Trade Expenditure and Trade Utility Functions Notes

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These notes derive the useful concepts of trade expenditure functions, the closely related trade indirect utility function, and the dual to it, the trade direct utility function. These are useful concepts to use in approaching empirical work and in modeling distortions of various kinds, especially tax and trade barrier policy. For trade policy purposes it is useful to define the balance of trade function, following the introduction of the other functions.

The trade expenditure function is most directly defined as the difference between the expenditure function of the representative agent and the GDP or revenue function:

$$E(p, v, u) \equiv e(p, u) - r(p, v).$$

The trade expenditure function inherits all the properties of the expenditure and revenue functions: concave and homogeneous of degree one in p, convex in v. And (where defined)

$$E_p = m = e_p = r_p.$$

Also $E_v = -w$.

The trade indirect utility function is defined by imposing the international budget constraint. Suppose that a transfer b is made (which can be equal to zero of course). Then

$$V(p, b, v) \equiv u : E(p, v, u) = b.$$

Roy's Identity obtains, $V_p = -V_b m$. Demonstrating this requires noting that $V_b = 1/E_u$ while $V_p = -E_p/E_u$. Shephard's and Hotelling's Lemmas imply

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that $E_p = m$. The other properties of the trade indirect utility function follow from the definition. V is quasi-convex in p, homogeneous of degree zero in p, b. And $V_v = V_b w$, and V is quasi-concave in v.

A very illuminating alternative derivation uses along the way the trade direct utility function. Thus, define the utility of the representative agent of a trading economy as:

$$M(m, v) \equiv \max_x \{U(x) | x - m \in T(x - m, v)\}.$$

I use M in honor of James Meade (Nobel laureate), who invented this function. The first order conditions equate marginal rates of substitution with marginal rates of transformation. The vector m is an arbitrary transfer of goods from the rest of the world to the home country. Autarky equilibrium delivers $M(0, v)$ in utility, and the autarky consumption and production bundles are implied by the first order conditon. M_m is the vector of marginal utilities of excess demand, equal to the marginal utilities of consumption. In considering trade policy interaction between two or more countries it is often useful to work in trade space using the trade direct utility function.

The alternative definition of the trade expenditure function is given by:

$$E(p, v, u) \equiv \min_m \{pm | M(m, v) \geq u\}.$$

Then the equilibrium utility is given by the indirect utility function

$$V(p, v, b) \equiv \{u | E(p, v, u) - b = 0\}.$$

In analyzing trade policy, b becomes active rather than a parameter. For example, it is usual to assume that the government redistributes any revenue as a lump sum, whereby b is a function of the tax rates and the optimal decisions of the agents allocating consumption and production. One can analyze trade policy with either the trade indirect utility function or the trade expenditure function. The latter is often much more convenient to use because it controls income effects. The latter is convenient sometimes, especially in analyzing a tariff setting interaction game between two countries.

1 Properties of the Trade Expenditure Function

The trade expenditure function inherits the properties of the expenditure and revenue functions so it is homogeneous and concave in p and convex

in v. By Shephard's and Hotelling's Lemmas, its first derivatives are equal to the vector of excess demands. The Slutsky decomposition applies to the trade expenditure function.

2 Nontraded Goods

Many trade models suppress the direct representation of nontraded goods. This is an entirely legitimate operation for many purposes (fortunately since all economies have a large portion of resources in the nontraded goods sector). Suppose the nontraded goods sector has price vector h. Then we may define the trade expenditure function as:

$$E(p, v, u) \equiv \max_h [e(p, h, u) - r(p, h, v)].$$

The right hand side defines a well behaved maximization problem with an interior solution. The first order conditions imply market clearance for nontraded goods.

The derivative properties of E are qualitatively the same as before: concave and homogeneous of degree one in prices, with first derivatives equal to the vector of excess demands. But the quantitative derivatives now include inverse matrices involving the response of nontraded goods markets to changes in traded goods prices. When the sign of off-diagonal elements of E_{pp} is important, the nontraded goods structure will have to be explicitly considered. (Of course, in an applied general equilibrium simulation model it is necessary to explicitly model nontraded goods.)

3 The Balance of Trade Function

For purposes of trade policy analysis, the transfer to the private sector b is endogenous. The government collects revenue from its distortion of trade and spends it on public goods as well as redistributing it in the form of subsidies. We assume for simplicity, following a long-standing tradition, that the government expenditure is a pure lump sum transfer back to the private sector: i.e., it does not alter any economically relevant decision. In this case, the balance of trade function is

$$B(p, v, u; p^*) \equiv E(p, v, u) - (p - p^*)' E_p.$$

Here, the trade tax revenue is $b = (p - p^*)'E_p$.

