Ionospheric imaging by finite-element tomography

RW Penney & NK Jackson-Booth,

QinetiQ Ltd, Malvern, UK

Introduction

Being able to estimate the spatial distribution of ionospheric electron density is important to many geophysical and communications applications affected by space weather. In general, this 3D density profile is very difficult to measure directly, and must involve fitting techniques which try to reconcile relatively sparse measurements from ionosondes and dual-band GPS receivers with a parameterized model of the ionosphere.

Tomographic techniques have been successfully applied to ionospheric imaging [e.g., Pryse 2003, Bust & Mitchell 2008, Kunitsyn & Tereshchenko, 2010], but generally require considerable specialization to handle the irregular spatial sampling and other peculiarities of time-series derived from satellite navigation systems such as GPS.

Obtaining the best tomographic image quality requires careful consideration of how sparsely sampled sensor measurements correspond to the imaging model. In this paper we will discuss how overly simplistic interpolation techniques used implicitly within a tomographic reconstruction can severely limit the fidelity of an estimated electron-density profile. We will describe improved techniques that self-consistently interpolate sensor data within a tomographic fitting process, based on finite-element modelling techniques and tetrahedral electron-density grids.

Interpolation and imaging artefacts

Let us consider an idealized 2D imaging scenario, shown in Figure 1, which is reminiscent of the problem of estimating the ionospheric electron-density profile from dual-band GPS carrier-phase time-series. Each total electron content (TEC) measurement provides effectively only a line-integral of electron density [cf. Hernández-Pajares et al 1998], but with the measurements being in quasi-random orientations associated with complex satellite motion relative to a set of irregularly spaced receiver ground stations.

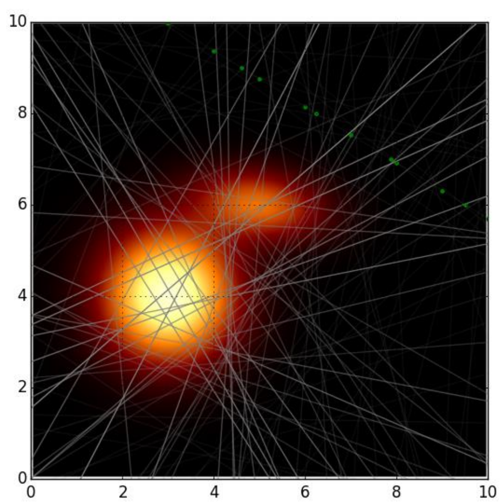

Figure 1 – Illustration of a simulated tomographic imaging scenario, where a set of 100 rays (grey lines) are used to estimate a 2D electron-density profile consisting of two overlapping Gaussian (orange/red) clouds. The green dots show the cell boundaries with which one of the rays intersects, on an 11x11 rectangular grid.

Suppose that we model the electron density using a rectangular grid of fitting parameters (cf. Figure 2). Clearly, each observation will cover multiple grid cells, and will generally have a rather complex pattern of lengths and vertex offsets within each of the cells through which it passes. Fitting the parameters of one's model to the available measurements requires one to define an interpolation scheme which allows one to integrate the fitted electron density along an arbitrary line across the grid. Similar issues apply to models that use a set of basis functions, such as polynomials or spherical harmonics.

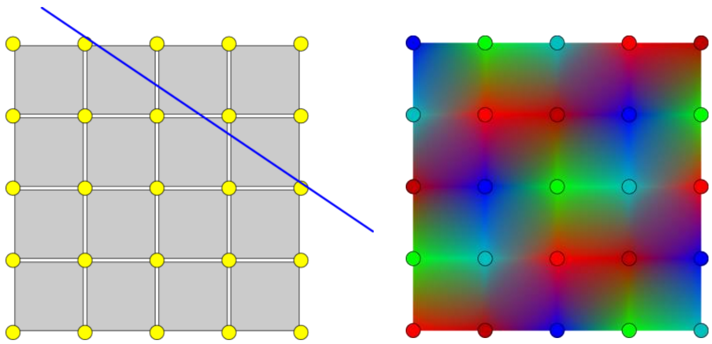

Figure 2 – Illustration of the two phases of tomography in which an interpolation function is needed. The left-hand image relates to the interpretation of each TEC line-integral as a slice through a grid of interpolation weights. The right-hand image relates to the subsequent reconstruction of a 2D density profile after having estimated those interpolation weights.

