# The Specific Factors Continuum Model, with Implications for Globalization and Income Risk

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**Abstract**

This paper embeds the specific factors model in the goods continuum approach of Dornbusch, Fischer and Samuelson (1977, 1980) and applies it to analyze the effect of globalization on income risk. Globalization amplifies sector specific income risk induced by idiosyncratic sectoral technology shocks, but tends to reduce income risk to both mobile and immobile factors induced by aggregate technology shocks that differ by country. Aggregate risk bears most heavily on the poorest specific factors.

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This paper develops a specific factors continuum model of trade and applies it to analyze the effect of globalization on income risk. The goods continuum trade model project of Dornbusch, Fischer and Samuelson (DFS) is completed by adding the specific factors model to their continuum versions of the Ricardian (1977) and Heckscher-Ohlin (1980) models. As with applications subsequently based on the DFS models, future applications of the specific factors continuum model can be anticipated. The application of this paper addresses the question of whether increasing openness to trade amplifies income risk induced by technology shocks. The short answer is that globalization damps aggregate risk (from economy-wide productivity shocks that vary between nations, associated with aggregate absolute advantage risk) but amplifies idiosyncratic risk (arising from a set of sector specific absolute advantage shocks, associated with comparative advantage risk). Mobile factor incomes become less risky while immobile factor incomes become more risky if idiosyncratic risk dominates aggregate risk.

The key feature of the DFS continuum approach is its clean solution for the boundary between nontraded and traded goods sectors. The extensive margin of trade[1] in general equilibrium is characterized as a smooth reduced form function of trade costs and other exogenous variables. Similarly clean solutions and tractable comparative statics of the extensive margin arise in the specific factors model. The DFS models have no intensive margin because there is at most infinitesimal import-competing production. In contrast, in the specific factors model there is always import-competing production, with important comparative static action on both the intensive and extensive margins of trade. Action on both margins is a useful property because the intensive margin adds richer income distribution effects and because recent empirical research demonstrates big variation on both margins (for example, Hummels and Klenow, 2005).

Extending the goods continuum approach to the specific factors case requires handling

[1]Extensive margin is used here to denote a boundary between sectors, not between firms within a sector. The techniques of this paper can indeed extend to apply to the margin between heterogeneous firms within a sector, as an early version of this paper showed.

a second continuum, that of the sector specific factors. Tractable solutions emerge after specializing the model. First, the specific factor (capital) is treated as ex ante allocable anywhere but sector specific once allocated (following Neary, 1978).[2] Second, the neoclassical two factor production functions in each sector are assumed to be identical up to a Hicks neutral productivity shock, realized after the allocation of capital has occurred. The mobile factor, labor, is allocated ex post to equalize its value of marginal product across the continuum of sectors. The Ricardian continuum model is contained as a special case, arising when either the share of capital goes to zero or the allocation of capital is ex post efficient.

Equilibrium is first characterized with arbitrary allocations of capital, but more can be said by imposing rational expectations equilibrium allocation of capital. Assuming that the productivity shocks in each sector are independent draws from a common distribution that may differ across countries, the equilibrium allocation implies equal prospects in each sector. Thus attitudes toward risk play no role in ex ante allocation, a great simplification.

Income risk due to productivity shocks is powerfully affected by globalization in the model, with results that modify previous insights in the literature.[3] Productivity risk as it affects incomes in the model decomposes naturally into idiosyncratic (a given home sector's productivity relative to its foreign counterpart may be high or low relative to other sectors) and aggregate (the average across all sectors of home vs. foreign productivity) components. These are familiar from Eaton and Kortum (2002) as comparative and absolute advantage components respectively.[4] Idiosyncratic equilibrium income risk (arising from idiosyncratic

[2]Nothing in the model hangs on the details of difference between the two types of factors as long as they are not perfect substitutes. (With perfect substitutes, the ability of the mobile factor to shift locations washes out the sectoral differences in returns to the specific factor and the model becomes Ricardian despite potential specificity.) Specificity could alternatively be associated with a labor type with high relocation costs, due either to psychic attachment or sector specific skill acquisition.

[3]Globalization is often thought to have increased personal income risk because it reduces the non-traded share of the economy that is sheltered from external shocks. Offsetting this, a standard argument is that income risk is decreased by the ability to trade because internal productivity shocks induce price movements that partially offset their effect on income. Whether open markets have less aggregate income risk than closed ones depends on whether internal or external shocks predominate. These intuitive partial equilibrium ideas have not been developed in a general equilibrium setting that allows for endogenous response to shocks in other sectors, and they do not address the effect of small changes in trade costs as opposed to opening or closing trade.

[4]Eaton and Kortum's absolute advantage parameter is treated here as a separate potentially random

productivity risk) to sector specific factors is increased by globalization because on the extensive margin it increases the range of sectors exposed to trade while on the intensive margin it amplifies the effect of good or bad luck in location in the traded goods sectors.

In contrast, globalization tends to damp income risk from aggregate shocks. Aggregate productivity shocks induce multi-factoral terms of trade (defined below) shocks with equiproportional effect on all factors within countries. The damping effect of globalization arises from enhanced action on the extensive margin: bigger inter-sectoral response of the mobile factor reduces the factoral terms of trade shifts needed to reach equilibrium following the aggregate productivity shock.[5] On the intensive margin there may be an offset because the effect of globalization on the variance of the multi-factoral terms of trade depends on a balance of forces. A symmetric trade cost reduction raises export sector specific incomes while it reduces import sector specific incomes, hence has ambiguous effect on the multi-factoral terms of trade. This ambiguity is carried through to the cross effect of aggregate shocks on the derivative of the multi-factoral terms of trade with respect to trade costs. Theory and evidence[6] suggest that the damping effect of the extensive margin predominates and globalization reduces aggregate income risk.

For thinking about income risk and ex post inequality, the specific factors model has several other advantages. First, random productivity draws across sectors combined with ex post immobility of ex ante identical factors help to rationalize a portion of the tremendous heterogeneity of sectoral returns to capital and wages (understanding that the identity of the sector specific factor is subject to the purpose of the analyst). The sectoral premium for export sector employment is well documented.[7] Second, the redistribution game of trade

draw.

[5]A force acting from reduced shocks to the extensive margin of trade is well recognized (see for example Bergin and Lin, 2008) but is shut down in this paper because the restrictions of the model imply that in equilibrium all allocation choices have equal prospects, so increased risk has no allocation consequences.

[6]Besedes and Prusa (2006a,b) report evidence that exporting winks on and off, consistent with a high response of the extensive margin to small changes in conditions.

[7]This regularity was given prominence by Katz and Summers (1989). The export premium is well documented in the US and other developed countries. Sparser available evidence finds the same pattern in poorer countries as well — see Milner and Tandrayen (2006) on sub-Saharan Africa and Tsou, Liu and Huang (2006) on Taiwan. Many other important dimensions of wage and capital returns heterogeneity and

policy in the now-standard political economy setup (Grossman and Helpman, 1994; empirically confirmed by Goldberg and Maggi, 1999) is based on the specific factors model. The link suggests the way toward an international political economy of the risk-sharing aspect of trade policy.[8]

A related literature treats the effect of globalization on income distribution. The factor proportions model applications surveyed in Feenstra (2004) have income distributions of low dimension without sectoral premia. In the Heckscher-Ohlin continuum model application of Feenstra and Hanson (1999), globalization raises the skill premium by increasing the average skill intensity of the production mix in North and South through reallocation on the extensive margin. In contrast the specific factors model generates sectoral wage premia. The Cobb-Douglas production function in the text neutralizes globalization's effect on the average immobile factor premium. In the general case developed in the Appendix, globalization can raise or lower the average immobile factor premium in both countries depending on whether the average immobile factor intensity of production rises or falls, itself ordinarily determined by whether the elasticity of substitution is less or greater than one.

Papers by Blanchard and Willman (2008) and Costinot and Vogel (2008) are similar to this paper in featuring continuum income distributions with heterogeneous workers who sort into industries of varying skill intensity. In contrast to the present paper these models generate rich descriptions of wage variation within sectors, in accord with empirical evidence. But they do not explain locational rents to otherwise observationally identical factors. The specific factors model complements the heterogeneity/matching story with a story of relocation costs. Also related is a paper by Helpman, Itskhoki and Redding (2008) that generates high dimensional income distributions due to workers' differential abilities interacting with a costly screening technology used by heterogeneous firms to select from applicants.

The model is also related to a literature featuring idiosyncratic productivity shocks.

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the accompanying income risk that have been emphasized in recent literature, such as wage premia and employment mobility within sectors or even within firms, are associated with heterogeneous productivity across firms.

[8] See Eaton and Grossman (1985) for analysis of 'optimal' tariffs as insurance.

Eaton and Kortum (2002) derive the equilibrium implications of the Ricardian continuum model with sectoral productivity shocks. They solve the many country Ricardian continuum model by imposing a Frechet distribution on the idiosyncratic productivity shocks. In contrast, this paper imposes no restriction on the idiosyncratic shocks distribution and adds aggregate risk.

Section 1 presents the basic production model for given allocations of the specific factors. Section 2 derives the global equilibrium of the two trading countries. Section 3 deals with the comparative statics of aggregate income risk. Section 4 derives the ex post distributional implications of idiosyncratic income risk. Section 5 discusses the equilibrium ex ante allocation of the specific factor. Section 6 concludes with speculation on future work.

# 1 The Production Model

Each good has an identical potential production function $F[L(z), K(z)]$ that is increasing, homogeneous of degree one and concave in the two factors, mobile and sector specific, where $L(z), K(z)$ are the amounts of labor and capital respectively allocated to sector $z$. Maximum potential output in sector $z$ is reduced by the realization of a random productivity draw $1/a(z)$, $a(z) \geq 1$. $F(\cdot)$ is assumed to be a Cobb-Douglas function in the text to generate parametric results.[9] The Appendix shows that the qualitative analysis holds with the general neoclassical production function.

There is a continuum of goods with production in sector $z$ given by

$$y(z) = [1/a(z)]L(z)^{\alpha}K(z)^{1-\alpha}, \forall z \in [0, 1]. \tag{1}$$

$\alpha \in [0, 1]$ is labor's share parameter. The aggregate supply of capital is $K$, each sector $z$ having sector-specific share $\lambda(z) = K(z)/K$ of the total supply. As a convenience to

[9]The restriction of identical shares across sectors implies constancy of labor's share of GDP, a well known empirical regularity.

economic intuition, despite the continuum of sectors, discrete concepts such as 'share' and 'fraction' here and henceforth are applied to densities like $\lambda(z)$ despite the violation of proper mathematical usage. The labels 'labor' and 'capital' are applied loosely for intuition, the important distinction being between mobile and immobile factors.

