Designing Fair Tiebreak Mechanisms: The Case of Penalty Shootouts

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Tiebreak Mechanisms

- **Tiebreak mechanisms** are used in many sports to determine the eventual winner when the regular game ends with a tie;
  - e.g., tennis, college football, hockey, and soccer.
- In general,
  - they are more structured and shorter than the regular match, and
  - they can be modeled similar to dynamic versions of *contests* or *all-pay auctions*.
- **Fairness** emerges as the main desideratum of design rather than **revenue maximization** or **efficiency** as in other economic settings.
- Proposed changes to the current system should be minimal for higher probability of adoption.

Contribution

This paper

1. models **soccer**'s tiebreak format, **fixed-order penalty shootouts** as a sequential mechanism;
2. introduces a new fairness notion in dynamic mechanism design, **sequential fairness**;
3. pinpoints the problems with the current mechanism in terms of fairness, in line with previous empirical studies;
4. characterizes the class of **sequentially fair** shootout mechanisms; and
5. makes practical design suggestions to be implemented in the field (so called "market design" approach).

Related Literature

- Association football: Soccer
  - Goalkeeper & kicker behavior in penalty kicks: Chiappori, Levitt, & Groseclose [2002], Palacios-Huerta [2003], Bar-Eli et al. [2007]
  - Unfairness/fairness of current shootout format: Apesteguia & Palacios-Huerta [2010], Kuchner, Lenz, & Sutter [2012]
  - New shootout scheme: Palacios-Huerta [2012]
- American football
  - New tiebreak scheme based on **ex-post fairness**: Che & Hendershott [2008]

Why Soccer?

- The leading sport in the world in terms of societal impact, e.g.,**participation**, **viewership**, **economic impact**, and **international discourse**:
  - 265 million people — equivalent of 85% of the US population — played soccer in organized leagues in 2007 around the world.
  - 3.2 billion+ people — 46.4% of the global population — saw some in-home television coverage of the 2006 FIFA World Cup.
  - Soccer accounts for 43% of sports industry's worldwide revenues, which summed up to $600 billion in 2009 — equivalent to the half of US exports.
    - Two most valuable teams in the Forbes 2014 top 50 list are soccer teams — Manchester United and Real Madrid.
    - Markets decline significantly after important soccer losses (Edmans et al. [2007]).
  - A 100-hour war took place between Honduras and El Salvador following a 1969 World Cup qualification game.

Institutional Background: Knockout & Tiebreaking

- Most major soccer tournaments — in the play-off rounds or throughout the whole tournament — use the **knockout format**:
  - at the national team level — **World Cup**, **European Championship**, **Copa América**, and all other continental/mini world nations cups etc, and
  - at the club level — continental **Champions League**s, continental secondary club cups such as **Europa League**, intra-country **Federation/Country Cups** etc.

Institutional Background: Knockout & Tiebreaking

- In the **knockout format**, when
  - a single game in a neutral field/in one team's home field or
  - the aggregate* of two games, one in home field and one in away field

  is tied, a **sequential tiebreak format** has been used to determine the team that survives that round, or if it is the final, **the champion of the tournament**:
  1. 30 minutes extra time is played in two halves immediately after, and then
  2. if the tie is still not broken, prior to 1970
     - a *coin toss* was used to determine the winner, or
     - a *second game* was played in a few days with the above tiebreak rules.

Institutional Background: Knockout & Tiebreaking

- In 1968 **European Championship**,
  1. the winner of the semi-final game was determined with a *coin toss* as Italy against the Soviet Union; and then
  2. the **champion** was determined through a *second game* after the final as Italy against Yugoslavia.
- Public outcry following the championship for the unfairness of the *coin toss* and impracticality of the *second game*
- A new scheme was instituted following the 30 minute extra time by FIFA.

