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Homework 4
Mathematics 4470.01
Due October 3, 2022

All homework solutions longer than one page **must be stapled**. A paper clip is not acceptable.
Remember that all homework solutions must be typeset in some way. You may print your answers on both sides of the page if you want.

1. Suppose that $x_{n+1} = x_n^4 + x_n^2$. We know that 0 is fixed point for this sequence. Use algebra (not numerical examples) to decide if 0 is attracting, repelling, or neither.

2. Suppose that $y_{n+1} = y_n^3 - y_n^2$. We know that 0 is fixed point for this sequence. Use algebra (not numerical examples) to decide if 0 is attracting, repelling, or neither.

3. Suppose that a printer has two states: working or broken. If the printer is working, then it has a 99% chance of working tomorrow. If the printer is broken, then it has a 35% chance of being broken tomorrow. In the long run, what fraction of the time is the printer working? Be sure to show explain your reasoning (though you can use a computer for eigenvalue-eigenvector computations).

4. Suppose that a printer has three states: working, broken and potentially reparable (because the technician has not yet visited to make a diagnosis), and broken and not reparable. If the printer is working, then it has a 99% chance of working tomorrow, and a 1% chance of being potentially reparable. If the printer is broken and potentially reparable, then it has a 35% chance of being broken and potentially reparable tomorrow, a 0.1% chance of being irreparable (because the technician showed up and could not repair it), and otherwise will be working tomorrow because the technician showed up and repaired it. If the printer is broken and not reparable, it stays broken forever.
In the very long run, it is guaranteed that the printer will break. Use an eigenvalue-eigenvector analysis (computer computation is fine) to determine how long that will be before it is more likely than not that the printer will be irreparably broken. After one year, how likely is the printer to be working?

5. Suppose that we divide the population into three age classes, 0–19, 20–39, and 40+, for the point of view of modeling reproduction. For simplicity, assume that there are no deaths in the first two classes, so that over 20 years everyone in the first class moves to the second, and everyone in the second class moves to the third. Assume as well that after another 20 years, everyone currently in the third class can be ignored. Denote the size of the three classes as x_n, y_n, and z_n, so that we have $y_{n+1} = x_n$ and $z_{n+1} = y_n$.
  (a) Suppose that $x_{n+1} = 0.5x_n + 0.5y_n + 0.15z_n$. Set up an appropriate matrix, find eigenvalues and eigenvectors (computer computation is fine), and determine how the population stabilizes over time.
  (b) Suppose instead $x_{n+1} = 0x_n + 0.75y_n + 0.45z_n$. Set up an appropriate matrix, find eigenvalues and eigenvectors (ditto), and determine how the population stabilizes over time.
  (c) Note that in each of these two scenarios, each woman gives birth to the same number of children (1.15), but in the second scenario, the births occur at an older age. Does that change in child-bearing habits change the total population in a significant way?

6. Sometimes transition probabilities are derived from observations. Suppose that we have a 7-state model in which we know that $p_{16} = p_{17} = 1$, and the following transitions were observed among the other states.

	1	2	3	4	5	6	7	Total
1	2	7	9	2	0	0	1	21
2	0	3	19	17	1	0	2	42
3	0	2	20	11	10	4	23	70
4	0	0	10	26	26	6	24	92
5	0	0	0	5	22	2	30	59

Note that this information is transposed from what we need. Use a computer for the eigenvalues and the eigenvector for the dominant eigenvalue.