The economy achieves efficient allocation of labor across sectors with price taking behavior by firms. The aggregate supply of labor is given by $L$, hence the resource constraint is $\int_0^1 L(z)dz \leq L$. The gross domestic product (GDP) function for this economy is given by

$$g = L^{\alpha} K^{1-\alpha} G, \tag{2}$$

where the GDP deflator $G$ is given by

$$G = \left[ \int_0^1 \lambda(z)(p(z)/a(z))^{1/(1-\alpha)} dz \right]^{1-\alpha}. \tag{3}$$

Here $p(z)$ denotes the price of good $z$. (2) is derived by (i) solving the value of marginal product of labor condition for labor demand in each sector $z$, (ii) solving the labor resource constraint for the equilibrium wage, and (iii) using the Cobb-Douglas property $wL = \alpha g$ to solve for $g$. 'Real GDP' is given by $R \equiv L^{\alpha} K^{1-\alpha}$. The Cobb-Douglas GDP function has the constant elasticity of transformation (CET) property.[10] It is convenient in what follows to use $P(z) = p(z)/a(z)$, the efficiency price of good $z$, as the price argument in the GDP function.

The GDP production shares are given by

$$s(z) = \lambda(z) \left\{ \frac{P(z)}{G} \right\}^{1/(1-\alpha)}. \tag{4}$$

A country produces all goods for which it has a positive specific endowment under the Cobb-Douglas assumption because the mobile factor has a very large marginal product in

[10] The elasticity of transformation is equal to $\alpha/(1-\alpha)$.

any sector where its level of employment is very small.

Capital is allocated to sectors prior to the realization of productivity shocks. The ex post returns differ by sector due to different productivity shock realizations and the efficient allocation of the labor that ensues.[11] The equilibrium ex post specific returns distribution exhibits higher returns in export sectors (those receiving relatively high productivity realizations) than in import competing sectors (those receiving relatively low productivity realizations), as shown in Section 4. In equilibrium each location choice has equal ex ante prospects for the specific returns distribution, as discussed in Section 5.

There is a foreign economy with identical potential production functions in each sector, but differing productivity realizations $a^*(z)$ possibly drawn from a different productivity distribution. The foreign economy is also characterized by differing specific factor endowments $K^*(z)$ and labor endowment $L^*$.

The foreign economy has GDP function and GDP share equations generated analogously to the home economy, all foreign variables being denoted by $^*$'s. The foreign efficiency prices are denoted $P^*(z)$. The foreign GDP function is $g^* = (L^*)^{\alpha}(K^*)^{1-\alpha}G^*$ where

$$G^* = [\int_0^1 \lambda^*(z)(P^*(z))^{1/(1-\alpha)}]^{1-\alpha}.$$

The absolute advantage of the home country in sector $z$ is given by $A(z) \equiv a^*(z)/a(z)$. Sectors are ordered so that lower values of $z$ are associated with higher home absolute advantage, $A'(z) < 0$. $A'(z)$ gives the Ricardian comparative advantage ranking of sectors locally around $z$. A shift in the entire $A(z)$ schedule is a shift in home absolute advantage in the aggregate, specialized below to an equiproportional shift.

In the specific factors model, in contrast to the Ricardian, home relative labor productivity in $z$ is affected by the specific factor endowments as well as absolute advantage. Define $\Lambda(z) \equiv A(z)^{1/(1-\alpha)}\lambda(z)/\lambda^*(z)$. $\Lambda(z)$ could in principle have a different ordering from $A(z)$,

[11] Under the sector specific skill interpretation of specificity, the range of productivity shocks is restricted such that skilled workers never choose to become mobile unskilled workers, hence even the sector with the worst shock continues to produce.

but Section 5 shows that the equilibrium allocation of capital implies $\lambda(z) = \lambda^*(z)$, hence $\Lambda(z) = A(z)^{1/(1-\alpha)}$ has the same ordering as $A(z)$, so its slope gives comparative advantage.

# 2 Trade Equilibrium

Trade equilibrium will be derived in two stages. First, the equilibrium goods prices and extensive margins of trade will be derived from market clearance and arbitrage conditions, both given the multifactoral terms of trade. Then the equilibrium factoral terms of trade will be derived from the trade balance condition.

Trade is costly, with parametric markup factor $t > 1$. Arbitrage implies that for goods exported by the home country, $p^*(z) = p(z)t$. For goods exported by the foreign country, $p(z) = p^*(z)t$. In terms of efficiency prices, $P^*(z) = P(z)t/A(z)$ for home exports while $P^*(z) = P(z)/tA(z)$ for home imports.

International trade occurs in equilibrium for a range of goods where relative productivity differences are large enough to cover the trade cost. Home exports are in the interval $z \in [0, \bar{z})$ and foreign exports in the interval $z \in (\bar{z}^*, 1]$. Non-traded goods are in the interval $[\bar{z}, \bar{z}^*]$ where productivity differences are too small to overcome trade costs.

Tastes are identical across countries and characterized by a Cobb-Douglas utility function with parametric expenditure 'share' for good $z$ given by $\gamma(z)$. Most of the paper treats the $\gamma$'s as given, but the model is extended to allow $\gamma$ to be perturbed by a zero mean taste shock.

## 2.1 Goods Market Equilibrium

Equilibrium prices must clear markets for each good. For non-traded goods $s(z) = \gamma(z) = s^*(z); z \in [\bar{z}, \bar{z}^*]$, the local supply share equals the local expenditure share. For traded goods, due to the iceberg trade costs, market clearance is given by $s(z)g + s^*(z)g^* = \gamma(z)(g + g^*)$.[12] This implies for traded goods:

$$\frac{s(z)}{\gamma(z)}\frac{g}{g+g^*} + \frac{s^*(z)}{\gamma(z)}\frac{g^*}{g+g^*} = 1. \tag{5}$$

(5) and the ordering of $z$ implies that $s < \gamma \iff s^* > \gamma$.

It is convenient to choose the foreign GDP deflator as the numeraire, $G^* = 1$. Then $G$, the equilibrium value of the home GDP deflator, is interpreted as the multifactoral terms of trade.[13]

Now solve for the (transform of the) equilibrium efficiency unit price for traded goods using the share equation (4) and the GDP equation (2) in the market clearance equation (5):

$$P(z)^{1/(1-\alpha)} = \frac{\gamma(z)}{\lambda(z)}\frac{GR/R^* + 1}{G^{-\alpha/(1-\alpha)}R/R^* + t^{1/(1-\alpha)}/\Lambda(z)}, z \in [0, \bar{z}); \tag{6}$$

$$P(z)^{1/(1-\alpha)} = \frac{\gamma(z)}{\lambda(z)}\frac{GR/R^* + 1}{G^{-\alpha/(1-\alpha)}R/R^* + 1/t^{1/(1-\alpha)}\Lambda(z)}, z \in (\bar{z}^*, 1]. \tag{7}$$

For non-traded goods the efficiency price transforms are given by

$$P(z)^{1/(1-\alpha)} = \frac{\gamma(z)}{\lambda(z)}G^{1/(1-\alpha)}, z \in [\bar{z}, \bar{z}^*] \tag{8}$$

$$P^*(z)^{1/(1-\alpha)} = \frac{\gamma(z)}{\lambda^*(z)}, z \in [\bar{z}, \bar{z}^*]. \tag{9}$$

The equilibrium production shares are functions of equilibrium $G$, using the equilibrium

[12] The expression for market clearance follows from material balance with iceberg melting trade costs. For example, in the range $z \in [0, \bar{z}]$, market clearance is given by

$$y(z) - x(z) = t[x^*(z) - y^*(z)]$$

where $x(z), x^*(z)$ denote consumption of good $z$ in the home and foreign countries. The equation implies that for each unit imported by the foreign economy, $t > 1$ units must be shipped from the home economy, $t - 1$ units melting away en route. Multiply both sides by $p(z)$, use $p^*(z) = p(z)t$ and utilize the GDP and expenditure share definitions to obtain the text expression.

[13] The Cobb-Douglas production function restriction is useful in getting sharp results that the Appendix shows extend more generally. In the general case, $G$ is replaced by the home relative wage.

prices from (6)-(9) in (4). The home traded goods equilibrium GDP shares are given by:

$$s(z) = \gamma(z)\frac{GR/R^* + 1}{GR/R^* + (Gt)^{1/(1-\alpha)}/\Lambda(z)}, z \in [0, \bar{z}); \tag{10}$$

$$s(z) = \gamma(z)\frac{GR/R^* + 1}{GR/R^* + (G/t)^{1/(1-\alpha)}/\Lambda(z)}, z \in (\bar{z}^*, 1]. \tag{11}$$

Crucially, export intensity $s(z)/\gamma(z)$ is decreasing in $z$ for traded goods. For non-traded goods, $s(z) = \gamma(z) = s^*(z), z \in [\bar{z}, \bar{z}^*]$. Foreign traded goods shares are given by

$$s^*(z) = \gamma(z)\frac{GR/R^* + 1}{G^{-\alpha/(1-\alpha)}t^{-1/(1-\alpha)}\Lambda(z)R/R^* + 1}, z \in [0, \bar{z});$$

$$s^*(z) = \gamma(z)\frac{GR/R^* + 1}{G^{-\alpha/(1-\alpha)}t^{1/(1-\alpha)}\Lambda(z)R/R^* + 1}, z \in (\bar{z}^*, 1].$$

The extensive margins of trade are determined by market clearance in the home and foreign export cutoff sectors $\bar{z}, \bar{z}^*$. Thus $s(\bar{z}) = \gamma(\bar{z})$ and $s^*(\bar{z}^*) = \gamma(\bar{z}^*)$ solve for the home and foreign export cutoff sectors as implicit functions of $G$ and $t$:

$$G = \Lambda(\bar{z})^{1-\alpha}/t \tag{12}$$

and

$$G = \Lambda(\bar{z}^*)^{1-\alpha}t. \tag{13}$$

A rise in $t$ at given $G$ lowers $\bar{z}$ and raises $\bar{z}^*$, shrinking the extensive margin of trade, by $\Lambda_z < 0$. (12)-(13) imply that $\bar{z}^*$ is implicitly a function $Z^*(\bar{z}, t)$ that is increasing in $\bar{z}$ and $t$ in equilibrium:

$$Z^*(\bar{z}, t) = \bar{z}^* : \Lambda(\bar{z}^*) = \Lambda(\bar{z})/t^{2/(1-\alpha)}. \tag{14}$$

## 2.2 Factoral Terms of Trade

The factoral terms of trade are determined by the international budget (balanced trade) constraint. Specifying the constraint requires new terms for aggregate expenditure and GDP shares for home imports and exports.

