Gravity, Productivity and the Pattern of Production and Trade*

James E. Anderson
Boston College and NBER

revised October 26, 2009

Abstract

The effects of geography and productivity on the global pattern of production are captured here in a specific factors gravity model. Simple enough for sharp results, the model is yet rich enough to contain the high dimensional productivity frictions in production and distribution of a many country world. The starting point is the international incidence of productivity frictions inferred from gravity. Sellers' and buyers' incidence both reduce real income. Sellers' incidence shocks reduce sectoral skill premia. Bigger sellers' incidence by country (sector) reduces equilibrium shares of world (national) GDP. In contrast to the generalized Ricardian gravity model of Eaton and Kortum (2002), relative factor endowments play a role and import-competing production and wage premia in exporting are featured.

JEL Classification: F10, D24

*I am grateful to Christian Broda for helpful comments on the first version of this paper at the NBER ITI Summer Institute, 2007. I thank participants in seminars at Drexel University, Brandeis University and Oxford University for comments on earlier versions, and Susanto Basu for helpful discussions of productivity.

Geography obviously has big effects on the global pattern of income and production. But geography has been difficult to capture in models simple enough to apply yet rich enough to reflect the high dimensionality of distribution frictions in a many country world. A key difficulty is that frictions in the productivity of both distribution and production impact production and expenditure patterns through their incidence on buyers and sellers. This paper applies the incidence measures of the structural gravity model to a many-countries-many-goods specific factors general equilibrium model of resource allocation. It provides a very sharp characterization of the effect of gravity's incidence on on production patterns, real incomes and internal income distribution.

The structural gravity model yields convenient operational measures of buyers' and sellers' incidence of productivity frictions in distribution, aggregated over trade partners at the sectoral level. See Anderson and Yotov (2008) for details and evidence on the incidence of trade costs of 2 digit sectors in Canada's provinces.[1] Their evidence shows that buyers' or sellers' incidence is weakly, sometimes negatively correlated with Total Factor Productivity (TFP) type measures, so the latter may be very misleading when the purpose of the analysis requires incidence. Incidence falls mostly (3 to 5 times more) on the supply side and its variation is large across countries and across sectors within countries. Most strikingly, sellers' incidence falls over time (1992-2003) despite constant gravity coefficients, due to changing shipment shares. This finding suggests the importance of modeling causation running from incidence to shipment shares, the focus of this paper.

The model features the incidence of TFP frictions that combine sectoral distribution frictions with Hicks neutral productivity frictions in production. In contrast to TFP, which is defined at constant prices, incidence decomposes TFP into its impact on buyers' and sellers' prices throughout the global economy. The applied literature has by default used partial equilibrium TFP measures of the effects of geography. For example, Redding and Venables (2004) link the pattern of production to TFP-type measures in distribution while

[1]See Anderson (1979) and Anderson and van Wincoop (2003, 2004) for further discussion of the structural gravity model.

the large literature on openness and comparative performance has used measures that at best (e.g., land-lockedness) instrument for TFP in distribution.

The main focus of the paper is the effect of sellers' incidence on resource allocation. Resource allocation across sectors in each national economy can be modeled without regard to distribution over trade partners within sectors under trade separability (Anderson and van Wincoop, 2004): national varieties form separable groups in preferences and technology in each sector, and distribution requires resources in the same proportions as does production (iceberg trade costs). Full general equilibrium arises when the intra- and inter-sectoral modules are mutually consistent.

Standard general equilibrium production theory yields only weak results in the form of a positive correlation between sellers' prices and sectoral output. The specific factors production model developed here provides much sharper results in closed form solutions that provide intuitive insight into how geography shapes economic outcomes. In addition to this modeling advantage, specific factors provide descriptive realism in that export intensity is correlated in the model with sectoral wage premia.

There are two factors of production in the main version of the model. Unskilled labor is intersectorally mobile. Skilled labor is mobile prior to production as well, but it acquires sector specific attributes between allocation and use in production. After sector specific skills are acquired, productivity shocks are realized, the ex post efficient allocation of unskilled labor occurs, resulting in production and consumption at world market clearing prices. Neither factor of production is internationally mobile. The market structure is competitive in the main body of the paper; monopolistic competition is treated in the Appendix. The tractability of the model makes it suitable for adaptation to empirical work on the one hand and to theoretical refinement on the other hand. Extensions of the basic model here to many specific factors, intermediate goods, selection to exporting and monopolistic competition retain the main qualitative properties of the simpler model.

The equilibrium (multi-) factoral terms of trade are negatively related across countries to country size and to the sellers' incidence of TFP, and positively related to allocative efficiency. Effective country size is the ratio of potential GDP to aggregate sellers' incidence of TFP frictions, as if frictions melt away a portion of potential GDP. Equilibrium Gross Domestic Product (GDP) is increasing in effective country size with elasticity between zero and one, due to the terms of trade effect of country size. Real incomes are additionally reduced by the buyers' incidence of TFP. Looking across sectors within a country, the sector specific skill premium is reduced by high sellers' incidence of productivity frictions. Equilibrium production shares are increasing in sector specific factor endowments and goods/variety taste parameters, and decreasing in the equilibrium sellers' incidence of TFP frictions.

The efficient ex ante allocation of skilled labor is characterized under risk neutrality. Looking across countries, given ex ante efficiency, higher variance of the incidence of productivity shocks lowers ex post production efficiency, as is intuitive. It is plausible that the national variance of the incidence rises with the mean, implying that ex post inefficiency is larger for economies with higher average trade costs. Within countries, higher sectoral variance raises ex ante efficient skill allocations under risk neutrality because sector specific factor returns are convex in productivity.

The closest related model is that of Eaton and Kortum (2002). They embed gravity in a Ricardian model of trade featuring productivity differences resulting from draws from nationally differing Frechet distributions. In equilibrium the model is observationally equivalent to the one good/many varieties gravity model (see Anderson and van Wincoop, 2004). Costinot and Komunjer (2007) extend the Eaton-Kortum framework to a multi-good setting. The specific gravity model here nests the Costinot-Komunjer model as a special case when the efficient allocation of skilled labor is made after the realization of productivity draws.[2]

The specific factors model has several advantages in descriptive power relative to the generalized Ricardian model. First, unlike the Ricardian model it is consistent with import-competing production, action on the intensive margin and the attendant political economy

[2]Inessentially, the nesting applies to a monopolistic competition version of the Eaton-Kortum-Costinot-Komunjer models and the specific gravity model.

of protection. Second, it features an empirically realistic income distribution. Ex post specificity combines with productivity shocks to generate the well documented phenomenon of sectorally heterogeneous returns to otherwise identical skilled labor, positively correlated with export intensity.[3] Finally, the model allows a role for relative factor endowment differences in explaining the pattern of production and trade. An important aspect is the ex ante allocative efficiency of he ex post specific factor. Other advantages not exploited here include linkage to the interest group political economy model of trade policy. The tractability of the model suggests that it is a useful platform for future development, especially for empirical applications.

A less closely related recent literature that seeks to explain the pattern of production by international differences in endowments and technology lacks an appropriate general treatment of trade costs. Davis and Weinstein (2001) use the multi-cone Heckscher-Ohlin continuum of products model, but effectively assume that all the incidence of trade costs is on the demand side. Romalis (2004) considers the role of uniform trade costs in resource allocation using the multi-cone Heckscher-Ohlin continuum model, but in a North-South model with M identical countries in each half of the world. Trade costs disappear from his empirical work via a substitution that is valid only using the high degree of uniformity of the model. Trefler's HOV model (1995) allows for technology differences and home bias in preferences, but the home bias is not connected with gravity.

Section 1 sets the stage by describing and distinguishing between TFP and its incidence in a global economy. It describes how the supply side incidence of trade and productivity frictions fit into a standard general model of production, yielding the usual weak correlation relationship between sellers' incidence and production patterns. Section 2 sets out the specific factors model of production. Section 3 derives and characterizes the world equilibrium

[3] Anderson (2009) applies the specific factors production model to the two country homogeneous products case with uniform trade costs, focusing on the comparative static effect of globalization on income distribution. The present paper applies the same production model to many countries and differentiated products with differentiated trade costs and focuses on characterizing cross section production and trade patterns as well as income distribution. The general equilibrium comparative statics of the specific gravity model are complex and left to future research.

reduced form pattern of production and trade. Section 4 analyzes efficient ex ante allocation of specific factors facing random productivity draws in the world economy. Section 5 extends the discussion to treat intermediate products trade and the implications of selection into exporting. Section 6 concludes. The Appendix develops the endogenous determination of varieties in monopolistic competition, and fills out the connection of the model with the Costinot-Komunjer model. It also reviews selection into exporting.

1 TFP Frictions, Incidence and General Equilibrium

Each country produces and distributes goods to its trading partners. Production for given resources is reduced from its maximal potential by a (Hicks neutral) productivity deflator $a_k^j \geq 1$ for product k in country j. Thus for product k in country j, $a_k^j - 1$ more factors are used than needed with the ideal practice. Distribution to destination h requires additional factors to be used, in the proportion $T_k^{jh} - 1$ to their use in production: the metaphor of iceberg-melting distribution costs.