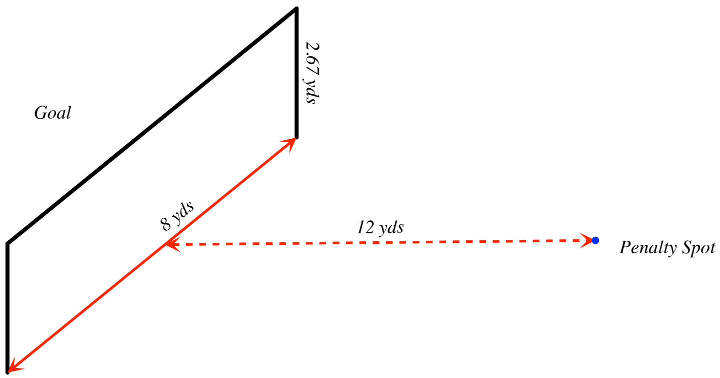
Institutional Background: Penalty Shootouts

- In the **penalty shootout** scheme:
  - A coin toss determines* the first team to kick the **penalty shots**.
  - A total of five rounds of **penalty shots** are taken such that the order of kicking in each round stays the same,i.e. in the order 1 2 - 1 2 - 1 2 - 1 2 - 1 2 ....
  - The winner is determined as follows:
    - if one team leads after the five **regular rounds,** the team that has the lead is the winner; and
    - if the tie is not broken after five rounds, in the same order, **sudden death** shots are taken, until one team scores in a round and the other team fails. The team that scored is the winner.

Institutional Background: The Penalty Shot and Shootouts

- A **penalty shot** during the regular game is the punishment given for a foul against an attacking team player in the 18-yard box around the defense goal.
- For each **penalty shot**:
  - The kicker shoots the ball from 12 yard distance to the goal.
  - The goal is defended only by the goalkeeper of the opponent team, who is supposed to wait on the goal line until the shot is taken.
- In the **tiebreak shootout** scheme, a different player of each team takes a shot sequentially. If all 11 players of each team take shots, then in the same order from the beginning, they continue with 12th round on.
- 18.6% of the World Cup knockout games since 1978 have been decided through penalty shootouts.

Institutional Background: Penalty Shootouts



Problem: First-Mover Advantage?

1. The team which starts kicking first more often, upto 64% of the time (Apesteguia & Palacios-Huerta [2010])
2. In a more extended dataset, the first-mover advantage persists, though more subtly, (at least 53% of the time in a time series) (Kuchner, Lenz, & Sutter [2012])
3. In a survey it was found that a great majority of Spanish La Liga players would prefer their team to kick first in a shootout (Apesteguia & Palacios-Huerta [2010])

Desideratum of Design: Fairness

- Although ***ex-ante***, the shootout mechanism is fair, ***ex-post***, i.e. after the coin toss, it is not, for the very same reason why tiebreaking using only a coin toss is deemed *unfair* in the first place.
- We utilize an **Aristotelian** approach to fairness in capturing this ex-post problem.
- **Aristotelian Justice** rests on a two-part principle: **equals need to be treated equally** and **unequals unequally**.
- In our setting, we utilize it through a **sequential** definition, at the beginning of every round of the shootout as long as the shootout is tied upto that point:
  1. opponents with equally skilled players should have the same chance of winning the shootout; and
  2. team with a better player should have a higher chance of winning the shootout, given that everything else is the same.

Kicker Preferences: Good versus Bad Shots

- What is a **good** versus **bad** penalty shot according to a kicker?
- Soccer players are among the highest paid workers in the world. Taking a penalty shot from a 12 yd distance against a defenseless goalie protecting a 8 yd-wide goal is generally perceived as the **simplest and most effortless act** in soccer.
- Yet, chance plays a role (but not so much the talent level of the goalkeeper), and the empirical scoring probability is less than 80%.
- Infamously, Roberto Baggio, one of the best players of his time, missed the last penalty shot that cost Italy the World Cup against Brazil in 1994. His shot went up high out. Years after this incident, he said:

Kicker Preferences: Baggio

*"As for the penalty, I don't want to brag but **I've only ever missed a couple of penalties in my career. And they were because the goalkeeper saved them not because I shot wide**. That's just so you understand that there is no easy explanation for what happened at Pasadena.... Unfortunately, and I don't know how, the ball went up three meters and flew over the crossbar. ... **I failed that time. Period. And it affected me for years. It is the worst moment of my career. I still dream about it. If I could erase a moment from my career, it would be that one**."*

Model: Kicker Preferences

Thus, we can safely infer that a kicker's utility has two components:

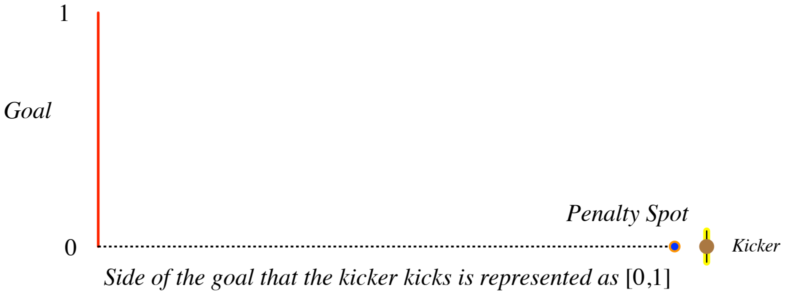
1. A team win is preferred to a team loss ($V_L < V_W$) .
2. The penalty shot's outcome affects the player's utility:
   (a) A **scored goal** is valued highest (U_G).
   (b) If a decent penalty was taken but not ended up as a goal (say, **saved by the diving goalie**), this can have a face-saving value ($U_S = 0^* < U_G$).
   (c) Taking a bad penalty (say, **sending the ball wide out**) is the worst individual outcome ($U_O < U_S = 0 < U_G$).

- For $a \in \{W, L\}$, $b \in \{G, S, O\}$:

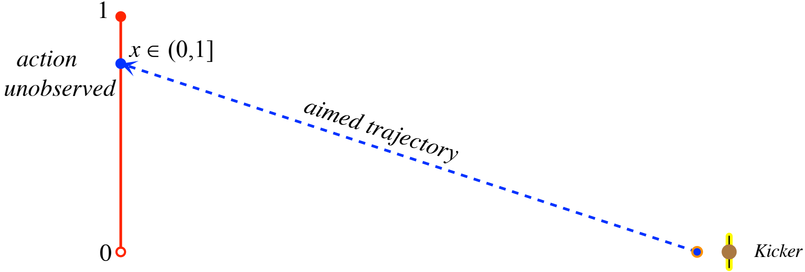
$$\text{Utility of a Kicker } = V_a + U_b$$

- Goalkeeper waits in the middle prior to kick on the goal line. He is not allowed to move before the kick.
- Closeness of the penalty spot and delay in the reaction time of goalkeeper makes it necessary for him to choose a side to dive* prior to the kick and dive simultaneously with the kick.
- In this study, we model him as a **robot** that chooses either side with probability 1/2 with a fixed probabilistic skill set for reaching and saving.

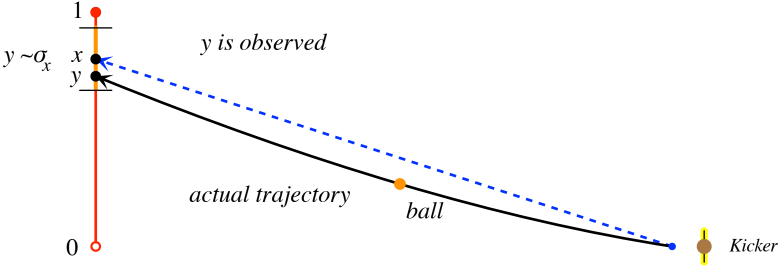
Model: Goal


1
Goal
0
Penalty Spot
Kicker
Side of the goal that the kicker kicks is represented as [0,1]

Model: Hidden Action (Aimed Spot)



Model: Where the Ball Arrives is Observed

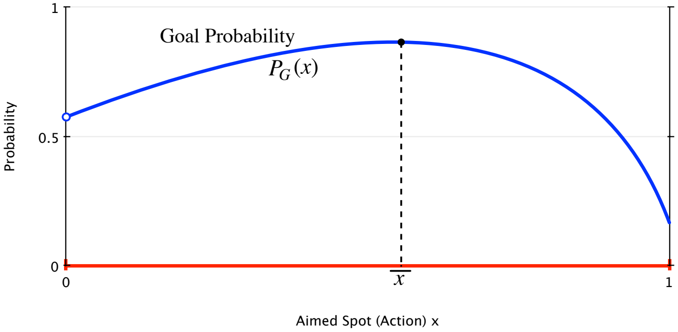


Model: Kick is a Goal, Save, or Out

- **Robot goalkeeper** dives to the side of the goal the kicker shoots with probability $1/2$.
- Given the machine and the kicker's ability probability density function to aim at x , σ_x we obtain summary statistics to represent **goal** and **out** probabilities.