Define the traded goods aggregate expenditure shares for home exports $\Gamma(z) \equiv \int_0^z \gamma(x)dx$ and for home imports $\Gamma^*(z^*) \equiv \int_{z^*}^1 \gamma(x)dx$, where $x$ is a variable of integration. The equilibrium home GDP shares for each good (10) and (11) add up to the GDP shares for exports $X$ and imports $M$:

$$S^X(\bar{z}, G; R/R^*, t, \cdot) \equiv \int_0^{\bar{z}} s(x)dx = \int_0^{\bar{z}} \gamma(x) \frac{GR/R^* + 1}{GR/R^* + (Gt)^{1/(1-\alpha)}/\Lambda(x)} dx$$

and

$$S^M(\bar{z}^*, G; R/R^*, t, \cdot) \equiv \int_{\bar{z}^*}^1 s(x)dx = \int_{\bar{z}^*}^1 \gamma(x) \frac{GR/R^* + 1}{GR/R^* + (G/t)^{1/(1-\alpha)}/\Lambda(x)} dx$$

The sectoral GDP share aggregators $S^X$ and $S^M$ differ in their response to trade costs — a fall in $t$ for given $(\bar{z}, \bar{z}^*, G)$ raises export sector income but but lowers import competing sector income. This implication foreshadows much of the succeeding analysis of distribution and responsiveness to shocks.

Balanced trade requires $S^X + S^M - \Gamma - \Gamma^* = 0$. Solve for $G$ as an implicit function of $\bar{z}$ using $\bar{z}^* = Z^*(\bar{z}, t)$ to substitute for $\bar{z}^*$:

$$B(z; R/R^*, t, \bar{z}, \cdot) = G : S^X(\bar{z}, G; R/R^*, t, \cdot) + S^M[Z^*(\bar{z}, t), G; R/R^*, t, \cdot] - \Gamma(\bar{z}) - \Gamma^*[Z^*(\bar{z}, t)] = 0. \tag{15}$$

Based on the properties of the model, $B(z)$ rises to a maximum at the equilibrium $\bar{z}$:

$$\frac{B_z}{B} = \frac{s(z) - \gamma(z) - [s(z^*) - \gamma(z^*)]Z_z^*}{-S_G^X G - S_G^M G}$$

which is equal to zero at $\bar{z}$. $B_z/B$ has the sign of the numerator because the denominator is positive: $S^X$ and $S^M$ are decreasing in $G$.[14]

(12), (13) and (15) are displayed in Figure 1.[15] The intersection of (12) at the maximum of $\ln B(z;\cdot)$ determines the equilibrium $\bar{z}, \ln G$. The intersection of (13) with the tangent line at E gives $\bar{z}^*$.

**Proposition 1** *Provided trade costs are not too high, a unique trading equilibrium exists on $z \in [0,1]$.*

If trade equilibrium exists, it is unique because the properties of (15) imply that $\ln B(z)$ has a unique maximum due to $\Lambda(z)$ decreasing in $z$. The equilibrium allocates home and foreign labor to maximize world income in terms of the numeraire, an instance of the invisible hand.

Autarky prevails when the trade cost is too large. If $t$ is too large for a given $\Lambda(z)$ schedule, the two downward sloping schedules in Figure 1 are too far apart and there is no value of $\ln G$ for which both $\bar{z}$ and $\bar{z}^*$ are in the unit interval. If $\Lambda(z)$ is too large relative to a given $t$, both the downward sloping schedules in Figure 1 are shifted upward and there is no trade because the foreign disadvantage is too large to overcome the trade cost.[16] Autarky also arises when country sizes are sufficiently unequal for given trade cost and $\Lambda(z)$. Drawing on a result proved in the Appendix, a rise (fall) in $R/R^*$ shifts $\ln B(z)$ down (up), and the range of exports of the foreign (home) country can vanish. Equilibrium $G$ is unable to fall (rise) enough to permit trade.

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[14]
$$\frac{S^X_G G}{S^X} = \frac{GR/R^*}{GR/R^* + 1} - \int_0^{\bar{z}} \frac{s(z)}{S^X} \frac{GR/R^*}{GR/R^* + (Gt)^{1/(1-\alpha)}/\Lambda(z)} dz \in (-1, 0),$$

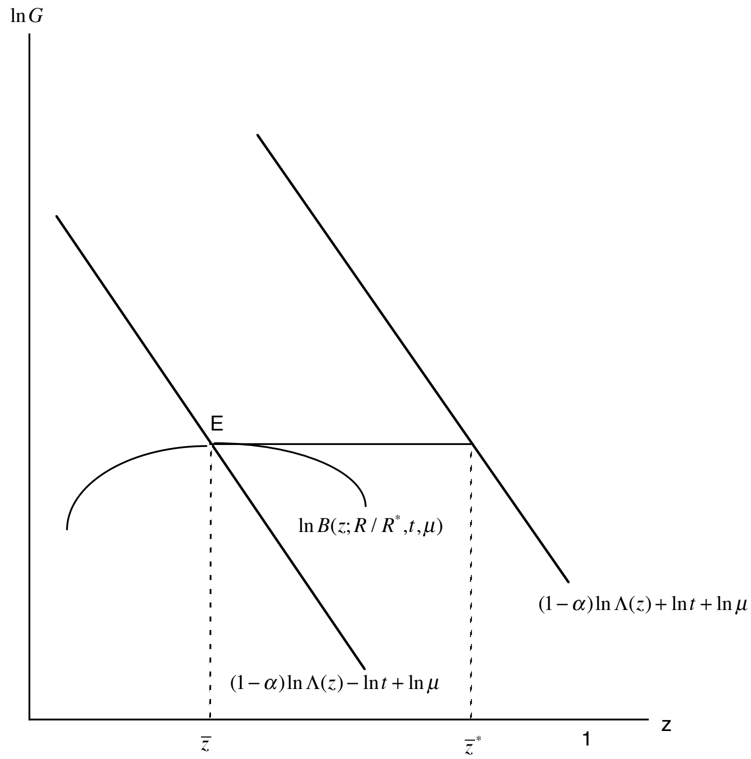
and
$$\frac{S^M_G G}{S^M} = \frac{GR/R^*}{GR/R^* + 1} - \int_{\bar{z}^*}^{1} \frac{s(z)}{S^M} \frac{GR/R^*}{GR/R^* + (G/t)^{1/(1-\alpha)}/\Lambda(z)} dz \in (-1, 0),$$

where the sign is due to the integrals being averages of elements that all exceed $GR^*/(1 + GR/R^*)$ except at the limits $\bar{z}$ and $\bar{z}^*$ respectively.

[15] The straight line cutoff schedules are literally correct in the constant elasticity case for $\Lambda(z)$.

[16] When the equilibrium allocation of capital $\lambda(z) = \gamma(z) = \lambda^*(z)$ obtains, $\Lambda(z) = A(z)^{1/(1-\alpha)}$, hence $(1-\alpha)\ln\Lambda(z) = \ln A(z)$.

Figure 1. Equilibrium Factoral Terms of Trade




It is instructive to note the points of difference with the familiar Ricardian continuum model that is nested in the specific factors model of this paper. The Ricardian (and Heckscher-Ohlin) continuum model(s) shut down import-competing production whereas the specific factors model ordinarily has diversified production. When ex post reallocation of capital occurs, both types of labor are used in the same proportions in every sector that produces and low productivity sectors shut down. $(1-\alpha)\ln\Lambda(z)$ is replaced by $\ln A(z)$ in Figure 1, $\ln G$ is replaced by $w$, the home relative wage, and the balanced trade equilibrium condition (15) that implies $\ln B(z)$ is replaced by $\ln w = \ln \Gamma^*(z)R/\Gamma(z)R^*$. The Ricardian balanced trade equilibrium condition is upward sloping in $z$ throughout, in contrast to the $\ln B(z)$ function that reaches a maximum at $\bar{z}$.[17] The Ricardian model also emerges as a special case of the Cobb-Douglas model when $\alpha = 1$. The endowments model is the opposite extreme where $\alpha = 0$.

# 3 Aggregate Shocks, Income Risk and Globalization

Factor incomes are given by the derivatives of the GDP function (2) with respect to endowments, $g_L = \alpha(K/L)^{1-\alpha}G$ for mobile labor and $g_K = (1-\alpha)(L/K)^{\alpha}G$ for the average return to sector specific capital. The distribution of the sector specific returns relative to $g_K$ is increasing in $G$ for all sectors $z$, with an elasticity that is largest for the most disadvantaged factors. The details are presented in Section 4.

Aggregate productivity risk is introduced as a shift variable in national productivity draws. Thus the sectoral productivity draws combine the idiosyncratic draw $\bar{a}(z)$ with a home country shock $\epsilon$ and similarly for the foreign country: $a(z,\epsilon) = \bar{a}(z)/\epsilon$ and $a^*(z,\epsilon^*) = \bar{a}^*(z)/\epsilon^*$. The national shocks $\epsilon, \epsilon^*$ generate an aggregate absolute advantage shock $\mu = \epsilon/\epsilon^*$.

Productivity risk is very simply connected to income risk in two cases that provide context. In a closed economy, the shock $\epsilon$ changes GDP by multiplicative factor $\epsilon$: $g = R\epsilon[\int_0^1 (p(z)/\bar{a}(z))^{1/(1-\alpha)}dz]^{1-\alpha}$. Relative equilibrium goods prices and the real cost of living index are invariant to $\epsilon$. Both nominal and real incomes of all factors move by the factor $\epsilon$. In an open world economy with common national shocks, $\epsilon = \epsilon^*$, hence real incomes in both countries shift by the common multiplicative factor $\epsilon^*$. The degree of openness, the size of $t > 1$, has no effect on exposure to income risk with common aggregate productivity shocks

[17] $B(z)$ need not be globally concave, but the portions to the left and right of $B_z = 0$ must be upward and downward sloping respectively.

because there is no absolute advantage shock.

The more relevant and interesting case is when $\mu \neq 1$, aggregate productivity risk induces aggregate absolute advantage risk. Relative prices must change in equilibrium to clear markets with the shocks $\epsilon$ and $\epsilon^*$. The effect on the multifactoral terms of trade $G$ is characterized as in the trade equilibrium of Section 2, after multiplying $\Lambda(z)$ by $\mu$. Both home and foreign GDP shift with the common shock factor $\epsilon^*$ while the relative price effect of the absolute advantage shock induces GDP changes in the home country that are proportional to changes $G$. As for real incomes, the true cost of living index in both countries changes with $G$.