The cost at destination h is given by $p_k^{jh} = a_k^j T_k^{jh} \widetilde{p}_k^j$, where $\widetilde{p}_k^j$ is the unit cost of production using ideal practice, or the 'efficiency unit cost'. The a's and T's thus combine in a bilateral productivity friction measure $t_k^{jh} \equiv a_k^j T_k^{jh}$ that contains both distribution and productivity frictions. This useful simplification is exploited everywhere in what follows.[4]

The TFP friction in each sector k for each origin j is the appropriate aggregator of $\{t_k^{jh}\}$ across destinations for delivery h. Let y_k^{jh} denoted delivered product k from j to destination h. The appropriate aggregator $\bar{t}_k^j$ is derived as follows. Let $g^j(\widetilde{p}^j, v^j)$ be the maximum value Gross Domestic Product (GDP) function for country j, defined as:

$$\max_{\{y_k^{jh}\}} \sum_{h,k} \widetilde{p}_k^j t_k^{jh} y_k^{jh} | \sum_{h,k} t_k^{jh} y_k^{jh} = f_k^j(\{v_k^j\}) \forall k, \sum_k v_k^j = v^j,$$

[4]The decomposition of t into T's and a's is always available, but mostly a distraction here. The metaphor of iceberg melting trade costs extends to productivity frictions that 'melt' resources before the shipments begin their journey to market.

where v^j is the endowment vector, f_k^j is a degree one homogeneous concave production function and v_k^j is the vector of inputs in sector k. The first order conditions imply that the t's disappear as active arguments in the GDP function. Sectoral TFP friction is defined by the uniform friction that preserves the value of sectoral shipments at destination prices: $\bar{t}_k^j = \sum_h t_k^{jh} y_k^{jh} / \sum_h y_k^{jh}$. Aggregate TFP friction for country j is similarly given by $\sum_{k,h} \bar{t}_k^j \widetilde{p}_k^j y_k^{jh} / \sum_{k,h} \widetilde{p}_k^j y_k^{jh}$. This is equal to the ratio of GDP at delivered prices to the value of output at factory gate efficiency unit prices.

TFP measurement takes efficiency unit prices $\widetilde{p}_k^j$ as given. While conceptually clean and useful for analyzing productivity, TFP is misleading for purposes of understanding comparative economic performance and the pattern of production and trade. Proposition 1 below shows that real incomes depend on aggregate sellers' incidence while Proposition 3 shows that sectoral GDP shares are decreasing in sectoral sellers' incidence.

The key building block is incidence decomposition and aggregation at the sectoral level using the properties of the structural gravity model. Like sectoral TFP friction $\bar{t}_k^j$, bilateral frictions are aggregated, but using only their sellers' incidence portion. Buyers' incidence of bilateral frictions is similarly aggregated. The standard partial equilibrium one good incidence analysis breaks the trade friction t into sellers' incidence Π and buyers' incidence P with $t = \Pi P$. The decomposition uses the hypothetical frictionless equilibrium price p^* such that with actual equilibrium volume and buyers' price p and sellers' price $\widetilde{p}$, $\widetilde{p}\Pi = p^*$ and $p = p^* P$.

The structural gravity model yields a set of Π_k^j's and P_k^h's such that if the actual trade costs were replaced by hypothetical trade costs $\{\bar{t}_k^{jh} = \Pi_k^j P_k^h\}$, all total shipments at delivered value Y_k^j and all total expenditures at delivered value E_k^h would remain constant while the bilateral shipments would shift to their frictionless equilibrium values (given the Y's and E's). It is thus a proper generalization of the partial equilibrium incidence analysis. The Appendix reviews the details.

With efficiency unit production cost $\widetilde{p}_k^j$ in country j, it is as if there was an average ('world') destination price for goods k delivered from j, $p_k^j = \tilde{p}_k^j \Pi_k^j$. Similarly, on the demand side it is as if a single composite good k shipped to h from a world market at markup P_k^h. Anderson and van Wincoop (2004) call Π's and P's outward and inward multilateral resistances, respectively.

Total shipments and expenditures are taken as given in the calculation of Π's and P's in conditional general equilibrium. The full general equilibrium requires that the allocations of resources (resulting in Y's) and expenditure at the upper level (E's) for given incidences be consistent with the allocations (Y's and E's) that generate those same incidences in the lower level. The separation into intra-sectoral and inter-sectoral modules is valid if the structure is trade separable.

Trade separability requires a separable structure on demand and supply sides of the world economy along with iceberg trade costs. On the demand side, each product group k has a natural aggregator with aggregate price index P_k^h for country h. P_k^h is used for the price index as well as the buyers' incidence here because in equilibrium they are the same. Expenditure across groups is allocated with the expenditure function $e(P_1^h, ..., P_M^h, u^h)$, concave and homogeneous of degree one in the P's. On the supply side, aggregate production in each sector is allocated across trade partners with perfect substitutability, so each sector's product forms a natural separable group $y_k^j = \sum_h y_k^{jh}$.[5] On the demand side, $E_k^h = e_k^h P_k^h$. On the supply side, $Y_k^j = \tilde{p}_k^j g_k^j = s_k^j g^j$ where s_k^j is the GDP share in j accounted for by good k and $g_k^j \equiv \partial g^j / \partial \tilde{p}_k^j$.

The balanced trade budget constraint $\sum_k Y_k^j = \sum_k E_k^j$ is given by

$$g(\tilde{p}^j, v^j) = e^j(P_1^j, ..., P_M^j, u^j) \forall j. \tag{1}$$

Given the prices, (1) is solved for the real incomes $u^j, \forall j$.

General equilibrium requires meeting the system of budget constraints (1), the consis-

[5]This assumption can be relaxed to allow imperfect substitutability among the destinations, but this relaxation is inessential for present purposes.

tency requirements between upper and lower level allocations $Y_k^j = \widetilde{p}_k^j g_k^j, \forall k, j$ and $E_k^j = e_k^h P_k^h, \forall (k,h)$, and the sectoral market clearance requirements imposed in Appendix equations (24)-(25).

Impose CES preferences on the sub-expenditure functions, consistent with the structural gravity model. The CES price index P_k^h for goods class k in location h is defined by

$$P_k^h \equiv \sum_j [(\beta_k^j \widetilde{p}_k^j t_k^{jh})^{1-\sigma_k}]^{1/(1-\sigma_k)}, \tag{2}$$

where $(\beta_k^j)^{1-\sigma_k}$ is a quality parameter for goods from j in class k[6] and σ_k is the elasticity of substitution parameter for class k.

The expenditure share for class k in h, by Shephard's Lemma, is given by

$$\frac{\partial P_k^h p_k^{jh}}{\partial p_k^{jh} P_k^h} = \left\{ \frac{\beta_k^j \widetilde{p}_k^j t_k^{jh}}{P_k^h} \right\}^{1-\sigma_k}.$$

The share of expenditure on k from all origins at destination h is given by

$$\theta_k^h = e_{P_k^h} \frac{P_k^h}{e^h}.$$

It simplifies the model inconsequentially to assume the upper level preferences are Cobb-Douglas, so that $\theta_k^h = \theta_k \forall h$, a parameter. Expenditure on k in h is given by $E_k^h = e_{P_k^h} P_k^h$, equal to $\theta_k e^h$ in the Cobb-Douglas case.

Market clearance with balanced trade requires that at delivered prices excess supply is equal to zero:

$$s_k^j g^j - \sum_h \theta_k \left\{ \frac{\beta_k^j \widetilde{p}_k^j t_k^{jh}}{P_k^h} \right\}^{1-\sigma_k} g^h = 0, \tag{3}$$

$\forall k, j$. Substituting in (2), (3) determines the set of efficiency unit costs, $\widetilde{p}_k^j$, one for each k and j. (3) is homogeneous of degree zero in $\widetilde{p}$'s, understanding that the P's are homogeneous

[6] In monopolistic competition models, $(\beta_k^j)^{1-\sigma_k}$ is endogenous, equal to the proportion of all varieties of class k that are produced by j. See the Appendix for a full treatment.

of degree one in the unit costs in their representation as CES cost of living indexes. Thus relative unit costs only are determined.

