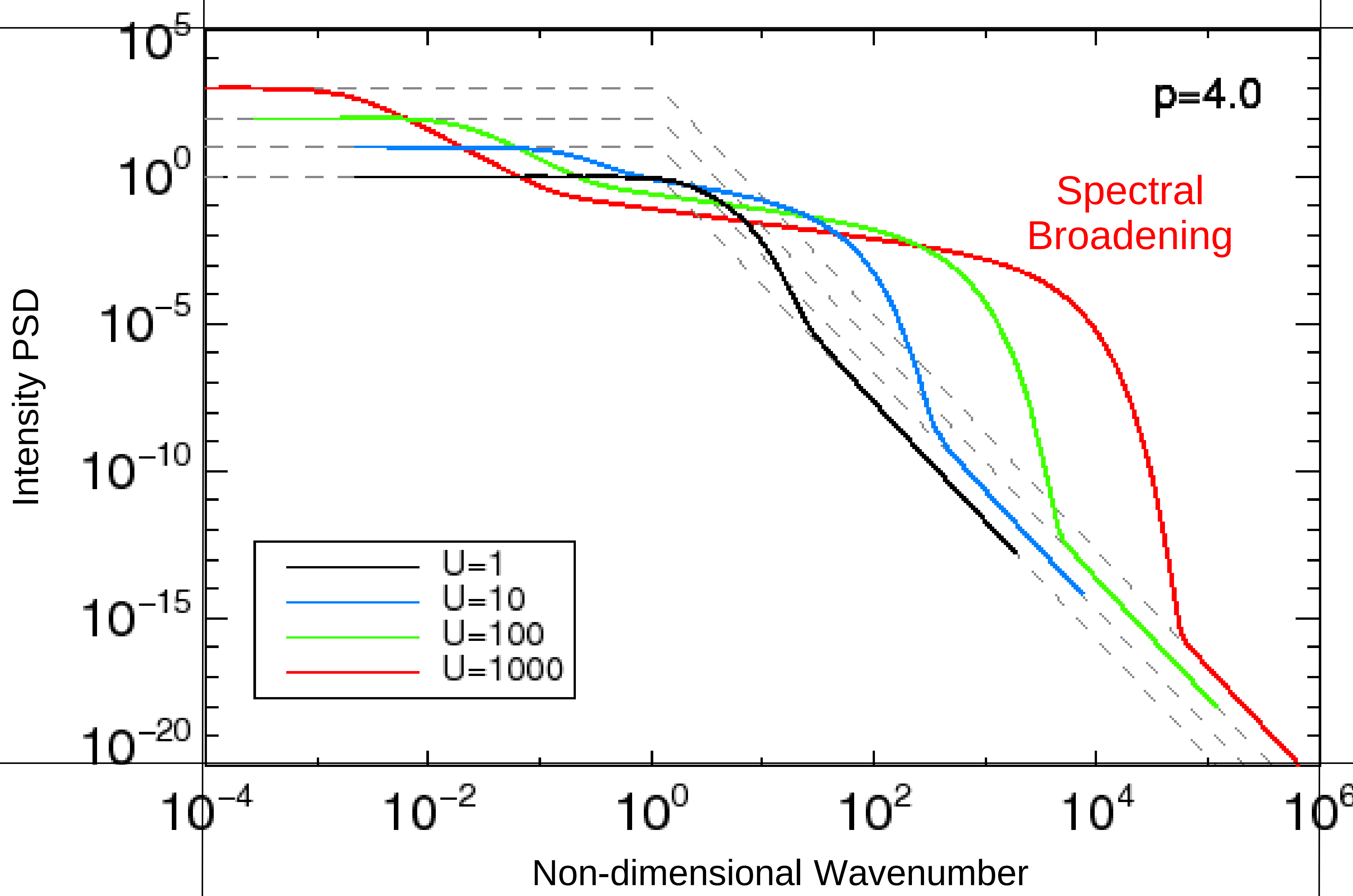
Single-Component Power-Law with p=2.5



Significant departures from power law behavior occur when the scatter is strong.

Prominent features are low frequency enhancement and gradual return to power law

Single Component Power-Law with p=4.0



Significant departures from power law behavior occur when the scatter is strong.

Prominent features include spectral broadening and abrupt return to power law