The variance of $G$ is locally proportional to the square of the comparative static response of $G$ to aggregate shock $\mu$, so the analytic issue is how globalization shifts that comparative static response. First the elasticity of $G$ with respect to $\mu$ is derived and then the response of this elasticity to a fall in trade costs is characterized.[18]

In terms of Figure 1, a favorable home aggregate shock $\ln \mu > 0$ shifts $\ln \Lambda(z)$ to the right and $\ln B$ rises, hence both $G$ and $\bar{z}$ rise. The shift in $\ln B$ at given $\bar{z}$ is given by:[19]

$$\frac{\mu B_\mu}{B} = -\frac{\mu S^X_\mu + \mu S^M_\mu}{G S^X_G + G S^M_G} \in (0, 1). \tag{16}$$

[18]The Appendix analyzes the very similar case of relative endowment shocks and the case of aggregate demand side shocks in the form of transfers.

[19]The sign and magnitude restrictions come from previous expressions for $S^X_G, S^M_G$ and

$$\mu S^X_\mu = S^X \int_0^{\bar{z}} \frac{s(z)}{S^X} \frac{(Gt)^{1/(1-\alpha)}/\mu\Lambda(z)}{GR/R^* + (Gt)^{1/(1-\alpha)}/\mu\Lambda(z)} dz$$

$$= S^X - S^X \int_0^{\bar{z}} \frac{s(z)}{S^X} \frac{GR/R^*}{GR/R^* + (Gt)^{1/(1-\alpha)}/\mu\Lambda(z)} dz.$$

The second expression implies that $\mu S^X_\mu < -G S^X_G$. Also

$$\mu S^M_\mu = S^M \int_{\bar{z}^*}^1 \frac{s(z)}{S^M} \frac{(G/t)^{1/(1-\alpha)}/\mu\Lambda(z)}{GR/R^* + (G/t)^{1/(1-\alpha)}/\mu\Lambda(z)} dz$$

$$= S^M - S^M \int_{\bar{z}^*}^1 \frac{s(z)}{S^M} \frac{GR/R^*}{GR/R^* + (G/t)^{1/(1-\alpha)}/\mu\Lambda(z)} dz.$$

The second expression implies that $\mu S^M_\mu < -G S^M_G$. Taken together the inequalities imply that $\mu B_\mu / B \in (0, 1)$.

For discrete changes in $\ln \mu$ on $d \ln G / d \ln \mu$ is evaluated at an intermediate value of $z$ where $B_z > 0$:

$$\frac{d \ln G}{d \ln \mu} = \frac{(B_\mu \mu / B)(1-\alpha)\Lambda_z z / \Lambda - B_z z / B}{(1-\alpha)\Lambda_z z / \Lambda - B_z z / B} \in (0,1). \tag{17}$$

$d \ln G / d \ln \mu \in (0,1)$ because $B_\mu \mu / B \in (0,1)$ and $B_z > 0$ because $\ln B$ on Figure 1 rises at constant $\bar{z}$.

Noting that $d \ln \mu = d \ln \epsilon - d \ln \epsilon^*$, (17) implies that the ability to trade provides partial insurance on domestic supply shocks, since the elasticity of $G$ with respect to $\epsilon$ is less than one, its autarky value. But trade also imports the effect of foreign supply shocks $d \ln \epsilon^*$ both directly via the common shock effect and via its effect on aggregate absolute advantage $\mu$ and the factoral terms of trade $G$. The net elasticity is $1 - d \ln G / d \ln \mu > 0$, foreign shocks tend to increase variation in GDP relative to its autarky value.[20] As the aggregate shocks approach being perfectly correlated (i.e., approach the common shocks case), these offsetting effects approach perfect balance so that the variance of GDP is unaffected by the opening of trade.

Now turn to the more relevant and difficult question of how a fall in trade costs — globalization — affects the variance of $G$ induced by productivity shocks $\mu$.

**Proposition 2** *Globalization acting on the extensive margin reduces the variance in $G$ induced by shocks in relative endowments, absolute advantage and transfers. Globalization acting on the intensive margin has ambiguous effects on variance.*

The proof of Proposition 2 is in the Appendix. The intuition for the strong result on the extensive margin is very simple — lower trade costs increase the responsiveness of the extensive margin to aggregate shocks, hence changes in the extensive margin absorb more of

[20] Newbery and Stiglitz (1984) argued that price responses to aggregate domestic supply shocks tend to offset the direct impact of the shock on real income and thus provide a kind of insurance. Cole and Obstfeld (1991) note that this partial insurance feature of terms of trade responses implies smaller scope for international asset trade to provide gains from risk-sharing. The presence or absence of risk-sharing assets has no impact on resource allocation under the restrictions of the present model, so in contrast to Newbery and Stiglitz there is no impact of globalization that affects the efficiency of trade. (I am grateful to Jonathan Vogel for pointing out this connection.)

the impact leaving less to fall on the factoral terms of trade.[21] The intuition for ambiguity on the intensive margin is also simple. With constant $\bar{z}$ and $\mu$, a symmetric fall in trade costs raises specific factor earnings in export sectors and reduces specific factor earnings in import-competing sectors, hence the effect on the multi-factoral terms of trade $G$ depends on details of the schedule of the sector specific factor returns. Introducing the aggregate shock $\mu$, all earnings rise with $\mu$, but the fall in $t$ amplifies the effect in export sectors and damps it in import-competing sectors.[22]

A strong response on the extensive margin has considerable empirical evidence behind it according to Besedes and Prusa (2006a,b), who show that over short time intervals bilateral trade in highly disaggregates sectors winks on and off. The theory and evidence taken together suggest that globalization implies aggregate absolute advantage risk dampening.

The aggregate-risk-damping property of globalization on the extensive margin should obtain in a wider class of models than the specific factors continuum model.[23] The Appendix shows that the same qualitative results obtain with the general neoclassical production function: lower trade costs increase the responsiveness of the extensive margin to aggregate shocks and thus damp down factoral terms of trade responses, all else equal. More complex

[21]In Dornbusch, Fischer and Samuelson (1977), a fall in trade costs (globalization) similarly reduces the variance of the factoral terms of trade through its action on the extensive margin.

[22]Formally, differentiating $B_t t/B$ with respect to $\mu$ leads to ambiguous results because the sign of $B_t$ is ambiguous.

[23]In the Ricardian model all action is on the extensive margin and the effect of globalization on variance is unambiguous. The trade balance equation implies $w/w^* = \Gamma(\bar{z})L^*/\Gamma^*(\bar{z}^*)L$ while the export cutoff equation is $w/w^* = \mu A(z)/t$. The response to shocks to absolute advantage shocks is given by

$$\frac{d \ln w/w^*}{d \ln \mu} = \frac{A_z/A}{A_z/A - [\gamma(\bar{z})/\Gamma(\bar{z}) + Z_z^*\gamma(\bar{z}^*)/\Gamma(\bar{z}^*)]} \in (0,1).$$

This elasticity is reduced by a fall in $t$: globalization reduces the variance of the factoral terms of trade due to aggregate technology shocks. Compared to

$$\frac{d \ln G}{d \ln R/R^*} = -\frac{(1-\alpha)\Lambda_z/\Lambda}{(1-\alpha)\Lambda_z/\Lambda - B_z/B} \in (-1,0)$$

the Ricardian term $[\gamma(\bar{z})/\Gamma(\bar{z}) + Z_z^*\gamma(\bar{z}^*)/\Gamma(\bar{z}^*)]$ is less complex than $B_z/B$ in the specific factors case. In both cases the responsiveness to shocks on the extensive margin is driven by $Z_z^*$, a response that is damped by higher trade costs. The intuition is simple — lower trade costs increase the responsiveness of the extensive margin to aggregate shocks, hence changes in the extensive margin absorb more of the impact leaving less to fall on the factoral terms of trade.

general equilibrium production structures bring in additional forces that modify the global economy's responsiveness on the extensive margin but seem unlikely to fundamentally change this insight.

Real aggregate income risk is damped still further by globalization because the cost of living deflator moves in proportion to the factoral terms of trade with a factor of proportionality that is larger the more open is the economy.

**Proposition 3** *Globalization reduces the real income risk due to factoral terms of trade risk.*

**Proof of Proposition 3** *The log of aggregate real income is defined by* $\ln R + \ln G - \ln C$ *where the log of the true cost of living deflator in numeraire units* $\ln C = \int_0^1 \gamma(z) \ln[P(z)\bar{a}(z)\epsilon^*/\epsilon]dz$*. Substitute the logarithm of (6)-(8) into* $\ln C$ *and differentiate with respect to* $\ln G$*. The preceding comparative static shocks to nominal income via changes in G affect log real income by*[24]

$$1 - \frac{d\ln C}{d\ln G} = 1 - (\Gamma + \Gamma^*)\frac{GR/R^*}{1 + GR/R^*} > 0. \tag{18}$$

*The variance of real income is locally proportional to the square of the preceding expression, which is decreasing in open-ness* $\Gamma + \Gamma^*$. $\|$

Propositions 2 and 3 together along with the interpretive discussion make a plausible case that globalization reduces real income risk due to aggregate shocks because globalization amplifies the damping effect of the extensive margin.

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[24]
$$1 - \frac{d\ln C}{d\ln G} = 1 - (\Gamma + \Gamma^*)\left[(1-\alpha)\frac{GR/R^*}{1 + GR/R^*} + \alpha(\rho^X + \rho^M)\right] > 0,$$

where

$$\rho^X \equiv \int_0^{\bar{z}} \gamma(z) \frac{GR/R^*}{GR/R^* + (Gt)^{1/(1-\alpha)}/\Lambda(z)dz} \in (0,1)$$

and

$$\rho^M \equiv \int_{\bar{z}^*}^1 \gamma(z) \frac{GR/R^*}{GR/R^* + (G/t)^{1/(1-\alpha)}/\Lambda(z)dz} \in (0,1).$$

Use (10)-(11) to substitute in $\rho^X, \rho^M$ and simplify using (15) to obtain the expression below.

# 4 Idiosyncratic Shocks and Internal Distribution

The specific factors income distribution of each country is determined by sector-specific absolute advantage shocks that generate sector specific premia, facing them with comparative advantage risk. To simplify the analysis, aggregate absolute advantage risk is suppressed as shocks $\epsilon$ and $\epsilon^*$ are assumed to be perfectly correlated: $\mu = 1$. The focus is on home income distribution because foreign distribution is a mirror image. The analysis is extended to treat idiosyncratic taste shocks along with idiosyncratic absolute advantage shocks.