Using the structural gravity methods of the Appendix, incidence is inferred from observed E's, Y's and t's. Connecting (3) to structural gravity expressions, $s_k^j g^j = Y_k^j$ while $\theta_k g^h = E_k^h$ (imposing balanced trade). Connecting from gravity back to (3), the Appendix shows that the complex second term on the left hand side of (3) that describes the actual allocation of goods to destinations can be replace with a much simpler term representing a hypothetical shipment of k from j to a world market at the uniform trade cost Π_k^j. Using Appendix equation (22),

$$Y_k^j = (\beta_k^j \widetilde{p}_k^j \Pi_k^j)^{1-\sigma_k} Y_k,$$

where $Y_k \equiv \sum_k Y_k^j$. $(\beta_k^j \widetilde{p}_k^j \Pi_k^j)^{1-\sigma_k}$ is recognized as the CES share equation for a hypothetical world market where buyers face a uniform markup on $\widetilde{p}_k^j$, and the adding up constraint on the shares imposes a normalization on the Π's for given $\widetilde{p}$'s: $\sum_j (\beta_k^j \widetilde{p}_k^j \Pi_k^j)^{1-\sigma_k} = 1$. Since $Y_k = \sum_h E_k^h$, replace Y_k with $\theta_k \sum_h g^h$. Then substitute $(\beta_k^j \widetilde{p}_k^j \Pi_k^j)^{1-\sigma_k} \theta_k \sum_h g^h$ for the second term on the left of (3) and divide through by $\sum_j g^h$. Define $\omega^j \equiv g^j / \sum_j g^j$, the world GDP share of country j. Then (3) is re-written as:

$$s_k^j \omega^j = (\Pi_k^j)^{1-\sigma_k} (\beta_k^j \widetilde{p}_k^j)^{1-\sigma_k} \theta_k. \tag{4}$$

(4) implies that sellers' incidence of TFP frictions on average reduces the equilibrium efficiency unit prices $\widetilde{p}_k^j$. The global equilibrium decomposes into a set of national equation systems (4). g^j is convex in $\widetilde{p}^j$. It can be shown that $\widetilde{p}^j$ is negatively associated with Π^j, in the sense that $\widetilde{p}^{j\prime}[d \ln \widetilde{p}^j + ((\sigma_k - 1)/\sigma_k)) d \ln \Pi^j] = 0$.[7] Pushing the interpretation of the preceding expression harder, the 'average' elasticity of $\widetilde{p}$'s with respect to Π's is between 0 and -1, as it would be in a partial equilibrium analysis of (4). Finally, the convexity of the

[7] Impose the normalization of prices $\sum_j g^j = 1$. (4) can be rewritten using Hotelling's Lemma as $g_{p_k}^j(\cdot) = (\Pi_k^j)^{1-\sigma_k} (\beta_k^j)^{1-\sigma_k} (\widetilde{p}_k^j)^{-\sigma_k} \theta_k$. Differentiate this system totally with respect to $\widetilde{p}^j$ and Π^j. Multiplying the differential vector by $\widetilde{p}^j$ and utilizing homogeneity properties of g^j, $\widetilde{p}^{j\prime} g_{pp}^j = 0'$. Then the inner product yields the expression above.

GDP function implies that $dy^j \cdot d\widetilde{p}^j \geq 0$, so higher sellers' incidence Π tends in some loose average way to induce lower supply looking across sectors.

In contrast to the very limited insight available from general technology, something like partial equilibrium reasoning is valid in a special case of the specific factors model that may stand in for approximate validity in a wider class of neoclassical production models for which the full effects of simultaneous determination of prices in (4) precludes sharp results.

2 The Specific Factors Model

Unskilled labor is intersectorally mobile but in fixed supply to the economy. The sector specific factor can be regarded for all purposes of analysis in this section and the next as an aggregate of many types of factors that form a separable group in the production function, including a variety of skilled labor types and capital. But for simplicity of exposition, reduce this aggregate to a single factor, skilled labor. Skilled workers are in fixed total supply to the economy prior to their acquisition of sector specific skills, after which they are in fixed supply to each sector. Section 4 treats the allocation decision. Both factors are internationally immobile. Deliveries are to final demand only until Section 5 introduces intermediate goods. To ease notational clutter, the country superscript is suppressed where possible.

Supply is defined as the activity y_k^j needed to achieve delivered output vector $(y_k^{j1}, ..., y_k^{jN})$. Thus $y_k^j = \sum_h t_k^{jh} y_k^{jh}$. Turning to the technology, supply in class k is given by

$$y_k^j = f^{kj}(L_k^j, K_k^j), \forall k,$$

where f^{kj} is a concave homogeneous of degree one potential production function giving the activity level of the ideal technology. Labor L_k^j is mobile across sectors while K_k^j is the specific skill endowment.

More restrictive production functions serve several important modeling purposes. Assuming identical production functions across sectors and countries (up to a productivity scalar

a) ensures that the fully efficient equilibrium will be Ricardian (because the equilibrium relative factor intensities will be identical), and thus the model will nest the Eaton-Kortum and Costinot-Komunjer models. Imposing Cobb-Douglas structure on $f(\cdot)$ results in a closed form solution for g with very convenient properties. First, aggregate factor shares are stable, consistent with observed nearly constant shares across periods of time when the composition of GDP has altered tremendously. Second, Stolper-Samuelson forces are shut down: the average skill premium is independent of international forces in the model. This is analytically convenient for thinking about a world in which skill premia seem to be rising simultaneously in both rich and poor countries. Third, the GDP function has a constant elasticity of transformation. Finally, when extending the interpretation to include many types of sector specific factors it is more natural to impose that the natural aggregator is the same in each sector. Temporarily, it eases notation to drop the country superscript.

Let $K = \sum_k K_k$ and let $f_k = L_k^{\alpha} K_k^{1-\alpha}$ where α is the parametric share parameter for labor. Then

$$g = L^{\alpha} K^{1-\alpha} G \tag{5}$$

where G is given by

$$G = [\sum_k \lambda_k (\widetilde{p}_k)^{1/(1-\alpha)}]^{1-\alpha}, \tag{6}$$

and $\lambda_k = K_k/K$, the proportionate allocation of specific capital to sector k.[8] GDP is the product of real activity in production and distribution $R = L^{\alpha} K^{1-\alpha}$ and the real activity deflator G. G is convex and homogeneous of degree one in the efficiency unit costs, the $\widetilde{p}$'s.

The elasticity of transformation is equal to $\alpha/(1-\alpha)$, the ratio of labor's share to capital's share.[9] The GDP share for any good k is given by:

$$s_k = \widetilde{p}_k y_k / g = \frac{\lambda_k \widetilde{p}_k^{1/(1-\alpha)}}{\sum_k \lambda_k \widetilde{p}_k^{1/(1-\alpha)}}. \tag{7}$$

[8] Solve the labor market clearance condition for the equilibrium wage, then use the Cobb-Douglas property $wL/\alpha = g$.

[9] The CET form is commonly used in applied general equilibrium modeling. The micro-foundations provided here may prove useful in this context.

3 World Trade Equilibrium

The equilibrium is solved for the specific factors Cobb-Douglas case by using (7) to substitute for s_k^j in the market clearance equations (4).

3.1 Equilibrium Prices

Solve for the equilibrium efficiency unit costs as:

$$\widetilde{p}_k^j = \left(\frac{D_k^j}{\omega^j \lambda_k^j (\Pi_k^j)^{\sigma_k - 1}} \right)^{(1-\alpha)/\eta_k} (G^j)^{1/\eta_k} \tag{8}$$

where $D_k^j \equiv \beta_{kj}^{1-\sigma_k} \theta_k$ and $\eta_k \equiv \alpha + \sigma_k(1-\alpha) > 1$ in the empirically relevant case $\sigma_k > 1$.

(8) implies that equilibrium efficiency unit costs are increasing in the demand side driver D_k^j and decreasing in the supply side drivers $\omega^j \lambda_k^j (\Pi_k^j)^{\sigma_k - 1}$. Intuitively, bigger country size ω^j and bigger sectoral allocations of specific factors λ_k^j reduce unit costs. Also, the higher the incidence of productivity frictions Π_k^j, the lower must the unit cost be to compensate. The GDP deflator G^j in the general equilibrium is the (multi-) factoral terms of trade. A rise in the factoral terms of trade raises unit cost, all else equal.

The demand shifter D_k^j is the product of a k specific component θ_k reflecting tastes in the global economy for good k and a national origin 'quality' parameter $\beta_{kj}^{1-\sigma_k}$ reflecting tastes within goods class k for varieties from origin j. In monopolistic competition, β_k^j is endogenously determined by the entry of firms in zero profit equilibrium.

The factoral terms of trade G^j are solved based on the equilibrium $\widetilde{p}$'s. The equilibrium GDP shares are expressed as 'reduced form' equations in the international equilibrium using (8) in (7).

$$s_k^j = \lambda_k^j \left(\frac{D_k^j}{\lambda_k^j (\Pi_k^j)^{\sigma_k - 1} \omega^j} \right)^{1/\eta_k} (G^j)^{(1-\sigma_k)/\eta_k}. \tag{9}$$

Use the adding up condition on the shares (9). Next, define the parametric 'real potential

GDP' $R^j \equiv (L^j)^\alpha (K^j)^{1-\alpha}$, and note that $\omega^j = R^j G^j / \sum_j R^j G^j$. Then

$$1 = \sum_k \lambda_k^j [D_k^j / \lambda_k^j (\Pi_k^j)^{\sigma_k - 1} R^j]^{1/\eta_k} (G^j)^{-\sigma_k/\eta_k} (\sum_j G^j R^j)^{1/\eta_k}, \forall j.$$

The natural normalization for the price system is $\sum_j G^j R^j = 1$. Subject to the normalization there is a unique solution because the right hand side of the sum of shares equation above is decreasing in G^j.

