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Homework 1  
Mathematics 4470.01  
Due September 9, 2022

All homework solutions longer than one page **must be stapled**. A paper clip is not acceptable. Remember that all homework solutions must be typeset in some way. You may print your answers on both sides of the page if you want.

1. Find all fixed points for these recursion relations, and test each fixed point for stability.

(a)  $x_n = \frac{x_{n-1}}{1 + x_{n-1}}$ .

(b)  $x_n = x_{n-1}^2 + 6$ .

(c)  $x_n = x_{n-1}^2 + 0.7x_{n-1} + 0.2$ .

(d)  $x_n = x_{n-1}^2 + 0.7x_{n-1} - 0.2$ .

2. Suppose that your credit card balance is  $m$ , and you are fortunate that there is no interest charge(!). Suppose that the minimum monthly payment is 1.5% of the balance or \$25, whichever is higher. (Under no circumstances do you need to pay more than the outstanding balance. In other words, if the balance is \$10, your payment is \$10.)

(a) Suppose that  $m = \$100$ . How many months does it take until the balance is \$0?

(b) Suppose that  $m = \$1000$ . How many months does it take until the balance is \$0? Do you expect this answer to be about ten times the answer to part (a), less than ten times the answer, or more than ten times the answer?

(c) Suppose that  $m = \$10^6$ . Do you expect your answer to be about a thousand times as large as your answer to part (b), more than a thousand times, or less? What is the actual answer?

3. Continuing with the previous problem, suppose that you charge \$50 each month. If your initial balance is \$100, what will happen? What about if your initial balance is \$1000?

4. BLOOD ALCOHOL MODEL, from our text.

We make these assumptions:

- Typical alcohol content of various products:

Product	Gms of Alcohol
12 oz regular beer	13.6
12 oz light beer	11.3
4 oz red wine	10.9
4 oz rosé wine	10.0
1.5 oz 80-proof vodka	13.4

- The average human can metabolize 12 gms of alcohol per hour.
- An average female weighing  $K$  kg has about  $0.65K$  liters of fluid in her body.
- An average male weighing  $K$  kg has about  $0.68K$  liters of fluid in his body.

- One is legally drunk if there is more than 1 gm of alcohol per liter of bodily fluid.
  - Assume that an average female weighs 80 kg, and an average male weighs 90 kg.
- (a) Assume that an average female arrives at a party and drinks a six-pack of regular beer, and all of the alcohol is absorbed into her body instantly. How many hours will it take before she is legally sober?
- (b) Assume that an average male arrives at a party and drinks a six-pack of regular beer, and all of the alcohol is absorbed into his body instantly. How many hours will it take before he is legally sober?
- (c) Repeat both calculations for light beer.