The distribution of equilibrium capital returns across sectors depends partly on the allocation of capital to sectors. The possible distributions are restricted by imposing the rational expectations equilibrium capital allocation. Intuitively, the allocation of capital $\lambda(z)$ matched to the allocation of expenditures $\gamma(z)$ smooths out all cross-sector differences that are know in advance of the productivity draws. The supporting argument is deferred to Section 5.

Under the equilibrium allocation the pattern of positive (negative) premia for export (import competing) sectors emerges. Moreover, the distribution of premia are ordered by export intensity $s(z)/\gamma(z)$. Intuitively this is because the highest comparative advantage sectors would have the largest premia even in the absence of a mobile factor, but their premia are increased by their ability to draw more of the mobile factor to their favored sector.

In each sector, the specific return is residually determined as $r(z)\lambda(z)K = (1-\alpha)p(z)y(z)$. Then the return in $z$ relative to the economy-wide average is given by $r(z)/g_K = s(z)/\lambda(z)$. Using the share functions (10) and (11) in the relative returns equation yields:

$$r(z)/g_K = \frac{\gamma(z)}{\lambda(z)} \frac{(GR/R^* + 1)}{GR/R^* + (Gt)^{1/(1-\alpha)}/\Lambda(z)}, z \in [0, \bar{z}); \tag{19}$$

$$r(z)/g_k = \frac{\gamma(z)}{\lambda(z)}, z \in [\bar{z}, \bar{z}^*]; \tag{20}$$

$$r(z)/g_K = \frac{\gamma(z)}{\lambda(z)} \frac{(GR/R^* + 1)}{GR/R^* + (G/t)^{1/(1-\alpha)}/\Lambda(z)}, z \in (\bar{z}^*, 1]. \tag{21}$$

(19)-(21) show that export sectors tend on average to have higher returns and import competing sectors to have lower returns than non-traded goods sectors. The implication emerges cleanly with the ex ante equilibrium allocation of capital $\lambda(z) = \gamma(z) = \lambda^*(z) \Rightarrow \gamma(z)/\lambda(z) = 1, \forall z$. In this case $\Lambda(z) = [a^*(z)/a(z)]^{1/(1-\alpha)}$, the (transform of) home absolute advantage in good $z$. Figure 2 illustrates the implications. The effect of idiosyncratic taste shocks is captured in (19)-(21) by $\gamma(z)/\lambda(z) \neq 1$, where $\lambda(z)$ is equal to the expected value of the expenditure share and $\gamma(z)$ is its realized value. All else equal, returns are increased by a positive shock to demand share. Incorporating taste shocks, the relative returns schedule of Figure 2 is the center of a confidence band generated by the distribution of taste shocks. Summarizing the implications for central tendency:

**Proposition 4** *(a) With the equilibrium allocation of capital and no taste shocks, capital premia rise with export intensity and fall with import intensity. (b) The relative returns of trade-exposed specific factors fall with $G$ everywhere, and most for the least productive sectors. (c) Globalization at given factoral terms of trade reduces the specific factor income of import-competing sectors by more the less relatively productive the sector, increases the specific factor income of exporting sectors by more the more productive the sector, while non-traded sectors are completely insulated from globalization.*

Proposition 4(a) follows because $s(z)/\gamma(z)$ is decreasing in $z$, hence the capital return is falling in $z$ for traded goods and equals the average capital return for non-traded goods.

Propositions 4 (b) and (c) draw comparative static implications of the model for specific factor income distribution. Improvements in the factoral terms of trade $G$ act on $r(z)$ via its direct relationship to $s(z)$. Examining (10) and (11), $s(z)$ is decreasing in $G$ for both exports and imports while for non-traded goods $s(z)$ is independent of $G$. Increases in the factoral terms of trade $G$ thus redistribute specific factor income from traded goods to non-traded goods.[25] Within the traded sectors, returns relative to the mean (equal to the non-traded

[25]
$$\frac{\partial \ln r(z)/g_K}{\partial \ln G} = \frac{\partial \ln s(z)}{\partial \ln G} = -\frac{1}{1+GR/R^*} - \frac{\alpha}{1-\alpha}\frac{H(z)}{GR/R^* + H(z)} < 0$$
where $H(z) \equiv \Lambda(\bar{z})/\Lambda(z) \in [0,1], z \leq \bar{z}; H(z) \equiv \Lambda(\bar{z}^*)/\Lambda(z) \geq 1, z \geq \bar{z}^*$; and $H' > 0$. The export cutoff

skilled wage) are given by $r(z)/g_K = s(z)/\lambda(z)$.

The intuition is that the sectoral return relative to the mean is given by $s(z)/g_K$ while the responsiveness of supply shares to changes in the factoral terms of trade is biggest for the lowest share sectors because the general equilibrium supply elasticity is given by $G_{pp}p/G = [1 - s(z)]\alpha/(1-\alpha)$. Proposition 4 (b) and its intuition extend from the Cobb-Douglas to the general neoclassical case, as the Appendix argues, but with mild qualification.

Proposition 4 (c) follows because

$$\frac{\partial \ln r(z)/g_K}{\partial \ln t} = -\frac{1}{G^{-\alpha/(1-\alpha)}t^{-1/(1-\alpha)}\Lambda(z)R/R^* + 1}\frac{1}{1-\alpha} < 0, z \leq \bar{z};$$

and

$$\frac{\partial \ln r(z)/g_K}{\partial \ln t} = \frac{1}{G^{-\alpha/(1-\alpha)}t^{1/(1-\alpha)}\Lambda(z)R/R^* + 1}\frac{1}{1-\alpha} > 0, z \geq \bar{z}^*.$$

For non-traded goods, sector specific factor incomes are invariant to $t$. For exported goods, a fall in $t$ increases relative income by more the more productive the sector, while for imported goods the relative income is reduced by more the less productive the sector.

Globalization is modeled here as decreases in symmetric trade costs. On the extensive margin, globalization widens inequality as it narrows the range of non-traded goods $[\bar{z}, \bar{z}^*]$ that is sheltered from external competition. Thus globalization redistributes specific factor income to exports from both non-traded goods and imported goods and to non-traded goods from imported goods for any given factoral terms of trade $G$.

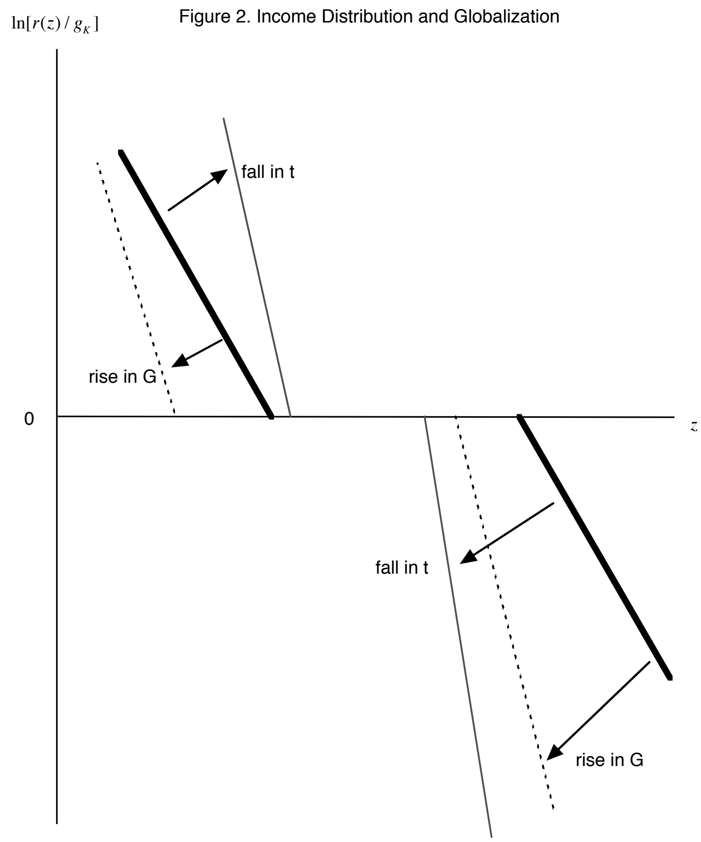
Notice that inequality increases in both countries, and that this property does not require restricting the distributions of productivity draws. It is a feature of factor specificity and the assumed equilibrium allocation of factors. The effect of globalization on the factoral terms of trade is ambiguous, but any improvement due to the fall in trade costs will redistribute income to non-traded sector specific factors from traded sector specific factors.

Figure 2 illustrates the effect of a rise in $G$ and a fall in $t$ on the distribution of $r$

equations are used above to simplify the derivatives of (10) and (11).

with equilibrium capital allocation $\lambda(z) = \gamma(z) = \lambda^*(z)$. A log-linear form for $A(z)$ is imposed for simplicity. A 1 percent rise in $G$ lowers the $\ln r(z)/g_K$ schedules for traded goods by $-1/[(1-\alpha)(GR/R^* + 1)]$. A 1 percent fall in $t$ raises export relative incomes by the (absolute value of the) expression on the right hand side of (4) and lowers import sector relative incomes by the expression on the right hand side of (4). The figure is drawn assuming that $G < t$ so that a one percent fall in $t$ has a bigger impact than a one percent rise in $G$ for import competing sectors, but this ranking is arbitrary and without significance for the analysis.

Figure 2. Income Distribution and Globalization



The implications for globalization and sector specific income risk due to idiosyncratic shocks are summarized as:

**Proposition 5** *(a) Globalization increases sector specific income risk from idiosyncratic productivity shocks. (b) Globalization has no effect on sector specific income risk from idiosyncratic taste shocks.*

When aggregate risk is present, the ex post distribution in Figure 2 is shifted up or down and the cutoffs $\bar{z}$ and $\bar{z}^*$ shift back and forth depending on the realization of aggregate shocks to absolute advantage, country size or transfers, but the profile retains its shape. Proposition 2 showed that globalization reduces the variance of the factoral terms of trade due to aggregate shocks, thus tending to offset the globalization-induced increase in exposure to idiosyncratic risk. Both forces hit the poorest skilled workers the hardest. The size of the reduction in exposure to aggregate income income risk varies by sector in proportion to the square of

$$\frac{\partial \ln r(z)/g_K}{\partial \ln G}.$$

With the equilibrium allocation and no taste shocks, Proposition 4 (b), illustrated by Figure 2, shows that this offset in aggregate risk is most important for the poorest factors, least important for the richest factors and irrelevant for the middle non-traded sector factors. Proposition 4 (c), also illustrated by Figure 2, shows that globalization increases idiosyncratic risk and is likewise most important for the poorest factors ex post, least important for the richest factors and irrelevant for the middle income non-traded sector specific factors.