The solution has a closed form when $\sigma_k = \sigma, \forall k$:

$$G^j = (\Lambda^j / R^j)^{1/\sigma}, \tag{10}$$

where

$$\Lambda^j \equiv \Big\{ \sum_k \lambda_k^j \Big\{ \frac{D_k^j}{(\Pi_k^j)^{\sigma-1} \lambda_k^j} \Big\}^{1/\eta} \Big\}^\eta. \tag{11}$$

Λ^j is an efficiency measure analyzed further below. G^j is decreasing in R^j interpreted as *relative* country size.[10] In contrast, G^j is increasing in both absolute and relative Λ^j.[11]

Λ^j decomposes into two intuitive components: a measure of harm from j's incidence of TFP frictions and a measure of j's efficiency of skill allocation, the λ's. Aggregate incidence of TFP friction is derived by equating the right hand side of (11) with the same function evaluated at a uniform value of $\bar{\Pi}^j$ and solving for $\bar{\Pi}^j$.

$$\bar{\Pi}^j = \left(\frac{\Lambda^j(\{\Pi_k^j\})}{\Lambda^j(1)} \right)^{1/(1-\sigma)}$$

where $\Lambda^j(1)$ denotes evaluation with $\Pi_k^j = 1, \forall k : \Lambda^j(1) = [\sum_k \lambda_k^j (D_k^j / \lambda_k^j)^{1/\eta}]^\eta$.[12] Then the

[10] If the normalization is not imposed, (10) should be multiplied by $[\sum_k (\Lambda^j/R^j)^{1/\sigma} R^j]^{1/(\sigma-1)}$. This form of the right hand sid of (10) implies that the equilibrium G^j is homogeneous of degree zero in $\{R^j\}$ — a scalar expansion of the world economy leaves all prices and shares unchanged.

[11] Extending the same analysis as in the previous footnote, G^j is homogeneous of degree $1/\sigma^2(\sigma-1)$ in the Λ's. All G's and thus GDP's rise in proportion with a uniform rise in efficiency.

[12] $\bar{\Pi}^j$ contrasts with aggregate TFP friction $\bar{t}^j$ because sectoral incidence differs from sectoral TFP frictions and because the implicit functions differ. Locally there is a close relationship because $\partial \ln \bar{\Pi}^i / \partial \ln \Pi_k^i = s_k^i = \partial \ln \bar{t}^i / \partial \ln \bar{t}_k^i$.

efficiency measure Λ^j decomposes into:

$$\Lambda^j(\{\Pi_k^j\}) = (\bar{\Pi}^j)^{1-\sigma}\Lambda^j(1).$$

The first term on the right implies, intuitively, that efficiency falls as the incidence of TFP frictions rises. $\Lambda^j(1)$ measures the efficiency of matching λ's to D's relative to a frictionless equilibrium. The equilibrium allocation of skilled labor to sectors analyzed in Section 4 implies that the hypothetical equilibrium ex post allocation of λ's maximizes Λ^j subject to $\sum_k \lambda_k^j = 1$. With no productivity shocks, frictionless trade and efficient allocation, $\lambda_k^j = D_k^j$ and hence $\Lambda^j = 1$. Ex ante allocations however determined will generally not satisfy $\lambda_k^j = D_k^j$, so $\Lambda^j(1) < 1$. How far $\Lambda^j(1)$ falls below 1 reflects aspects of possibly efficient ex ante allocation facing uncertain Π's as well as the deviations of realized productivity draws from expected draws. Thus the decomposition above is arbitrary in assigning an efficiency meaning to $\Lambda^j(1)$. Nevertheless, Λ^j has an unambiguous efficiency interpretation and $\bar{\Pi}^j$ also has a clear interpretation as an efficiency measure.

3.2 Equilibrium Income Patterns

The model yields strikingly simple links from the incidence of productivity frictions to the equilibrium cross section pattern of aggregate real incomes, income distribution and the pattern of production and trade.

Proposition 1 *(a) The factoral terms of trade G^j is decreasing in the sellers' incidence of TFP frictions, increasing in the relative efficiency of allocation $\Lambda^j(1)$ and decreasing in real potential GDP R^j:*

$$G^j = \left(\frac{\Lambda^j(1)}{R^j(\bar{\Pi}^j)^{\sigma-1}}\right)^{1/\sigma}.$$

(b) Real national income is increasing in effective potential GDP $R^j/\bar{\Pi}^j$ and in the efficiency

of allocation of λ's; and it is decreasing in national average buyers' incidence $\bar{P}^h$:

$$u^h = \Lambda^h(1)^{1/\sigma}\left(\frac{R^h}{\bar{\Pi}^h}\right)^{1-1/\sigma}\frac{1}{\bar{P}^h}.$$

G^j is decreasing in relative R^j due to the familiar effect of country size on the terms of trade. Real income is less-than-unit elastic in effective country size $R^h/\bar{\Pi}^h$ for the same reason. Also intuitive, allocative efficiency improves the factoral terms of trade. In equilibrium, sellers' and buyers' incidences tend to be negatively correlated, so comparative static shifts in their product are damped.

Proposition 1 suggests that gravity plays a powerful and previously unappreciated role in accounting for comparative national economic performance. Incidence varies across countries in ways that cannot be captured by partial equilibrium measures of geography. Sellers' and buyers' incidences each matter. Over time, even though geography is constant, its incidence is endogenously shifting. Evidence from Anderson and Yotov (2008) suggests weak or even negative correlation between buyers' and sellers' incidence and between the incidence measures and TFP type measures of distribution frictions. Over time, Anderson and Yotov report significant changes in incidence despite constant gravity coefficients, with real income impact averaging around 1/3 of measured TFP over the same period.

Considering the effect of unequal elasticities, something like Proposition 1 should continue to hold. $\bar{\Pi}^j$ is solved from

$$\sum_k \lambda_k^j [D_k^j/(\lambda_k^j \bar{\Pi}^j)^{1-\sigma_k} R^j]^{1/\eta_k}(G^j)^{-\sigma_k/\eta_k} = \sum_k \lambda_k^j [D_k^j/(\lambda_k^j \Pi_k^j)^{1-\sigma_k} R^j]^{1/\eta_k}(G^j)^{-\sigma_k/\eta_k}$$

where G^j is the equilibrium factoral terms of trade solved from adding up (9). Intuitively, higher Π's reduce G and less efficient allocation of λ's reduce G. These effects carry through to real incomes as in Proposition 1(b).

Equilibrium wages have simple patterns in the model. The unskilled wage (using $w = g_L$)

is given by $w^j = \alpha(R^j/L^j)G^j$. In the equal elasticity case,

$$w^j = \alpha \left(\frac{K^j}{L^j}\right)^{1-\alpha} \left(\frac{\Lambda^j}{R^j}\right)^{1/[1+(\sigma-1)\eta]}, \forall j.$$

The national average return to skills is $\bar{r}^j = g^j_K$. Based on the preceding discussion,

Proposition 2 (a) *Unskilled wages are increasing in the skilled to unskilled endowment ratio. The average skill returns are decreasing in the same ratio. Both factor incomes are increasing in the relative efficiency of sector specific allocations, and decreasing in country size. (b) The average skill premium $\bar{r}^j/w^j - 1 = [(1-\alpha)/(\alpha)](K^j/L^j)^{\alpha-1} - 1$ is independent of international forces.*

The preceding algebra does not require nonzero bilateral trade flows.[13] For present purposes, this property means that substitution on the extensive margin between traded and nontraded goods plays no central role but occurs in the background.

Proposition 2 (b) is a useful neutrality property of the model with respect to income distribution. In contrast, the distribution of sector specific factor incomes is powerfully affected by international forces. Sector specific factor returns are given by $r^j_k = g^j_{\lambda^j_k}/K^j$. Use the properties of the special Cobb-Douglas GDP function to yield

$$r^j_k = \bar{r}^j \frac{s^j_k}{\lambda^j_k}.$$

The properties of the national average returns to skill, $\bar{r}^j$, are given above. The sector specific part of the preceding expression will be developed following the analysis of equilibrium production shares.

[13] Section 5 validates this claim in the presence of traded inputs and selection into trade.

3.3 Equilibrium Production and Trade Patterns

In the equal case $\sigma_k = \sigma$ the reduced form production share equations simplify to

$$s_k^j = \frac{\lambda_k^j (D_k^j/\lambda_k^j (\Pi_k^j)^{\sigma-1})^{1/\eta}}{(\Lambda^j)^{1/\eta}}. \tag{12}$$

As compared to (9), (12) eliminates the effect of country size on the equilibrium pattern of production. Replacing Λ^j with $\Lambda^j(1)(\bar{\Pi}^j)^{1-\sigma}$ in (12):

Proposition 3 *In the equal elasticities case (with $\sigma > 1$) and uniform Cobb-Douglas production functions, the equilibrium production share is*

1. *increasing in the capital allocation share λ_k^j;*

2. *increasing in the demand 'parameter' D_k^j; and*

3. *decreasing in the relative sectoral incidence of inverse TFP $\Pi_k^j/\bar{\Pi}^j$.*

The negative association between shipment shares and Π's of Proposition 3.3 helps explain the same empirical finding of Anderson and Yotov. They offer complementary causation flowing from given shares to the Π's for the special case of uniform border barriers.