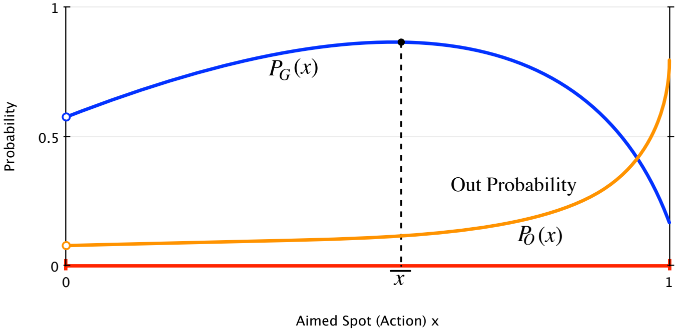
Model: Goal Probability

- **Goal probability function** $P_G(\cdot)$ is a twice differentiable strictly concave function of **aimed spot** x with the internal maximum at $\overline{x}$.
- $P_G(x) > 1/2$ for all $x \in (0, \overline{x}]$.



Model: Out & Save Probabilities

- **Out probability function** $P_O(\cdot)$ is a strictly increasing, twice differentiable strictly convex function of **aimed spot** x for all $x \in (0, 1]$.
- **Save probability function** is $1 - P_G(\cdot) - P_O(\cdot)$.



Model: Game

- Each team, $T1$ or $T2$, consists of an infinite number of kickers with the same skill set. Suppose $Ti, 1 - Ti, 2 - ... Ti, r - ....$ is the sequence in which team Ti's kickers take shots.
- Each kicker takes at most one penalty shot to maximize his expected utility.
- Shots are taken sequentially. **Aimed spot** is unobservable by others, but the **actual spot** the ball arrives and the result of the shot as **goal**, **save**, or **out** is observable.
- Suppose there are $r = 1, ..., n$ **regular rounds** of kicks and rounds thereafter (used only if tie continues at the end of round n) $s = n + 1, ...$ are referred to as **sudden death rounds**.
- We set $n = 2$ in the rest of the paper.

Model: Shootout Mechanism

- Kick sequence is determined according to some format we refer to as a **tiebreak mechanism**: $\phi(h^{r-1}, g_{T1} : g_{T2})$ is the probability of a $T1$ kicker kicking first in round r for the realized order history h^{r-1} until round r, when the score is $g_{T1} : g_{T2}$ under mechanism ϕ.

Solution Concept: Perfect Bayesian Equilibrium

- **Assessment** $(\mathbf{x}, \mu)$ is a **PBE** for a game induced by mechanism ϕ, if for **strategy profile** $\mathbf{x} = (x_k)$, each $x_k = (x_k^I)$ maximizes each kicker k's expected payoff given x_{-k} at any information set I that k can move; and the beliefs μ are **Bayesian** when possible.
- Hence, Bayesian **beliefs** do not play any role in optimization (except for **equilibrium refinement** under the current **fixed order** mechanism later).

Solution Concept: State-Symmetry ≈ Markov-Perfection

- We focus on **state-symmetric (SS) equilibria** (analogous* to **Markov-perfect equilibria** when we eliminate beliefs):

  An assessment is **state-symmetric** if

  1. strategies in regular rounds only depend on the **round number**, **kicking order**, and **score difference**; and
  2. strategies in sudden death rounds only depend on the current **kicking order** and **score difference.**

- We prove that **team symmetry** turns out to be redundant for equilibrium in the **regular rounds**, and hence, we drop it for regular rounds: i.e., kickers of different teams can also follow different strategies, but they don't at equilibrium.

Desideratum of Design: Sequential Fairness at Equilibria

- A mechanism ϕ is **sequentially fair** if *in all of its state-symmetric equilibria*, at any state $(h^{r-1}; g_{T1} : g_{T2})$ with a **tie**, i.e. $g_{T1} = g_{T2}$, each team has exactly 50% chance of winning.
- Anonymity throughout: state symmetry and the implicit assumption of equally talented teams.

Analysis: A Kicker's Optimization Problem

Kicker k's optimization problem:

$$\max_{x_k \in (0,1]} \underbrace{P_G(x_k) w_G + [1 - P_G(x)] w_{NG}}_{\text{Expected Team Payoff}} + \underbrace{P_G(x_k) U_G + P_O(x_k) U_O}_{\text{Expected Kick Payoff}}$$

for expected continuation values

- w_G when he **scores** and
- w_{NG} when he **does not score**

as functions of

1. **mechanism** ϕ,
2. **score difference** $g_{T1} - g_{T2}$,
3. **round number** r,
4. **kicking order** in round r, first or second, and
5. **others' strategy profile** x_{-k}.