# 5 Equilibrium Sector Specific Factor Allocation

The rational expectations equilibrium allocation of capital for the home and foreign economies is $\lambda(z) = \gamma(z) = \lambda^*(z)$. This follows because the production functions in all sectors are ex ante identical, and the only source of predictable difference across sectors is variation in $\gamma(z)$.

The relative return is given by (19)-(21). When $\lambda(z) = \gamma(z) = \lambda^*(z)$, the price prospects for every sector are identical ex ante. Then this is an equilibrium allocation because no agent has an incentive to deviate in his allocation. Any other allocation induces price variation that can be anticipated and arbitraged. Thus the equilibrium is unique. The point is simplest to see for non-traded goods. Away from the allocation $\lambda(z) = \gamma(z) = \lambda^*(z)$, the home non-traded goods market clears ex post with

$$\left(\frac{P(z)}{G}\right)^{1/(1-\alpha)} = \frac{\gamma(z)}{\lambda(z)}.$$

Consider a pair of sectors $z', z''$ with $\gamma(z')/\lambda(z') > 1 > \gamma(z'')/\lambda(z'')$. Some agents can reallocate from $z''$ to $z'$ and reap a certain gain in every realization of the random productivity draws that relocate the two sectors somewhere on $[\bar{z}, \bar{z}^*]$. Thus if $\gamma(z')/\lambda(z') \neq 1$ for any $z'$, the allocation is not an equilibrium.

A more complex version of the same reasoning applies to the sectors that end up as tradable. $\lambda(z')/\gamma(z') \neq 1$ complicates the left hand sides of the market clearing equations with ratios $\lambda(z)/\gamma(z)$ and $\lambda^*(z)/\gamma(z)$ that multiply the expressions for $s(z)/\gamma(z)$ and $s^*(z)/\gamma(z)$. If home workers anticipate 'structurally rational' foreign allocations such that $\lambda^*(z') = \gamma(z')$, then there are arbitrage gains unless $\lambda(z') = \gamma(z'), \forall z'$. Symmetrically, if foreign skilled workers anticipate home allocations $\lambda(z) = \gamma(z)$, then there is arbitrageable variation in $P(z)/A(z)$ unless $\lambda^*(z) = \gamma(z)$. Thus the only allocation where such arbitrage is not possible is $\lambda(z) = \gamma(z) = \lambda^*(z)$.[26]

Extending the model to incorporate taste shocks, each sector has mean taste parameter $\gamma^0(z)$ and receives additive zero mean shock from a common distribution, realizing $\gamma(z)$ that is not equal to $\lambda(z) = \gamma^0(z)$ in equilibrium.

Notice that no assumption is made about agents' attitudes toward risk. Thus the setup is compatible with concerns about distribution amplified by declining marginal utility of income.

[26]Multiple equilibria may be possible when expectations about foreign allocations do not settle down to 'structural rational expectations' as assumed here.

# 6 Conclusion

Globalization increases the dispersion of factor incomes in a model with idiosyncratic productivity shocks when specific factors are allocated prior to the realization of the shocks. In contrast, globalization tends to damp the income risk for both specific and mobile factors arising from aggregate shocks because it increases the responsiveness of the extensive margin to the shocks. The poorest specific factors benefit most from the reduction in aggregate risk.

The model features comparative static adjustment on both the intensive and extensive margins of trade. This stands in contrast to the Ricardian and Heckscher-Ohlin continuum models with an action on the extensive margin only and to the family of discrete goods models with action on the intensive margin only. The two active margins property may be useful for future research in light of recent evidence that both margins are important in inter-temporal movements of trade data.

The model extends naturally to incorporate political economy. In Grossman and Helpman (1994), tariffs of organized sectors vary with size, which in turn should vary with productivity draws; hence political economy motives appear likely to amplify income risk.

A useful extension of the model is to heterogeneous firms and workers. Workers develop both sector and firm specific skills and random productivity shocks have both sector and firm specific components. Mobility within sectors is empirically more important than between sectors. By allowing for intra-sector mobility at a cost, the extended model can generate within-sector dispersion of wages.[27]

The complementary work of Blanchard and Willman (2008) and Costinot and Vogel (2008) on income distribution based on worker heterogeneity suggests that a combination of ex ante heterogeneity and ex post locational premia can go far toward fitting the extremely rich empirical regularities of actual income distributions. Matching frictions are a promising way to dig more deeply into the structure of random productivities. The analytic simplicity of their models and the specific factors continuum model suggests that analytic solutions

[27] The working paper version of this paper sketched out more details.

may be feasible.

Institutional environments and regulation presumably affect the distribution of productivity shocks that are realized. International differences in institutions thus should have implications for the shape of the absolute advantage spectrum $A(z)$ and thus the pattern of trade and income distribution.

The static analysis of this paper is a platform for interesting dynamics. The specificity of factors is transitory. Adjustment to a longer run equilibrium will have interesting and important economic drivers. An earlier literature (for example, Neary, 1978) provides an analysis of adjustment to a one time shock. In the present setup it is natural to think of productivity draws arriving each period. Serial correlation in the draws would induce persistence in comparative advantage with potentially interesting implications for investment patterns and income distribution. Labor market evidence reveals that young workers are more likely to relocate in response to locational rents, suggesting overlapping generations models. Another important aspect of worker relocation is international migration subject to migration costs and accompanied by remittance flows. The simplicity of the model and in particular its consistent aggregation structure also suggest that it is applicable to analyze a range of macro-economic dynamic issues.

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# 8 Appendix: Derivation of Results and Extensions

The supporting results are in two sub-sections. First, the comparative statics in the background of Section 3 are derived. Second is a demonstration that the income distribution results hold for a general neoclassical production function identical across sectors. Differing production functions across sectors brings in Heckscher-Ohlin influences on distribution but also all the complexities of behavior towards risk and the availability and type of risk-sharing.

## 8.1 Comparative Statics

### 8.1.1 Relative Growth Shocks

*Relative endowment growth induces a rise in the growing country's export share of GDP, its world GDP share and a less than unit elastic fall in its factoral terms of trade.*

The effect of a change in $R/R^*$ on $\ln B$ at constant $\bar{z}$ is given by

$$\frac{(R/R^*)B_{R/R^*}}{B} = -\frac{(R/R^*)S^X_{R/R^*} + (R/R^*)S^M_{R/R^*}}{GS^X_G + GS^M_G} \in (-1, 0).$$

The reason is plain: $G$ and $R/R^*$ enter (15) multiplicatively but $G$ also appears separately in the denominators of $S^X, S^M$. A 1% rise in $R/R^*$ matched with a 1% fall in $G$ would raise $S^X + S^M$, necessitating a rise in $G$ to lower $S^X + S^M$. When $R/R^*$ rises discretely, the change in $\bar{z}$ has an impact on $\ln G$. Solving the cutoff equation to obtain the change in $\bar{z}$ implied by the change in $G$ and substituting to obtain the reduced form effect of relative endowments on $G$:

$$\frac{d\ln G}{d\ln R/R^*} = \frac{(R/R^*)B_{R/R^*}}{B}\frac{(1-\alpha)\Lambda_z/\Lambda}{(1-\alpha)\Lambda_z/\Lambda - B_z/B} \in (\frac{(R/R^*)B_{R/R^*}}{B}, 0). \tag{22}$$

The elasticity is further restricted within the negative unit interval because with $\ln B = \ln G$ shifted down at the initial value of $\bar{z}$, $B_z/B$ is positive. Thus, $GR/R^*$ increases with $R/R^*$ in equilibrium and more so with discrete changes.

### 8.1.2 Absolute Advantage Shocks

Aggregate relative productivity (absolute advantage) risk is introduced as a shift variable $\mu$ multiplying $\Lambda(z)$. $\mu$ is the ratio of a domestic productivity advance $\epsilon$ to a foreign productivity advance $\epsilon^*$: $\mu = \epsilon/\epsilon^*$. Thus $a(z,\epsilon) = \bar{a}(z)/\epsilon$ and $a^*(z,\epsilon^*) = \bar{a}^*(z)/\epsilon^*$.

In terms of Figure 1, $\ln\Lambda(z)$ is shifted by $\ln\mu$, hence both the cutoff schedules shift in the same direction. As for $B(\cdot)$:[28]

$$\frac{\mu B_\mu}{B} = -\frac{\mu S_\mu^X + \mu S_\mu^M}{GS_G^X + GS_G^M} \in (0,1).$$

For discrete changes,

$$\frac{d\ln G}{d\ln\mu} = \frac{(B_\mu \mu/B)(1-\alpha)\Lambda_z z/\Lambda - B_z z/B}{(1-\alpha)\Lambda_z z/\Lambda - B_z z/B} \in (0,1). \tag{23}$$

The restriction to the unit interval follows from the restriction on $B_\mu \mu/B$ and the property that the upward shift in $\ln B$ at constant $\bar{z}$ implies $B_z > 0$. Home relative productivity improvements raise the home relative income, but by less than the full amount of the improvement.

#### 8.1.3 Transfers

Transfers alter the balance of payments equilibrium condition (15) to

$$[\Gamma(\bar{z}) + \Gamma^*(\bar{z}^*)] - [1 - \Gamma(\bar{z}) - \Gamma^*(\bar{z}^*)]\beta = S^X(\bar{z},\cdot) + S^M(\bar{z}^*,\cdot)$$

where $\beta = b/g$ is the ratio of the transfer to income and $b$ is the international transfer in home prices.[29] A rise in $\beta$ due to borrowing shifts up the $\ln B$ function in Figure 1. In the neighborhood of equilibrium $\bar{z}$:

$$\frac{\beta B_\beta}{B} = -\frac{1 - \Gamma - \Gamma^*}{GS_G^X + GS_G^M} > 0.$$

For discrete changes there is also an effect of transfers on the trade cutoff equations

[28] The sign and magnitude restrictions come from

$$\mu S_\mu^X = S^X \int_0^{\bar{z}} \frac{s(z)}{S^X} \frac{(Gt)^{1/(1-\alpha)}/\mu\Lambda(z)}{GR/R^* + (Gt)^{1/(1-\alpha)}/\mu\Lambda(z)} dz$$

$$= S^X - S^X \int_0^{\bar{z}} \frac{s(z)}{S^X} \frac{GR/R^*}{GR/R^* + (Gt)^{1/(1-\alpha)}/\mu\Lambda(z)} dz.$$

The second expression implies that $\mu S_\mu^X < -GS_G^X$. Also

$$\mu S_\mu^M = S^M \int_{\bar{z}^*}^1 \frac{s(z)}{S^M} \frac{(G/t)^{1/(1-\alpha)}/\mu\Lambda(z)}{GR/R^* + (G/t)^{1/(1-\alpha)}/\mu\Lambda(z)} dz$$

$$= S^M - S^M \int_{\bar{z}^*}^1 \frac{s(z)}{S^M} \frac{GR/R^*}{GR/R^* + (G/t)^{1/(1-\alpha)}/\mu\Lambda(z)} dz.$$

The second expression implies that $\mu S_\mu^M < -GS_\mu^M$. Taken together the inequalities imply that $\mu B_\mu/B \in (0,1)$.