Sector specific factor returns can now be characterized drawing on Proposition 3. Using (12) and $r_k^j = \bar{r}^j s_k^j/\lambda_k^j$ yields

$$\frac{r_k^j}{\bar{r}^j} = \frac{[D_k^j/\lambda_k^j (\Pi_k^j)^{\sigma-1}]^{1/\eta}}{(\Lambda^j)^{1/\eta}}. \tag{13}$$

Then using $\Lambda^j = \Lambda^j(1)\bar{\Pi}^{1-\sigma}$:

Proposition 2 (c)-(d) *Sector specific factor returns are increasing in the national labor to human capital endowment ratio, decreasing in the sector specific allocation, decreasing in the relative sectoral incidence of TFP frictions and increasing in the sectoral demand parameter. (d) The distribution of skill premia is more dispersed the more inefficient is the sectoral allocation of human capital.*

(13) summarizes the properties of the inequality of specific factor returns in global equilibrium. Technology shocks affect the Π's primarily (exclusively under Cobb-Douglas upper

level preferences so that D_k^j is parametric.) Then for given allocations of skills, more dispersion of the incidence of productivity induces more ex post inequality.

The reduced form unit cost equations simplify when $\sigma_k = \sigma$. Using (11) in (10), substituting into (8) and simplifying yields

$$\widetilde{p}_k^j = \frac{(D_k^j/\lambda_k^j(\Pi_k^j)^{\sigma-1})^{(1-\alpha)/\eta}}{(\Lambda^j)^{\alpha/\eta\sigma}}(R^j)^{-(1-\alpha+\alpha/\sigma)/\eta}. \tag{14}$$

Using the decomposition $\Lambda^j = \Lambda^j(1)(\bar{\Pi}^j)^{1-\sigma}$, the implications of (14) for equilibrium 'competitiveness' in the cross section, interpreted as determinants of $\widetilde{p}_k^j$'s, are intuitive and sharp:

Proposition 4 *In the uniform elasticities case:*

1. *larger specific endowments lower costs;*
2. *larger world demand for a good raises its cost;*
3. *higher quality costs more;*
4. *higher sectoral sellers incidence of TFP frictions lowers unit costs;*
5. *bigger countries have lower costs.*
6. *higher national average sellers' incidence raises unit costs while better efficiency of allocation lowers unit costs.*

Proposition 4.3 states that in general equilibrium, higher quality goods have higher unit costs, all else equal. This is less obvious than it might seem. The CES model of preferences implies that some of each variety will be demanded, so it is not true that lower quality must have a lower price to be purchased by anyone.[14]

With unequal elasticities, (8) applies. Proposition 4 still applies for given G^j, with additional effects arising through effects of the exogenous variables on G^j. Compared to

[14] The interpretation of $\beta_{kj}^{1-\sigma_k}$ as a quality parameter is natural from examining the sub-utility function that lies behind the CES expenditure function: starting from equal consumption of each variety, the consumer's willingness to pay is higher the larger is $\beta_{kj}^{1-\sigma_k}$.

(8), the special case (14) implies that larger countries have uniformly lower unit production costs.

The model yields strong restrictions on the equilibrium pattern of trade. The ratio of net exports to GDP is given by $s_k^j - \theta_k$ in the Cobb-Douglas preferences case. Gross exports are more interesting. Production is given by $s_k^j g^j$. Own demand is given by (26). The ratio of gross exports to GDP in the special case of equal elasticities of substitution is given by

$$s_k^j\left(1 - \frac{E_k^j}{Y_k}\left\{\frac{t_k^{jj}}{\Pi_k^j P_k^j}\right\}^{1-\sigma}\right).$$

Imposing Cobb-Douglas upper level preferences, E_k^j/Y_k is replaced with ω^j. Using $\omega^j = (\Lambda^j/R^j)^{1/\sigma}R^j$ this reduces to

$$s_k^j\left(1 - \Lambda^j(1)\left(\frac{R^j}{\bar{\Pi}^j}\right)^{1-1/\sigma}\left\{\frac{t_k^{jj}}{\Pi_k^j P_k^j}\right\}^{1-\sigma}\right). \tag{15}$$

Then:

Proposition 5 *With equal elasticities the ratio of sectoral gross exports to GDP is increasing in s_k^j, which moves according to Proposition 3. For given s_k^j, sectoral gross exports to GDP is decreasing in Constructed Home Bias $\left\{\frac{t_k^{jj}}{\Pi_k^j P_k^j}\right\}^{1-\sigma}$ and global GDP share ω^j which itself is*

1. *increasing in allocative efficiency $\Lambda^j(1)$ and*

2. *increasing in effective relative country size $R^j/\bar{\Pi}^j$.*

The term Constructed Home Bias is coined by Anderson and Yotov (2008). It summarizes the implications of gravity for the prominent empirical regularity called home bias.

Proposition 5 combined with Proposition 2 (c) implies that *export intensity is positively correlated with sectoral earnings premia*, a well documented empirical regularity in rich and poor countries alike. For sharper results in a specific factors continuum model, see Anderson (2009).

4 Equilibrium Specific Factor Allocation

The specific factor allocations are presumably determined by optimizing behavior. Investments in sectors become specific once made, but are allocated from a given stock K so as to equalized anticipated returns. It is useful to consider the fully efficient equilibrium before proceeding to the more realistic equilibrium where investments are ex ante efficient but ex post inefficient due to the realizations of the productivity draws. This section concludes with discussion of allocation of multiple sector specific factors.

4.1 Fully Efficient Equilibrium

An instructive benchmark is the special case model when the specific factors are fully efficiently allocated. This arises if the specific factor becomes mobile; or equivalently, if the incidence of trade and productivity frictions is perfectly anticipated by agents selecting the specific factor investments. The identical Cobb-Douglas production function structure assumed here makes the production set effectively Ricardian when capital allocation adjusts efficiently.[15] Due to the love of variety structure of preferences, prices adjust in equilibrium to support diversification, avoiding the corner solutions that otherwise arise with Ricardian production.

The long run general equilibrium GDP shares reduce to[16]

$$s_k^j = \lambda_k^j = \frac{D_k^j (\Pi_k^j)^{1-\sigma}}{\sum_k D_k^j (\Pi_k^j)^{1-\sigma}}. \tag{16}$$

Supply adjusts to meet demand in absence of trade and productivity frictions. Trade and productivity frictions captured by the Π's redistribute sales through a CES structure, but the mechanism of an essentially demand driven equilibrium pattern of production remains.

[15] The Ricardian production set is the outer envelope of specific factor production sets for fixed sectoral allocations.

[16] Using (12) for s_k^j, $s_k^j/\lambda_k^j = 1$ can be solved for $\lambda_k^j = D_k^j (\Pi_k^j)^{1-\sigma} / \sum_k [\lambda_k^j (D_k^j / \lambda_k^j (\Pi_k^j)^{\sigma-1})^{1/\eta}]^\eta$. $\sum_k \lambda_k^j = 1$ implies that $\sum_k [\lambda_k^j (D_k^j / \lambda_k^j (\Pi_k^j)^{\sigma-1})^{1/\eta}]^\eta = \sum_k D_k^j (\Pi_k^j)^{1-\sigma}$. This yields the solution in the text.

Paralleling this feature, with efficient allocation the unit costs of (14) become invariant to k: $\widetilde{p}_k^j = \bar{p}^j, \forall k$, where $\bar{p}^j = (\Lambda^j)^{(1-\alpha-\alpha/\sigma)/\eta}(R^j)^{-(1-\alpha+\alpha)/\eta}$ and $\Lambda^j = \sum_k D_k^j(\Pi_k^j)^{1-\sigma}$. The equilibrium national shares of world sales in each sector k are given by $Y_k^j/Y_k = (\bar{p}^j)^{1-\sigma}(\beta_k^j\Pi_k^j)^{1-\sigma}$.

The specific gravity model takes the β's as given, while the Eaton-Kortum model endogenizes them. The connection between the two models is seen as follows. Using the preceding equation to obtain $Y_k^j/Y_k(\Pi_k^j)^{1-\sigma}$ and substituting back into the gravity equation (26), the bilateral trade flows are given by

$$X_k^{jh} = (\beta_k^j)^{1-\sigma_k} E_k^h (\bar{p}^j)^{1-\sigma_k} (t_k^{jh}/P_k^h)^{1-\sigma_k}.$$

For any sector, the Eaton-Kortum assumptions result in equilibrium β's such that the right hand side above is replaced with a gravity expression equivalent to (26), only with $1-\sigma$ replaced by $-\nu$ where ν is the dispersion parameter of the Frechet distribution. (See Eaton and Kortum, equation (11).) Demand side forces in the Eaton-Kortum model disappear into a constant term that cancels in equilibrium trade shares. Substitution is all on the extensive margin. In contrast, the Armington structure forces diversified production in each country by assuming that goods are differentiated by place of origin. Substitution is on the intensive margin. The distribution of the productivity penalties, the a's, is unrestricted.