Analysis: A Kicker's Optimization Problem

- First order conditions:

$$P'_G(x_k^*)\left(w_G - w_{NG} + U_G\right) + P'_O(x_k^*)U_O = 0$$

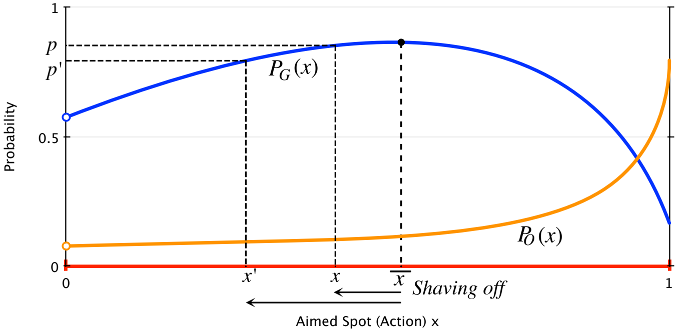
- If kicking out and goal being saved were not valued less than scoring a goal, then $x_k^* = \overline{x}$ optimal
- Since they are: $x_k^* < \overline{x}$: **kicks safer**
- The magnitude of $w_G - w_{NG}$ determines how much he *shaves off* an optimal kick.

  The smaller it is, the more he shaves it off

  $\Longrightarrow$ The **smaller** future benefit $w_G - w_{NG}$,
  the **smaller** Kicker k's probability of goal scoring.

Analysis: A Kicker's Optimization Problem

- x is obtained from a higher future gain $w_G - w_{NG}$ with respect to x'



- The first-kicking team is determined with an even coin toss in round 1 and the order does not change after round 1.
- Without loss of generality $T1$ goes first in every round.

Theorem (Fixed-Order is not fair)

*Generically, the **fixed-order mechanism** has multiple SS-equilibria and is **not sequentially fair**.*

Analysis: Current Mechanism: Fixed Order in Sudden Death Rounds:

- Continuation value of either team is the same in every round in SS-equilibria.
- Infinite game and T1 is never in the shoes of T2.
  $\Longrightarrow$ Multiple continuation values are possible: if $(p, 1-p)$ are continuation probabilities of winning then there is another equilibrium with $(1-p, p)$ are continuation probabilities of winning, and hence, not sequentially fair if $p \neq 1/2$.
- This symmetry is broken when we also consider regular rounds.
- T1 can collectively enforce the most **aggressive kicking equilibrium** for itself and **win more often**.
  - First kicker sets the tone of aggressiveness for his team.

- The class of **uneven score symmetric** mechanisms: A mechanism that is for all $(r; g_1 : g_2)$ such that $g_1 \neq g_2$ and $r \leq n$, we have $\phi(h^{r-1}; g_1 : g_2) = 1 - \phi(h'^{r-1}, g_2 : g_1)$.
- Team 1 and Team 2 are chosen to go first with the same probability if they were in *each other's shoes* in terms of score.

Design: Sequentially Fair Mechanisms, A Characterization

Theorem (Sequentially Fair Mechanisms)

*A mechanism ϕ is **sequentially fair** if and only if ϕ is **uneven score symmetric** in regular rounds and has $1/2 : 1/2$ winning probability in sudden-death rounds in all state-symmetric equilibria.*

Design: Sequentially Fair Mechanisms, Remarks

- **Exogenous mechanisms**, which set the order of agents based on external events, are **not sequentially fair** except the **coin-flip mechanism**, which sets the order with a fair coin toss in every round – not practical.
  - **Fixed order mechanism** would not be sequentially fair even if the sudden death rounds consisted of a coin toss.
  - **Alternating order (or tennis tiebreak service) mechanism** is not sequentially fair — it determines which team will go first with a coin toss in round 1, and if T1 goes first then the order is **1 2 - 2 1 - 1 2 - 2 1 - 1 2** is not sequentially fair.
  - **Prouhet-Thue-Morse mechanism**, i.e., **1 2 - 2 1 - 2 1 - 1 2 - 2 1** (Palacios-Huerto [2012]), is also sequentially unfair.