[29] The balance of payments constraint is given by $b = \Gamma(b+g) - S^M g - [S^X g - \Gamma^*(g+b)]$. This solves for the text expression.

$(1+\beta)\gamma(\bar{z}) = s(\bar{z})$ and $(1+\beta)\gamma(\bar{z}^*) = s(\bar{z}^*)$.

$$\frac{d\ln G}{d\ln\beta} = \frac{\beta B_\beta}{B}\frac{(1-\alpha)\Lambda_z z/\Lambda}{(1-\alpha)\Lambda_z z/\Lambda - B_z z/B} \in (\beta B_\beta/B, 0). \tag{24}$$

### 8.1.4 Globalization and Responsiveness to Shocks

The response of the world economy to aggregate shocks decomposes into responses on the intensive ($B_z = 0$) and extensive ($B_z \neq 0$) margins. The marginal response on the intensive margin is affected by trade cost declines through a balance of forces because $G$ itself is raised (lowered) by a fall in $t$ as the positive (negative) effect on export (import-competing) sector earnings predominates.

In contrast, the responsiveness of the factoral terms of trade to aggregate shocks via the extensive margin is ordinarily reduced by globalization. In each of (22)-(24) a reduction in $B_z > 0$ reduces the responsiveness of $G$ to shocks, and as argued in the text, $B_z$ is ordinarily reduced by a fall in $t$.

Proposition 2 addresses the question of how small changes in $t$ affect the variance of $G$. $Var(\ln G) \approx (d\ln G/d\ln\mu)^2 Var(\ln\mu)$. Then the variance of $G$ is shifted by a change in $t$ with the sign of

$$\frac{t}{d\ln G/d\ln\mu}\frac{\partial(d\ln G/d\ln\mu)}{\partial t} = \eta_1\frac{\partial(\ln B_\mu\mu/B)}{\partial t} - \eta_2(1-\eta_2)\frac{\partial(\ln(B_z z/B)}{\partial t} \tag{25}$$

where

$$\eta_1 = \frac{(B_\mu\mu/B)(1-\alpha)\Lambda_z z}{(B_\mu\mu/B)(1-\alpha)\Lambda_z z - B_z z/B} > 0$$

and

$$\eta_2 = \frac{B_z z/B}{(B_\mu\mu/B)(1-\alpha)\Lambda_z z - B_z z/B} < 0.$$

In evaluating (25) the response of the extensive margin to $t$, based on (12)-(13) for given $G$ is a part of the response of $B_z z/B$ to a change in $t$, with the other effects acting through the intensive margin.

**Proof of Proposition 2**

*(i) Ambiguity on the Intensive Margin*

$$\frac{\partial\ln B}{\partial\ln t} = \frac{tB_t}{B} = \frac{tS_t^X + tS_t^M}{-GS_G^X - GS_G^M}, \tag{26}$$

*where*

$$\frac{tS_t^X}{S^X} = -\frac{1}{1-\alpha}\int_0^{\bar{z}}\frac{s(z)}{S^X}\frac{(Gt)^{1/(1-\alpha)}/\mu\Lambda(z)}{GR/R^* + (Gt)^{1/(1-\alpha)}/\mu\Lambda(z)}dz \tag{27}$$

$$\frac{tS_t^M}{S^M} = \frac{1}{1-\alpha}\int_{\bar{z}^*}^1\frac{s(z)}{S^M}\frac{(G/t)^{1/(1-\alpha)}/\mu\Lambda(z)}{GR/R^* + (G/t)^{1/(1-\alpha)}/\mu\Lambda(z)}dz. \tag{28}$$

*Since $-tS_t^X/S^X < tS_t^M/S^M$ and $S^X > S^M$ ordinarily, $B_t$ can have either sign. Differenti-*

*ating (26) logarithmically with respect to $\mu$ yields*

$$\frac{\partial^2 \ln B}{\partial \ln t \partial \ln \mu} = \frac{tS^X_{t\mu}\mu + tS^M_{t\mu}\mu}{tS^X_t + tS^M_t} - \frac{GS^X_{G\mu}\mu + GS^M_{G\mu}\mu}{GS^X_G + GS^M_G}. \tag{29}$$

*Differentiating (27) and (28) with respect to $\mu$ yields results opposite in sign so the first term on the right hand side of (29) is ambiguous in sign, hence so is the sign of (29).*

*(ii) The Extensive Margin.*

*Differentiating $B_z/B$ with respect to $t$:*

$$\frac{\partial B_z/B}{\partial t}\frac{1}{B_z/B} = \frac{-Z^*_{zt}[s(z^*) - \gamma(z^*)]}{s(z) - \gamma(z) - Z^*_z[s(z^*) - \gamma(z^*)]} - \frac{S^X_{Gt} + S^M_{Gt}}{S^X_G + S^M_G}.$$

*The second term on the right hand side is another response on the intensive margin and is ambiguous in sign. The first term on the right hand side is the response of the extensive margin and is always positive because differentiating (14), $Z^*_{\bar{z}t} = -2\Lambda_z(\bar{z})/[(1-\alpha)\Lambda_z(\bar{z}^*)t^{2/(1-\alpha)}] < 0$ while for discrete changes in $\mu$, $s(z) - \gamma(z) > 0, s^*(z^*) - \gamma(z^*) > 0$. Then since $\eta_2 < 0$ in (25), the effect on the variance of $\ln G$ is positive.*‖

In the case of transfers there is an additional effect because higher trade costs raise $1 - \Gamma - \Gamma^*$ and thus increase the response of the factoral terms of trade to given transfers.

## 8.2 General Production Function Case

Replace the Cobb-Douglas production function with the general neoclassical degree one homogeneous and concave, twice differentiable potential production function $F(K(z), L(z))$.

Output in sector $z$ is given by

$$y(z) = \frac{\lambda(z)K}{a(z)} f[L(z)/\lambda(z)K] \tag{30}$$

where $f(\cdot) \equiv F[L(z)/K(z), 1]$, $\lambda(z)$ is the fraction of capital allocated to sector $z$ where it acquires sector specific properties, and $K$ is the total supply of capital.

Labor is mobile subsequent to the realization of shocks. Equilibrium allocation of labor satisfies the value of marginal product conditions for each sector:

$$w = \frac{p(z)}{a(z)} f'[L(z)/\lambda(z)K], \tag{31}$$

where $w$ is the wage rate and $p(z)$ is the price of good $z$. It is convenient in what follows to work with efficiency prices $P(z) \equiv p(z)/a(z)$. Solving for the labor demand yields $L(z) = \lambda(z)Kh(P(z)/w)$, where $h(\cdot) \equiv [1/f']^{-1}$ is the labor demand per unit of capital. $h' = -(f')^2/f'' > 0$ due to $f'' < 0$. Substituting into the production function, the supply function is given by

$$y(z) = \frac{\lambda(z)K}{a(z)} f[h(P(z)/w)]. \tag{32}$$

The supply side of the economy is closed with the labor market clearance condition. The aggregate supply of labor is given by $L$, hence market clearance implies:

$$\frac{L}{K} = \int_0^1 \lambda(z)h(P(z)/w)dz. \tag{33}$$

Applying the implicit function theorem to (33), the equilibrium wage is $w = W[\{P(z))\}, \{\lambda(z)\}, L/K]$.

Gross domestic product is given by $g = \int_0^1 p(z)y(z)dz$. This becomes the GDP function

$$g(\{P(z))\}, \{\lambda(z)\}, L, K) = K\int_0^1 \lambda(z)P(z)f[h(P(z)/W(\cdot)]dz \tag{34}$$

where $W(\cdot)$ is substituted for $w$ in (32) and the result used to substitute for $y(z)$.[30] The GDP share of sector $z$ is given by

$$s(z) = \frac{\lambda(z)P(z)f[h(P(z)/W(\cdot)]}{\int_0^1 \lambda(z)P(z)f[h(P(z)/W(\cdot)]dz}. \tag{35}$$

The GDP function is convex and homogeneous of degree one in prices, concave in $K, L, \{\lambda\}$ and homogeneous of degree one in $K, L$.

Let the foreign wage be the numeraire. Multiply and divide by the home wage rate in (35) to obtain prices in terms of home labor units $\widetilde{P}(z) = P(z)/w$. Then home GDP is given by $wg(\{\widetilde{P}(z)\}, \cdot)$. The GDP shares are given by (35) after dividing through by the unskilled wage $w$:

$$s(z) = \frac{\lambda(z)\widetilde{P}(z)f[h(\widetilde{P}(z))]}{\int_0^1 \lambda(z)\widetilde{P}(z)f[h(\widetilde{P}(z))]dz}.$$

The arbitrage conditions imply $\widetilde{P}(z)t/wA(z) = P^*(z), z \in [0, \bar{z})$ and $\widetilde{P}(z)/wA(z)t = P^*(z), z \in (\bar{z}^*, 1]$.[31] Finally, impose the equilibrium allocation $\lambda(z) = \gamma(z) = \lambda^*(z)$.

For given $w$, (5) determines the traded goods prices. Imposing the ex post ordering of sectors such that $A' < 0$, (5) yields the implication that equilibrium $\widetilde{P}(z)$ is falling in $z$ and $\widetilde{P}(z)/A(z)$ is rising in $z$, hence by properties of (35) home shares are falling and foreign shares are rising in $z$. For nontraded goods $s(z) = \gamma(z)$ determines home prices $\widetilde{P}(z)$ and $s^*(z) = \gamma(z)$ determines foreign prices $P^*(z)$. Finally, the entire schedule of equilibrium $\widetilde{P}(z)$ is increasing in $w$ with elasticity less than one.

The factoral terms of trade $w$ is determined by the trade balance equation. The shares are implicit functions of the factoral terms of trade and the exogenous shift variables along with $A(z)$. The analog to $B(z)$ is the solution for the wage $w = \omega(z, \cdot)$ from the balance of trade constraint $S^X(w, z; \cdot) + S^M(w, z, \cdot) - \Gamma(\bar{z}) + \Gamma^*(\bar{z}^*) = 0$. As with Figure 1, the export cutoff equation $w = A(z)/t$ must slice through $\omega(z, \cdot)$ at a maximum.