The Costinot-Komunjer extension of Eaton-Kortum to combine deterministic sector/country productivity with variety specific productivity draws from Frechet distributions results in the assignment of proportions of varieties within sectors as in Eaton-Kortum along with the assignment of sectoral allocations. The Appendix expands on the connection between the generalized Ricardian and specific factors models by analyzing monopolistic competition equilibrium when skill allocation is subsequent to productivity realizations.

The difference between the specific gravity and Eaton-Kortum/Costinot-Komunjer models is the specificity of skilled labor. Frechet distributions of productivity draws do not yield closed form predictions when factors are specific. Nevertheless, as preceding sections show, useful predictions about the pattern of production and trade can be made taking the λ's and β's as given.

Suppose that the productivity of distribution and production is random.[17] Investments in sectors must be made prior to the realization of the random variables, at which time the realized sector specific returns differ. Ex ante efficient equilibrium with risk neutral agents is characterized by equal expected rates of return.[18]

The ratio of the realized rate of return in sector k to the average realized rate of return is given by

$$\frac{r_k^j}{\sum_k \lambda_k^j r_k^j} = \frac{s_k^j}{\lambda_k^j}.$$

Simplify by assuming Cobb-Douglas preferences for choice between sectors, hence D_k^j is a parameter. For any pair of sectors k, k' in j, equal expected returns implies

$$\frac{\lambda_k^j / D_k^j}{\lambda_{k'}^j / D_{k'}^j} = E[(\Pi_k^j / \Pi_{k'}^j)^{1-\sigma}].$$

Here E denotes the expectation operator. Combined with the adding up constraint on λ's this implies:

$$\lambda_k^j = \frac{D_k^j E[(\Pi_k^j)^{1-\sigma}]}{\sum_k D_k^j E[(\Pi_k^j)^{1-\sigma}]}, \forall k, j. \tag{17}$$

Note that since the right hand side is convex in Π, *riskier sectors receive more investment* all else equal, by Jensen's Inequality. Intuitively, this occurs because r_k^j is inversely related to Π_k^j and thus is increased in expected value by mean preserving spreads in the distribution of Π_k^j. Of course, considerations of risk aversion, risk sharing and covariation of the Π's modify any such conclusions based on (17).

An empirically tractable form of the share equation emerges from considerations of ex

[17]Evidence from Anderson and Yotov (2008) indicates that bilateral trade costs T_k^{jh} are remarkably stable over time, while in contrast the sectoral productivity penalties a_k^j and the multilateral resistances Π_k^j appear to have significant randomness.

[18]There are important resource allocation implications of risk aversion, but these carry much additional complexity. Helpman and Razin (1978) develop the implications of international trade in securities in this setting when there is aggregate risk.

ante efficiency in monopolistic competition equilibrium. Realized D's differ from ex post efficient equilibrium (including rational expectations) D's by a white noise error term. The Appendix shows that realized shares are given by

$$s_k^j = \frac{\lambda_k^j \left(d_k d_j \epsilon_k^j (\Pi_k^j)^{1-\sigma}\right)^{1/\eta}}{\sum_k \lambda_k^j \left(d_k d_j \epsilon_k^j (\Pi_k^j)^{1-\sigma}\right)^{1/\eta}}, \tag{18}$$

where the d's are fixed effects and the ϵ's are realizations of a unit mean random error that is orthogonal to the other terms. The orthogonality property is due to the assumption of ex ante efficient allocation.

Now consider the implications of randomness for the efficiency of allocation given ex ante efficiency.Taking expectations of (11), the convexity of Λ in Π guarantees that riskier incidence lowers efficiency for a given allocation. Moving to a risk-reducing allocation helps to offset this but cannot fully do so. Moreover, variance plausibly rises with the mean, in which case higher average incidence of trade and productivity frictions imposes an added burden through greater expected ex post inefficiency of allocation. In empirical exercises the λ^*'s can be calculated and compared to actual λ's to decompose the inefficiency due to randomness into its avoidable and unavoidable components.

The full rational expectations equilibrium of the model requires that the expectations of Π's be equal to the expectations of the realized Π's obtained from (24)-(25) subject to (23).

Multiple specific factors introduce no new elements, with the minor exception that there may be differences in the efficiency of allocation of the different factors. The multiple specific factors form a natural aggregate, a concave and homogeneous of degree one function ϕ: $K = \phi(K^1, ..., K^M)$ where the superscript now refers to the specific factor type. Sectoral allocations yield $\phi_k = \phi(\lambda_k^1 K^1, ..., \lambda_k^M K^M)$. Then $\lambda_k K = K_k = \phi_k$ as before with λ_k being the exact index of allocations $\lambda_k \equiv \phi_k/\phi$. If all M specific factor types follow the same allocation rules, then multiple goods make no difference at all. But it is plausible that some factors follow different allocation rules (e.g., risk neutrality may be appropriate for some types of plant and equipment investment but implausible for human capital), in which case the structure of the index λ_k plays a role. Multiple specific factors of course also introduce richer ex post income distributions.

5 Intermediate Inputs and the Extensive Margin

Intermediate products trade comprises a large and growing share of world trade. A simple extension of the specific factors model of production encompasses intermediate products trade.

Vertical disintegration is apparent — an increasing share of components are imported, meaning some formerly potential trade becomes active. In the multi-country context, similar shifts in the qualitative pattern of trade arise as more of the potential bilateral trade links are activated by the choice of firms to initiate trade. The action on the extensive margin of trade introduced here also applies to final goods trade.

5.1 Specific Factors Production with Intermediates

Intermediate products enter for simplicity as just a single intermediate product, potentially produced as a variety at each location.[19] The CES aggregate of the varieties is an input into production of all final goods and the intermediate good at each location. To ease notation, suppress country indexes. The production function for product k in the Cobb-Douglas case is given by

$$f_k = L_k^{\alpha} K_k^{1-\alpha-\nu} M_k^{\nu} / a_k$$

where M_k is the quantity of the CES aggregate intermediate input used in sector k and sector m is the intermediate goods production sector.

Let P_m denote the price of the intermediate input used by the home country, a CES aggregate of the intermediate products purchased from all trading origins. Cost mini-

[19]The methods used here readily scale up to any number of intermediate product classes.

mization combines with the labor market clearance condition to yield the GDP function $g(\widetilde{p}, P_m, L, K, \{\lambda_k\})$ with a closed form given by

$$\{L^{\alpha} K^{1-\alpha}[(\sum_k \lambda_k \widetilde{p}_k^{1/(1-\alpha-\nu)})^{1-\alpha-\nu} P_m^{-\nu}]\}^{1/(1-\nu)} c. \tag{19}$$

Here, c is a constant term combining the parameters, while $\widetilde{p}_k$ is the 'efficiency unit cost' of output in sector k. For some sector $k = m$, $\widetilde{p}_m$ is the efficiency unit price of the intermediate product from sector m produced in the home country.

P_m^j is the buyers' incidence in j of intermediate goods, a CES price aggregate for country j of the elements of the vector $\{\widetilde{p}_m^i t_m^{ij}/\Pi_m^i\}$. All the earlier procedures for multilateral resistance apply. Higher buyers' incidence of TFP in intermediate inputs lowers GDP while lower sellers' incidence of intermediate products raises GDP.

Due to the separability of the GDP function, the reduced form equilibrium efficiency prices and production shares are independent of the incidence of trade costs on intermediate inputs P_m. This separability implies that all the production, trade and income distribution pattern results of Section 3 apply in the presence of intermediate goods.

Aggregate incidence of inverse TFP with intermediate products of the type modeled here is measured based on substituting for $\widetilde{p}$'s using (8) in the price term of the GDP function (19):

$$[(\sum_{k=1}^{n} \lambda_k (\widetilde{p}_k)^{1/(1-\alpha-\nu)})^{1-\alpha-\nu} P_m^{-\nu}]^{1/(1-\nu)}.$$

Evaluate this expression with the reduced form $\widetilde{p}$'s of (8) evaluated at the actual Π's and at a uniform $\bar{\Pi}$ that yields the same value of the expression. The equilibrium inverse TFP incidence measure has the same form as in Proposition 1 except that α is replaced with $\alpha' = \alpha + \nu$. A rise in buyer's incidence for the intermediate goods P_m lowers productivity with elasticity $-\nu/(1-\nu)$ given by the expression above. The key property is that final and intermediate productivity frictions decompose neatly due to the Cobb-Douglas structure of the model.

5.2 The Extensive Margin, Productivity and Trade Patterns

The production function for each industry k is comprised of the production functions of those firms that earn non- negative profits. The firms choose to enter production, commit a skilled labor force and then receive a Hicks-neutral productivity draw from a probability distribution. Those firms unlucky enough to receive draws too low to allow breaking even exit from production. The average productivity in industry k, $1/\bar{a}_k$, is determined by the cutoff productivity of the marginal firm in combination with the parameters of the productivity draw distribution. Average productivity is for present purposes taken as given.