Consolidating the public and private budgets, the social budget constraint is

$$B(p, v, u; p^*) = B^0.$$

Here, B^0 is a transfer from the outside world.

3.1 Properties of the Balance of Trade Function

The balance of trade function is not a minimum value function generally. Its properties derive from those of the trade expenditure function and the tax revenue, and have ambiguous concavity and homogeneity properties except in special cases. A particularly useful first derivative is the marginal cost of tariffs, $B_p = -(p - p^*)'E_{pp}$. Intuitively, the increase in p is expected to raise the amount of foreign exchange required to support u, all else equal. This is true for the case of one tariff only, but may be violated for many tariffs.

When the GDP function is not differentiable in p, then the trade expenditure and balance of trade functions similarly become correspondences. For many purposes this is a technical detail but sometimes it is necessary to keep track of differentiability. The old trade geometry of the 50's and 60's is sometimes very useful in developing intutition.

4 General Equilibrium

The technical description of a single economy can now be combined with the description of its trading partners to characterize a general trading equilibrium. For many comparative static purposes it is convenient to combine the social budget constraints with the worldwide system of compensated excess demands (market clearance conditions given the real incomes). Thus:

$$m^c(p, v, u) + m^{*c}(p, v^*, u^*) = 0$$

$$B(p, v, u) = B^0$$

$$B^*(p, v^*, u^*) = -B^{*0}$$

constitutes a system of equations to determine (p, u, u^*). Homogeneity of course implies that the vector of prices is determined only up to a scalar.

For graphical analysis, it is handy to solve the budget constraints for the real incomes and substitute into the compensated excess demand functions

to yield the uncompensated excess demand functions as functions of the relative price and the endowments. Uncompensated excess demand functions normally slope downward (outside of rather extreme Giffen good complications). In contrast, excess supply functions can fairly easily have upward sloping portions. With normality, a rise in the price of an exportable induces substitution effects tending to raise the quantity exported but income effects tending to reduce the quantity exported. Backward bending export supply can imply multiple equilibria.

The existence of trading equilibrium occupies a technical literature that is inactive since the 70's. Convex economies always have equilibria. Uniqueness requires restrictions on income effects. Non-existence is associated with non-convexity. Illustrative cases with scale economies can be drawn but CGE simulations suggest that non-existence is esoteric.

5 Factor Trade Direct and Indirect Utility Functions

It is sometimes handy to think about trade in goods as indirectly trade in factors. This leads naturally to linkage between factor prices indirectly through goods trade.

The direct factor trade utility function is defined by

$$\Phi(v + f) \equiv \max_{y} \psi(y)|y \in T(y, v + f). \tag{1}$$

The factor trade expenditure function is defined by

$$\Gamma(w, u, v) \equiv \min_{f} wf|\Phi(v + f) \geq u. \tag{2}$$

The indirect factor trade utility function is defined by

$$\Omega(w, v, b) \equiv u : \Gamma(w, u, v) = b. \tag{3}$$

These structures yield neoclassical factor demand (excess demand) functions with all the standard properties. $\Gamma_w = f(w, u, v)$. Gains from trade propositions can be deduced. See Neary and Schweinberger for details.

Neary and Schweinberger, Prop. 5, shows that

$$w^0 f(w^0, u^0, v) \leq w^0 f(w^0, u^1, v) \leq w^0 f(w^1, u^1, v)$$

The first inequality is based on the assumption of the proposition that the move involves gains from trade. The second inequality is based on minimization (of the implicit factor trade expenditure function).

If position 1 is free trade with no transfers, then $w^1 f(w^1, u^1, v) = 0$, so Prop. 5 implies that $(w^0 - w^1) f(w^1, u^1, v) \geq b^0$. If trade is not initially subsidized, then the difference in factor prices is positively correlated with the excess factor demand. If position 0 is autarky, then $b^0 = 0$. Then the proposition implies that the implicit factor trade vector is positively correlated with the difference in factor prices between autarky and trade. In other words, on average the economy will implicitly import expensive factors and export cheap ones.

Notice that this implicit factor trade proposition does not imply internationally equalized factor prices ($w = w^*$) or material balance($f + f^* = 0$). In the very special case of the Heckscher-Ohlin-Vanek model, such properties do obtain: trade in goods perfectly substitutes for trade in factors. The logic of the proposition contains the special case, suggesting that with imperfect substitutability, trade in goods tends to lower the price of scarce factors and raise the price of abundant factors.

6 References

Neary, J. Peter and Albert Schweinberger (1986), "Factor Content Functions and the Theory of International Trade", *Review of Economic Studies*, 53, 421-32.

Woodland, Alan (1980), "Direct and Indirect Trade Utility Functions" *Review of Economic Studies*, 47, 907-26.