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Homework 7
Mathematics 4470.01
Due November 4, 2022

All homework solutions longer than one page **must be stapled**. A paper clip is not acceptable.

Remember that all homework solutions must be typeset in some way. You may print your answers on both sides of the page if you want.

1. Solve the system of differential equations

$$\begin{aligned}x' &= -4x - 2y \\ y' &= x - y\end{aligned}$$

with initial conditions $x(0) = 5$ and $y(0) = 6$. I suggest using a computer algebra system, but it is possible to solve the problem without one.

2. Solve the system of differential equations

$$\begin{aligned}x' &= -4x - 5y \\ y' &= x - 2y\end{aligned}$$

with initial conditions $x(0) = 5$ and $y(0) = 6$. I suggest using a computer algebra system, but it is possible to solve the problem without one.

3. This problem concerns the system of differential equations

$$\begin{aligned}x' &= y^2 - x \\ y' &= x^2 - 8y\end{aligned}$$

- (a) Find the fixed points of the system.
(b) Use Jacobian analysis to determine the nature of each fixed point (that is, source, sink, saddle, spiral, and so forth).

4. (Richardson's Arms Race Model) Consider two competing countries, with an underlying sense of mutual distrust. Suppose that $x(t)$ and $y(t)$ represent the amount of money each of the two countries spends on arms. The simplest assumption is that each country's rate depends on the other's:

$$\begin{aligned}x' &= ay \\ y' &= bx\end{aligned}$$

Suppose that $x(0) = c$, $y(0) = d$, and a , b , c , and d are positive constants.

- (a) Solve and show that

$$ay^2 - bx^2 = ad^2 - bc^2.$$

- (b) Assume that $ad^2 - bc^2 > 0$. Show that under these assumptions, at time t , $y(t) > \sqrt{\frac{b}{a}}x(t)$. Use this inequality to show that $x' > \sqrt{ab}x$, and deduce that

$$\lim_{t \rightarrow \infty} x(t) = \lim_{t \rightarrow \infty} y(t) = \infty.$$