Profits are earned by inframarginal firms, and form part of the rents earned by the sector specific factors.[20] The average productivity is associated with an average price, a constant markup over the the average unit cost of extant firms. See Melitz (2003) for details. The Melitz model differs in having only one factor of production, but the essentials remain the same, illustrated in the Appendix development of the monopolistic competition model. This setup allows aggregation of the heterogeneous firm model into a representative firm model easily linked to the general equilibrium production theory of preceding sections.

The second key contribution of Melitz is to introduce a second cutoff due to fixed costs of exporting. Expanding the iceberg metaphor, part of the iceberg shears off and is lost as it leaves the home glacier, the remainder melting as it travels to its destination. There are two consequences for the allocation of trade and a further consequence for the allocation of resources. As for trade, some (many in practice) trade links are shut down completely because no firm exports, and secondly, firm selection contributes to trade volume in active links. As for resource allocation, firms choosing to export must hire additional unskilled workers to meet the fixed cost.[21] The resulting rise in the wage raises the cost of production for all firms. Now the conditions of trade have an effect on average productivity: lowering

[20]The division of rents between 'owners' and skilled workers is irrelevant to present purposes.

[21]If skilled workers are not completely firm specific, firms can also hire skilled workers within their sector with some loss of skills. The implications of this Darwinian force in the sectoral skilled labor market is developed further in Anderson (2009).

the variable cost of trade induces more firms to incur the fixed cost of trade and raises the average productivity of all surviving firms.

Helpman, Melitz and Rubinstein (2008) develop the implications of fixed costs of export for bilateral trade and the gravity model. For present purposes, note that the effect of action on the extensive margin is isolated in the multilateral resistance terms, Π_k^j for outputs and P_m^j for inputs in country j and sector k. The Appendix develops the implications of their model for multilateral resistance.

6 Conclusion

This paper provides a framework for integrating buyers' and sellers' incidence measures of TFP in production and distribution into a many country general equilibrium model. Differences in the incidence of TFP across goods and countries impact the cross section pattern of production and trade, with sharp results for the special case of the specific factors model.

Given buyers' and sellers' incidence measures for an appropriately disaggregated set of goods, countries (and possibly years) the specific factors model has testable implications for the pattern of production and trade. Deviations from predicted values may give useful clues.

The paper also points to future theoretical refinement. The model links trade frictions to income distribution, and points toward political economy, a link that appears worth exploring in light of concerns about globalization causing inequality. The model also points toward dynamics, as specific factors adjust.

The extreme simplicity of the model buys strong results, while hinting that the results hold in less restrictive cases. The restrictive assumptions about distribution are especially important to relax. The convention of gravity modeling is that the seller provides all the distribution services, so these accrue as income to the seller and form part of the income side of the budget constraint above. In reality, some distribution services are provided by buyers, hence GDP as modeled in this paper does not equal measured GDP. Moreover, gravity measurement picks up trade costs that are implicit and thus do not correspond to directly measurable trade costs, hence national product accounting deviates further from the theoretical model here. A tractable alternative approach that preserves the qualitative features of the present model is to treat distribution services as an intermediate input. More general treatments may still be tractable.

Finally, the analysis reveals important channels through which technology shocks in production and in distribution in one country are transmitted to the incidence of productivity in all trading partners. The specific factors structure suggests gradual adjustment to long run equilibrium. Future research might profitably explore these channels for their implications about inference of productivity and about the international transmission of shocks.

7 References

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8 Appendix

8.1 Sectoral Incidence

On the supply side, for the moment take as given the national output of each sector, and its allocation to all destinations. The value of shipments at *delivered* prices from origin h in product class k is Y_k^h.

Market clearance requires:

$$Y_k^j = \sum_h \left\{ \frac{\beta_k^j \widetilde{p}_k^j t_k^{jh}}{P_k^h} \right\}^{1-\sigma_k} E_k^h. \tag{20}$$

Now solve (20) for the quality adjusted efficiency unit costs $\{\beta_k^j \widetilde{p}_k^j\}$:

$$(\beta_k^j \widetilde{p}_k^j)^{1-\sigma_k} = \frac{Y_k^j}{\sum_h (t_k^{jh}/P_k^h)^{1-\sigma_k} E_k^h}. \tag{21}$$

Based on the denominator in (21), define

$$(\Pi_k^j)^{1-\sigma_k} \equiv \sum_h \left\{ \frac{t_k^{jh}}{P_k^h} \right\}^{1-\sigma_k} \frac{E_k^h}{\sum_h E_k^h}.$$

Divide numerator and denominator of the right hand side of (21) by total shipments of k and use the definition of Π, yielding:

$$(\beta_k^j \widetilde{p}_k^j \Pi_k^j)^{1-\sigma_k} = Y_k^j / \sum_j Y_k^j. \tag{22}$$

The right hand side is the global expenditure share for class k goods from country j. The left hand side is a 'global behavioral expenditure share', understanding that the CES price index is equal to one due to the normalization implied by summing (22):

$$\sum_j (\beta_k^j \widetilde{p}_k^j \Pi_k^j)^{1-\sigma_k} = 1. \tag{23}$$

The implication of (22) is that effectively each origin j ships to a world market with incidence of trade costs Π_k^j. The incidence of trade costs to sellers being given by the Π's, the incidence of bilateral trade costs on the buyers' side of the market is given by t_k^{jh}/Π_k^j, taking away the sellers' incidence. The average incidence of all bilateral costs to h from the various origins j is given by the buyers' price index P_k^h. The buyers' incidence is obtained by substituting for quality adjusted efficiency unit costs from (21) in the definition of the true cost of living index, using the definition of the Π's:

$$(P_k^h)^{1-\sigma_k} = \sum_j \left\{\frac{t_k^{jh}}{\Pi_k^j}\right\}^{1-\sigma_k} \frac{Y_k^j}{\sum_j Y_k^j}. \tag{24}$$

Collect this with the definition of the Π's:

$$(\Pi_k^j)^{1-\sigma_k} = \sum_h \left\{\frac{t_k^{jh}}{P_k^h}\right\}^{1-\sigma_k} \frac{E_k^h}{\sum_h E_k^h}. \tag{25}$$

These two sets of equations jointly determine the inward multilateral resistances, the P's and the outward multilateral resistances, the Π's, given the expenditure and supply shares and the bilateral trade costs, subject to the normalization (23). A normalization of the Π's is needed to determine the P's and Π's because (24)-(25) determine them only up to a scalar.[22] See Anderson and Yotov (2008) for analysis of the properties of multilateral resistance.

Bilateral trade flows are given by the gravity equation

$$X_k^{jh} = \left\{\frac{t_k^{jh}}{\Pi_k^j P_k^h}\right\}^{1-\sigma_k} \frac{Y_k^j E_k^h}{\sum_j Y_k^j}. \tag{26}$$

This follows from the CES expenditure setup using (22) to substitute for $(\beta_k^j \widetilde{p}_k^j)^{1-\sigma_k}$. The interpretation of (26) reveals that trade frictions modify the frictionless flow $Y_k^j E_k^h / \sum_j Y_k^j$ by a power transform of the relative incidence of trade costs.

The relationship between the incidence of trade frictions and productivity frictions in the

[22] If $\{P_k^0, \Pi_k^0\}$ is a solution to (24)-(25), then so is $\{\lambda P_k^0, \Pi_k^0/\lambda\}$ for any positive scalar λ; where P_k denotes the vector of P's and the superscript 0 denotes a particular value of this vector, and similarly for Π_k.

cross section is clarified by analyzing the special limiting case of frictionless trade, where $t_k^{jh} = a_k^j, \forall k, j$. The solution to (24)-(25) under the convenient normalization $P_k^1 = 1$[23] is $\Pi_k^j = a_k^j, \forall k, j$, and $P_k^h = 1, \forall k, h$. All the incidence of productivity is borne on the supply side. The reason is that in conditional general equilibrium the expenditure E_k^j on good k from source j is given. With a fall in a_k^j, market clearance is achieved with a rise in the efficiency unit cost, so that all the benefit accrues to suppliers of k from j. The further implication is that sectoral TFP is decomposable into a Hicks neutral production component and an equilibrium distribution incidence component. It is important to keep in mind that the comparative static incidence of a productivity improvement is still shared between buyer and seller; this decomposition applies in the cross section.

8.2 Monopolistic Competition

The special form of monopolistic competition and trade that is the focus of most of the literature has essentially no effect on the equilibrium of the model for given allocations of the specific factor. Endogenizing the allocation of the specific factor has the additional important effect of endogenizing the expenditure share parameters.

The CES preferences in each sector now contain a very large number of potential brands produced by firms in each country. Each firm is a monopolistic competitor, marking up price over cost by a constant proportion $\sigma/(\sigma - 1)$. The GDP shares have exactly the same form as in the text because differing elasticities act on the model exactly like differing technology frictions and become part of the Π's while common elasticities cancel out.

The development of a brand takes F units of skilled labor. The allocation of skilled labor is subject to the constraint $K^j = F \sum_k n_k^j + K_k^j$ where n_k^j is the number of firms in sector k and country j. The allocation share of skilled labor net of development requirements is given for each sector k by $\lambda_k^j = K_k^j/(K^j - F \sum_k n_k^j)$.

