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Homework 10
Mathematics 2216.01
Due September 28, 2022

Please note that there is no class on Monday, September 26.

1. Compute the integral

$$\int_0^1 \sqrt{1-x^2} dx$$

using a trigonometric substitution (no tables!) and perhaps a trigonometric identity.

2. Let n be a positive integer. Show using induction

$$\int_0^1 (1-x^2)^{n-\frac{1}{2}} dx = \frac{(2n)!\pi}{(n!)^2 2^{2n+1}}.$$

The case $n = 1$ is the previous problem.

3. Suppose that a , b , and c are positive integers. Suppose as well that $(a, b) = 1$, $a|c$, and $b|c$. Prove that $ab|c$.

4. Suppose that a and b are positive integers, and $d = (a, b)$. Find an example of integers a and b so that $(\frac{a}{d}, \frac{b}{d}) \neq 1$ and $(a, \frac{b}{d}) \neq 1$.

5. Suppose that a and b are relatively prime positive integers. Let d be a positive integer.

- (a) Suppose that $d \geq (a-1)(b-1)$. Show that it is always possible to find non-negative integers m and n so that $am + bn = d$.
- (b) Show that it is not possible to find non-negative integers m and n so that $am + bn = (a-1)(b-1) - 1$.