Design: Sequential Fairness in Sudden Death Rounds

- Are there uneven score symmetric mechanisms that give $1/2 : 1/2$ winning probabilities in sudden death rounds?
- Plenty

Theorem (Alternating Order in Sudden Death Rounds)

Any mechanism that is ***alternating order starting from any sudden death round*** *$r > n$ is* ***sequentially fair*** *in the sudden death rounds.*

Design: Sequential Fairness in Sudden Death Rounds

- Are there uneven score symmetric mechanisms that give $1/2$: $1/2$ winning probabilities in sudden death rounds?
- Plenty

Theorem (Alternating Order in Sudden Death Rounds)

Take any mechanism that is alternating order starting from any sudden death round $r \geq n$ is ***sequentially fair*** *in the sudden death rounds.*

Sequentially Fair Mechanisms

Corollary

Any mechanism that is ***uneven score symmetric*** *in regular rounds and* ***alternating order*** *in sudden death rounds is* ***sequentially fair****.*

But which mechanism to use in the field?

Selection & Market Design: Behind-First Mechanisms

- A continuum of sequentially fair mechanisms.
- **Behind-first mechanisms** are uneven score symmetric mechanisms in which when a team is behind at the end of a regular round then it kicks first in the next round.
- For a couple of different reasons **behind-first mechanisms** could be desirable.

- Mechanism ϕ **dominates** mechanism φ (**in terms of goal production**) if at each SS-equilibrium of ϕ, each kicker Ti, r kicks closer to the optimal penalty spot, i.e., kicks a shot **with a higher goal probability**, than he does at all SS-equilibria of φ.

Theorem

*A mechanism is **dominant sequentially fair** if and only if it is **behind-first** and **sequentially fair in sudden-death rounds**.*

Market Design: Rational Criterion — Goal Efficiency

- When a team is **ahead**, its kicker's future benefit $w_G - w_{NG}$ is higher when he kicks after the other team's kicker. Otherwise, the score gap would be even larger to begin with
  $\Longrightarrow$ his team's winning probability is even higher
  $\Longrightarrow$ less marginal value of his kick.

  So **ahead** team going last makes him more aggressive and more productive.

- When a team is **behind**, its kicker's future benefit $w_G - w_{NG}$ is higher when he kicks before the other team's kicker. Otherwise, the score gap would be even larger to begin with
  $\Longrightarrow$ his team's winning probability is even lower
  $\Longrightarrow$ less marginal value of his kick.

  So ahead team going last makes him more aggressive and more productive.

Market Design: Behavioral Criterion — Instant Rectifiability

- Some empirical behavioral studies proposed **competitive pressure** of chasing from behind as the reason why there could be first-mover advantage in penalty shootouts.
- Instant rectifiability could be a remedy to relieve **competitive pressure** in a rational environment along with sequential fairness.
- Behind-first mechanisms -by definition- are the only sequentially fair mechanisms that satisfy instant rectifiability.

Market Design: Practical Criterion — Minimum Order Switches & Coin Tosses

- A **minimum-switch SF mechanism** is such that there is no other SF mechanism that has fewer possible order switches before the sudden death rounds are reached.
- Such mechanisms minimize the number of coin tosses or order alternations, and hence, mistakes that can happen (while referee errors have no repercussions, rule violations cause the whole game to be replayed).
- A **minimum-switch behind-first mechanism** is a behind-first mechanism that only alters the kicking order of teams when the second kicking team falls behind.

Market Design: Practical Advice

- We advocate a variant of the minimum-switch behind first mechanism for soccer:
  1. A coin toss gives the right of first kick refusal to a team.
  2. The team which wins the coin toss accepts or defers to kick first.
  3. The kicking order does not change in regular rounds unless the second kicking team falls behind.
  4. Once the new order is established, the above rule is applied until the end of the regular rounds.
  5. In the sudden death rounds, an alternating order format is used starting with Round $n + 1$.

Ideas outside of the Box: Simultaneous-Kick Mechanism

- At each end of the field both, teams simultaneously take penalty shots.
- Sequentially rational?
  When **team-symmetry** requirement is dropped from the definition of SS-equilibria in the regular rounds, the answer is **no**.
- This mechanism has generically multiple equilibria (similar to fixed-order).
- We need **sequentiality** for the uniqueness of SS-equilibrium in a *symmetric format*.

Conclusions:

- Mechanism design approach to a practical and important sports competition design problem.
- Minimum changes in the rules would be easier to be adopted.
- Fairness first, efficiency second approach.
- Characterization of sequentially fair mechanisms.
- A mechanism in this class is advocated for its efficiency and practicality.
- Future Work: Field experiment among different tiebreak mechanisms.