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Homework 2
Mathematics 4470.01
Due September 16, 2022

All homework solutions longer than one page **must be stapled**. A paper clip is not acceptable.

Remember that all homework solutions must be typeset in some way. You may print your answers on both sides of the page if you want.

1. Each of these three sequences has 0 as a fixed point:

(a) $x_{n+1} = x_n - x_n^3$.

(b) $y_{n+1} = y_n + y_n^3$.

(c) $z_{n+1} = z_n + z_n^2$.

In each case, the first derivative test is inconclusive. Analyze each of these sequences algebraically (not graphically) to decide if 0 is a stable fixed point, unstable fixed point, or neither.

2. Define two sequences of numbers a_n and b_n recursively with the formulas $a_0 = 2$, $b_0 = 1$, and

$$a_{n+1} = 2a_n + 3b_n$$

$$b_{n+1} = a_n + 2b_n$$

for $n > 0$.

(a) Compute the next five terms of each sequence (up to and including a_5 and b_5), either by hand or using a computer.

(b) Find a matrix A so that

$$\begin{bmatrix} a_n \\ b_n \end{bmatrix} = A^n \begin{bmatrix} a_0 \\ b_0 \end{bmatrix}$$

(c) Diagonalize A (in other words, find a diagonal matrix D so that $A = PDP^{-1}$, either by hand or by using a computer algebra system) and use that calculation to find explicit formulas for a_n and b_n .

(d) Using those formulas, compute $\lim_{n \rightarrow \infty} \frac{a_n}{b_n}$, and compare your answer (to 6 decimal places) to the value of $\frac{a_5}{b_5}$.

3. Define a sequence with $x_1 = 1$, $x_2 = 3$, and $x_n = x_{n-1} + x_{n-2}$ if $n \geq 3$. Find an explicit formula for x_n . You may make use of the work that we did computing an explicit formula for the Fibonacci numbers.