The forces that shape the comparative static derivatives such as $\omega_\beta$ are different and more complex than in the Cobb-Douglas case. But the the property that globalization enhances the responsiveness of the extensive margin of trade and thereby reduces the variance of

[30]As allocation of the specific capital grows more efficient, the model converges onto a Ricardian model (since production functions are identical over $z$). Then in the limit $g = L \max_z p(z)/a(z)$.

[31]Division by $w$ is needed to convert prices to foreign efficiency units from home labor efficiency units.

income due to aggregate shocks carries through. The comparative static derivative with respect to exogenous variable $x$ is solved from differentiating the trade balance and export cutoff equations

$$\frac{d\ln w}{d\ln x} = \frac{\partial \ln \omega}{\partial \ln x} + \frac{\partial \ln \omega}{\partial z}\frac{dz}{d\ln x}$$

and

$$\frac{d\ln w}{d\ln x} = \frac{A_z}{A}\frac{dz}{d\ln x}.$$

The solution is

$$\frac{d\ln w}{d\ln x} = \frac{\partial \ln \omega}{\partial \ln x}\frac{A_z/A}{A_z/A - \partial \ln \omega/\partial z}.$$

The relevant coefficient for discrete changes is the second fraction on the right hand side. Compared to its counterpart in the Cobb-Douglas case, efficient allocation implies $(1-\alpha)\Lambda_z/\Lambda$ is replaced by $A_z/A$ and $B_z/B$ is replaced by $\partial \ln \omega/\partial z$. The latter has exactly the same structure as in the Cobb-Douglas case. Recalling the Cobb-Douglas case, differentiating $B_z/B$ with respect to $t$:

$$\frac{\partial B_z/B}{\partial t}\frac{1}{B_z/B} = \frac{Z^*_{zt}s(z^*) - \gamma(z^*)}{s(z) - \gamma(z) + Z^*_z[s(z^*) - \gamma(z^*)]} - \frac{S^X_{Gt} + S^M_{Gt}}{S^X_G + S^M_G}.$$

The first term is negative while the second term is ambiguous in sign for the same reason $B_t$ is ambiguous in sign. Disregarding the influence of the second term, $B_{zt} < 0$. The general case replaces $B$ with $\omega$ and $G$ with $w$, all other elements of the expression remaining the same qualitatively.

### 8.2.1 Income Distribution

The average capital return is related to the factoral terms of trade by $\bar{r} = w(1-\bar{\alpha})/\bar{\alpha}$ where $\bar{\alpha} \equiv \int_0^1 s(z)\alpha(z)dz$ is the average labor share in the economy. At a constant labor share, the average return is unit elastic with respect to the factoral terms of trade. Aggregate shocks will ordinarily change the average labor share, and general analytic results are precluded. More analysis follows below at the end of this section in the context of evaluating the effect of globalization on the average return on capital relative to labor.

Nominal income and real income move together in the general case, as in the Cobb-Douglas case. The log of the true cost of living index is $\ln C = \int_0^1 \gamma(z)[\ln \widetilde{P}(z) + \ln a(z)]dz + \ln w$. The cost of living index has elasticity with respect to the factoral terms of trade equal to

$$1 - \Gamma\int_0^{\bar{z}} \frac{\gamma(z)}{\Gamma}\frac{d\ln \widetilde{P}(z)}{d\ln w}dz - \Gamma^*\int_{\bar{z}^*}^1 \frac{\gamma(z)}{\Gamma^*}\frac{d\ln \widetilde{P}(z)}{d\ln w}dz \in (0,1)$$

because $\widetilde{P}(z)$ has elasticity with respect to $w$ between 0 and 1. The real wage thus has elasticity with respect to the factoral terms of trade between 0 and 1, and the average real capital return will as well unless the return is sufficiently responsive to the factoral terms of trade. Globalization increases $\Gamma$ and $\Gamma^*$ and by this channel it raises $d\ln Cd/\ln w$. In contrast to the Cobb-Douglas case, however, a change in $t$ has effects on the distribution of $d\ln \widetilde{P}(z)/d\ln w$ that are difficult to sign. On balance, globalization should ordinarily raise

$d \ln C/d \ln w$ and damp the real income response to underlying aggregate shocks, as it does in the Cobb-Douglas case.

Now turn to the idiosyncratic income distribution properties of the model. Ex post dispersion is induced by realizations of the productivity shocks. For simplicity in thinking about the ex ante personal income risk that is associated, suppress aggregate risk. The return to capital is residually determined in each sector. Thus $r(z) = [1-\alpha(z)]p(z)y(z)/\lambda(z)K$ is the sector specific return in $z$, where $\alpha(z) \equiv wL(z)/p(z)y(z)$ is labor's share in $z$. Replace $p(z)y(z)$ with $s(z)g$. The average return is $g_K = (g/K)\int_0^1 [1-\alpha(z)]s(z)dz = (1-\bar{\alpha})g/K$. Then the sector $z$ return relative to the mean is:

$$\frac{r(z)}{g_K} = \frac{s(z)}{\lambda(z)}\frac{1-\alpha(z)}{1-\bar{\alpha}}. \tag{36}$$

Using the value of marginal product conditions, $\alpha(z) = wh(\widetilde{P}(z))\lambda(z)K/s(z)g$. Replace $s(z)$ with $\widetilde{P}(z)f[h(\widetilde{P}(z))]\lambda(z)K/g$ in the labor share and relative returns conditions to yield:

$$\frac{r(z)}{g_K} = \frac{1}{1-\bar{\alpha}}\left(\frac{\widetilde{P}(z)f[h(\widetilde{P}(z))]K}{g} - \frac{h(\widetilde{P}(z))K}{g}\right).$$

This simplifies to

$$r(z) = \widetilde{P}(z)f[h(\widetilde{P})] - h(\widetilde{P}). \tag{37}$$

The distribution of capital returns across sectors is characterized by

$$\frac{dr(z)}{dz} = h'\widetilde{P}_z \leq 0.$$

For tradable goods sectors $\widetilde{P}_z < 0$ while for nontraded goods, $\widetilde{P}_z/\widetilde{P} = 0$. The return in the non-traded goods sectors is equal to the cutoff sector returns $r(\bar{z}) = r(\bar{z}^*)$. Thus Proposition 4 holds for the general case.

Next, consider the effect of changes in $w$, the factoral terms of trade, on the profile of specific factor returns. Differentiating (36) with respect to $\ln w$ at the efficient allocation $\lambda(z) = \gamma(z)$, the components that change are $s(z)/\gamma(z)$ and $[1-\alpha(z)]/(1-\bar{\alpha})$. As for the change in $s(z)/\gamma(z)$, differentiating (35),

$$\frac{\partial \ln s(z)/\gamma(z)}{\partial \ln w} = [1-s(z)][1+\widetilde{P}(z)\alpha(z)\eta(z)] > 0.$$

where $\eta(z) = h'(\cdot)\widetilde{P}(z)/h(\cdot) = -d\ln L(z)/d\ln w$, the elasticity of demand for labor in sector $z$.[32] The first term on the right hand side $1 - s(z)/\gamma(z)$ is increasing in $z$. The second term may be increasing or decreasing in $z$ in general. The change in $[1-\alpha(z)]/(1-\bar{\alpha})$ due to change in $w$ is similarly ambiguous in its impact on the distribution of response of $r(z)/g_K$ to change in $w$. The Cobb-Douglas case removes the ambiguity, yielding the implication in Proposition 5 that the poorest sectors are hit the hardest by changes in the factoral terms of

[32] The expression on the right uses $1+\widetilde{P}(z)\alpha(z)\eta(z) = 1 + h'(\cdot)\widetilde{P}(z)/f(\cdot)$ where the right hand expression is obtained from differentiating (35).

trade. The Cobb-Douglas logic remains active in the general case but qualified by possible offsetting influences from changes in the distribution of labor shares and demand elasticities.

Next, consider the comparative statics of globalization. A fall in $t$ raises $\bar{z}, \bar{z}^*, \Gamma(\bar{z})$ and $\Gamma^*(\bar{z}^*)$. It generally has ambiguous effects on $w$. As for the ex post distribution of capital income, the dispersion of returns relative to the average are increased. This arises for two reasons. First, export sectors experience a price rise while import competing sectors experience a price fall due to the fall in trade costs at constant factoral terms of trade. Second, the expansion of the extensive margin of trade raises the sector specific incomes of newly exporting sectors while lowering the sector specific incomes of newly import-competing sectors. Thus Proposition 6 holds for the general case. Globalization intensifies the impact of good or bad luck in the choice of location by capital. This is true for both countries. Viewed ex ante, personal income is made more risky by globalization when there is no aggregate productivity risk.

Globalization ordinarily would have some effect on the average capital return relative to the wage, but general analytic results are precluded. For the CES production function with $\sigma > 1$, globalization at constant terms of trade ordinarily raises both $\bar{\alpha}$ and $\bar{\alpha}^*$ and thus the average return relative to the wage ordinarily falls in both North and South. This property arises from consideration of

$$\bar{\alpha} = \int_0^{\bar{z}} s(z)\alpha(z)dz + \int_{\bar{z}}^{\bar{z}^*} \gamma(z)\alpha(z)dz + \int_{\bar{z}^*}^{1} s(z)\alpha(z)dz.$$

Export sectors experience rising $s$ and rising $\alpha$ while contracting sectors experience falling $s$ and falling $\alpha$. So the first and third terms on the right hand side of the above equation must rise. The middle term should ordinarily not change much because the mobile factor flows from import-competing to export sectors mainly. Finally, the fall in $t$ should intuitively raise $\bar{z}$ and lower $\bar{z}^*$, further raising $\bar{\alpha}$. For the case of $\sigma < 1$, the effects through $s$ and $\alpha$ in the first and third terms reverse in sign, and the effect of globalization should ordinarily raise the average return relative to the wage in North and South.

Relaxing the identical production functions assumption introduces a host of complications that might greatly qualify the results. The key mechanism of the paper that allows simple results is that the allocation of capital can be derived as $\lambda(z) = \gamma(z) = \lambda^*(z)$ because all industries then give equal prospects. Once the production functions differ, it is no longer possible to allocate such that there are identical prospects across sectors. The risk aversion of the capitalists and the availability and quality of risk sharing instruments then become crucial to characterizing the equilibrium allocation. In some circumstances, globalization might lower efficiency (Newbery and Stiglitz, 1984). See Helpman and Razin (1978) for an analysis of allocation with risk-sharing with limited assets.