The number of brands is determined in fully efficient equilibrium by the zero profit

[23] For allocations within sectors, only the relative multilateral resistances are relevant for allocation, so allocation is invariant to the normalization.

condition $r^j(Fn_k^j + K_k^j) + wL_k^j = p_k^j y_k^j$. Using the marginal revenue product functions for skilled and unskilled labor for the Cobb-Douglas production function in the zero profit condition and simplifying yields

$$n_k^j = \frac{K_k^j}{F}\left(\frac{\sigma}{\sigma-1} - 1\right) = \lambda_k^j \frac{K^j - F\sum_k n_k^j}{F(\sigma-1)}.$$

Sum and solve for $\sum_k n_k^j$, then substitute back into the right hand expression to yield

$$n_k^j = \lambda_k^j \frac{K^j}{F(\sigma-1)}\left(1 - \frac{1}{(\sigma-1)(1+F)}\right). \quad (27)$$

The supply of labor net of development requirements is in equilibrium given by

$$K^j\left(1 - \frac{1}{\sigma-1} + \frac{1}{(\sigma-1)^2(1+F)}\right)$$

Thus the GDP function remains exactly the same as in the text, with the understanding that K^j is replaced by the expression above for net skilled labor and λ's are defined as shares of net skilled labor.

Now consider the implications for the demand side of the model. For each sector k, the demand 'parameter' is

$$D_k^j = \theta_k n_k^j / \sum_j n_k^j \quad (28)$$

based on the Dixit-Stiglitz structure.

The efficient allocation in a frictionless world is $\lambda_k^j = \theta_k, \forall j, k$. This follows from solving (16) with the Π's equal to one.

For a world with trade frictions the equilibrium allocation is solved from $\lambda_k^j = s_k^j$, using (12) for s_k^j and then replacing D_k^j with a function of λ's by using (27) in (28).

$$D_k^j = \theta_k \frac{\lambda_k^j}{\bar{\lambda}_k} \frac{K^j}{\sum_j K^j}, \quad (29)$$

where $\bar{\lambda}_k \equiv \sum_j \lambda_k^j K^j / \sum_j K^j$. Substituting in (12) and simplifying using $s_k^j = \lambda_k^j$ yields, for goods that are produced,

$$1 = \frac{\theta_k / \bar{\lambda}_k (\Pi_k^j)^\sigma}{\sum_k \lambda_k^j \theta_k / \bar{\lambda}_k (\Pi_k^j)^\sigma}. \tag{30}$$

(30) only holds (goods are produced) for goods with the same Π's. The Π's being endogenous, describing the equilibrium is difficult.

Eaton and Kortum resolve this difficulty by imposing a Frechet distribution on the a's that differs nationally by a location parameter but has a common dispersion parameter. Eaton and Kortum predict the proportion of varieties that will be produced and exported in equilibrium by each country to each partner as a gravity equation. (See their equation (11).) Costinot and Komunjer extend the Eaton-Kortum approach by adding a deterministic country/sector specific component to productivity. Now the gravity model describes bilateral trade patterns in any sector while the country/sector productivity component shifts the country/sector production shares. Thus this generalized Ricardian approach is nested in the specific gravity approach when the specific factor is allocated after the productivity draws.

Admitting productivity shocks that are not revealed prior to the allocation of skilled labor, the efficient allocation is solved from using (27) in (28) and then substituting the result into (17). The pattern of production and trade predictions of the model remain those of the text for given λ's and D's, while the explantion of the λ's and D's is deeply implicit. An empirically tractable form of the share equation nevertheless emerges from considerations of ex ante efficiency. Realized D's differ from ex post efficient equilibrium (including rational expectations) D's by a white noise error term. Substituting the right hand side of (29) augmented by white noise into (12) yields

$$s_k^j = \frac{\lambda_k^j \left(f_k f_j \epsilon_k^j (\Pi_k^j)^{-\sigma} \right)^{1/\eta}}{\sum_k \lambda_k^j \left(f_k f_j \epsilon_k^j (\Pi_k^j)^{-\sigma} \right)^{1/\eta}}, \tag{31}$$

where the f's are fixed effects and the ϵ's are realizations of a unit mean random error that is orthogonal to the other terms. The orthogonality property is due to the assumption of ex ante efficient allocation.

8.3 Selection to Trade

Helpman, Melitz and Rubinstein (2008) derive the gravity model with selection. The exposition below reviews their model, and reformulates it to highlight the role of multilateral resistance in both intensive and extensive margins. It eases notational clutter to suppress the separate accounting for each goods class k, and to move the location indexes to the subscript position.

The model of the preceding subsection applies to determine the number of firms that enter, taken here as given along with the other variables of conditional general equilibrium.

The cost of a firm to serve its own market (assuming that $t_{ii} = 1$ for simplicity) is given by $\widetilde{p}_i$ times a_i, the inverse of the firm's productivity draw. The aggregate expenditure at destination j is E_j and the CES expenditure system allocates expenditure across origins. Sales by i to country $j \neq i$ are profitable only if $a_i \leq a_{ij}$ where a_{ij} is defined by the zero profit condition:

$$\sigma^{-\sigma}(\sigma - 1)^{\sigma-1} \left(\frac{t_{ij}\widetilde{p}_i a_{ij}}{P_k^j} \right)^{1-\sigma} E_j = f_{ij}.$$

Here, f_{ij} denotes the fixed bilateral export cost. Extending the iceberg metaphor, f is measured in uinits of the good, as if a chunk sheared off and was lost as the berg separated from the mother glacier. Note that the markup cancels in the numerator and denominator of the demand function facing the firm.

Define the selection variable $V_{ij}(a_{ij})$ where

$$V_{ij} = \int_{a_L}^{a_{ij}} a^{1-\sigma_k} dF(a)$$

for $a_{ij} \geq a_L$ while

$$V_{ij} = 0$$

otherwise. Here, F is the cumulative density function. The value of shipments to all destinations from location i is denoted Y_i.

Now derive the gravity model. The bilateral import value of shipments is given by

$$X_{ij} = \left(\frac{\widetilde{p}_i t_{ij}}{P_j}\right)^{1-\sigma} E_j n_i V_{ij}.$$

The total value of shipments is

$$Y_i = \sum_j X_{ij} = \widetilde{p}_i^{1-\sigma} n_i \sum_j (\frac{t_{ij}}{P_j})^{1-\sigma} V_{ij} E_j.$$

First, solve market clearance for $\widetilde{p}_i^{1-\sigma}$:

$$\widetilde{p}_i^{1-\sigma} = \frac{y_i/Y}{\Pi_i^{1-\sigma}}. \tag{32}$$

Here, y_i denotes the shipments of the average firm in country i, Y_i/n_i and $Y = \sum_i Y_i = \sum_j E_j$, while

$$\Pi_i^{1-\sigma} \equiv \sum_j (\frac{t_{ij}}{P_j})^{1-\sigma} V_{ij} E_j / Y \tag{33}$$

Substitution yields the bilateral flows as:

$$X_{ij} = (\frac{t_{ij}}{P_j \Pi_i})^{1-\sigma} V_{ij} Y_i E_j / Y,$$

where

$$P_j^{1-\sigma} = \sum_i (\frac{t_{ij}}{\Pi_i})^{1-\sigma} V_{ij} Y_i / Y. \tag{34}$$

The normalization condition for the Π's follows from manipulating (32) and summing:

$$\sum_i n_i (\Pi_i \widetilde{p}_i)^{1-\sigma} = 1. \tag{35}$$

The selection equation can be restated to highlight the role of multilateral resistance.

Selection is controlled by:

$$\sigma^{-\sigma}(\sigma-1)^{\sigma-1}(\frac{a_{ij}t_{ij}}{P_j\Pi_i})^{1-\sigma}E_j y_i/Y = f_{ij}. \tag{36}$$

There are three implications. First, notice that the gravity model with selection combines the effects of trade costs on the intensive margin with their effects on the extensive margin acting through V_{ij}. Higher fixed costs reduce volume while larger markets draw more entrants. Second, σ plays a role in selection. Incorporating variation across goods class, lower elasticity (higher markup) goods classes will have more firms selected into exporting, all else equal. Third, most importantly, the multilateral resistance variables incorporate both the productivity penalty imposed by the incidence of trade costs and the productivity gain garnered by the incidence of selection into trade.

The formal model is completed by specifying a distribution function for G. With the Pareto distribution used by Helpman, Melitz and Rubinstein, let the Pareto parameter be κ. Then

$$V_{ij} = \frac{\kappa a_L^{\kappa-\sigma+1}}{(\kappa-\sigma+1)(a_H-a_L)}W_{ij}$$

$$W_{ij} = max[(a_{ij}/a_L)^{\kappa-\sigma+1}-1, 0].$$

Helpman, Melitz and Rubinstein estimate selection with a Probit regression, then use these estimates to control for selection in the second stage gravity model regression with positive trade flows.