EXERCISES FOR CHAPTER II.4

1. Proj is proper

Exercise 1.1. Exercise II.4.8.

Exercise 1.2. Exercise II.4.10.

2. Finite morphisms

Exercise 2.1. Exercise II.4.6.

Exercise 2.2. Let k be a field. Suppose that $X \subset \mathbb{P}^{n+1}_k$ is a hypersurface and that $x \in \mathbb{P}^{n+1}_k$ is a k-point that is not contained in X. Let $f : X \to \mathbb{P}^n_k$ be the restriction of projection away from x. Prove that f is finite.

Exercise 2.3. Let $f : X \to Y$ be a finite morphism of Noetherian schemes such that $\deg_y(f) \leq 1$ for every point $y \in Y$. Prove that f is a closed embedding.

3. $\mathcal{O}_X$-modules

Exercise 3.1. Exercise II.5.1.

Exercise 3.2. Let X be a scheme. Let $x \in X$ be a point, let M be a $\kappa(x)$-module, and let $\mathcal{G}$ be the skyscraper sheaf at x with value M. Show that $\mathcal{G}$ is an $\mathcal{O}_X$-module and that for any $\mathcal{O}_X$-module $\mathcal{F}$

$$\mathcal{H}om_{\mathcal{O}_X}(\mathcal{F}, \mathcal{G}) \cong Hom_{\mathcal{O}_{X,x}}(\mathcal{F}_x, M).$$

Exercise 3.3. Let $f : X \to Y$ be a morphism of schemes. Prove that f^* commutes with tensor product.

Given an example such that f_* does not commute with tensor product. (However, there will always be a natural map $f_*\mathcal{F} \otimes f_*\mathcal{G} \to f_*(\mathcal{F} \otimes \mathcal{G})$.)

Exercise 3.4. Verify carefully that f_* and f^* are adjoint functors.