The simplest interpolation scheme would be a piece-wise constant function based on rounding down to the nearest grid corner [cf. Hernández-Pajares et al. 1999]. This allows one to represent the line-integral of electron density in terms of the weighted sum of geometrical segment lengths within each grid cell. Such an approach appears to be popular, but does not produce the highest fidelity reconstruction of the known electron density (Figure 3, left).

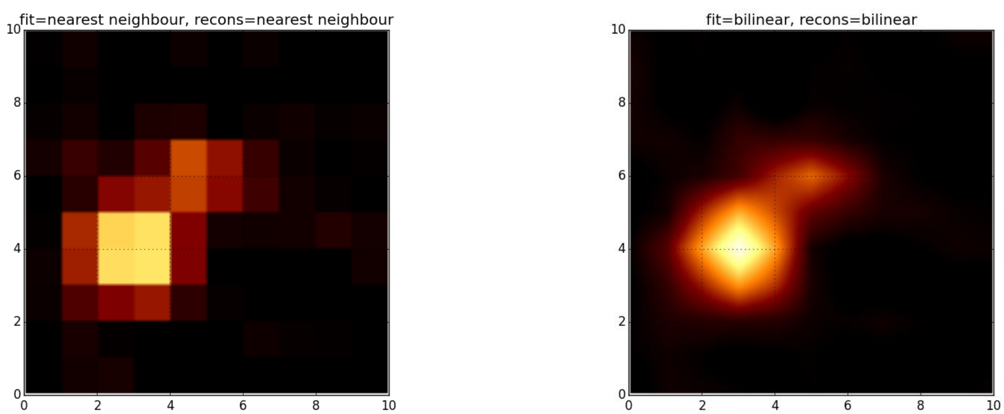

Figure 3 – Illustration of the effects of different interpolation functions on the fidelity of tomographic reconstruction for the scenario shown in Figure 1. The left-hand image shows the use of a piecewise-constant interpolator, while the right-hand image uses a bilinear interpolator.

In contrast, if one defines a smoother interpolation scheme, it is rather more subtle to integrate the modelled electron density along each of the measurement directions [cf. Kunitsyn & Tereshchenko 2010]. However, the benefits of doing so are that one can achieve much higher quality image construction from a model with exactly the same number of fitting parameters (Figure 3, right). Here, we use a set of bi-linear interpolators (essentially 2D quadratic functions), for which it is possible to analytically calculate the relationship between the densities at the grid vertices and the line-integral of electron density across any given ray.

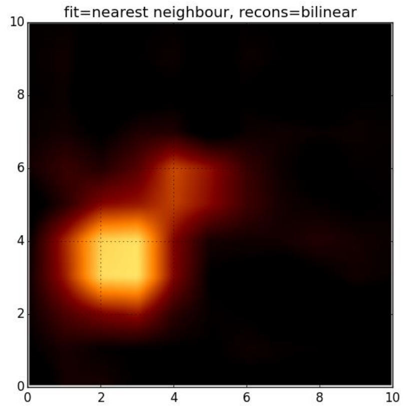

Figure 4 – Illustration of the artefacts associated with a tomographic fit that was generated using piecewise-constant interpolation, but where a smooth bilinear interpolator was used to render the reconstructed image. This has resulted in both a bulk error in the centres of the clouds, a broadening of their extent, and various spurious patches of density.

One might attempt to improve image quality by combining a simple piecewise-constant interpolation scheme while estimating parameters, with a smoother interpolation scheme after one has estimated density parameters on a regular grid. However, this can easily lead to artefacts that are a direct consequence of using different interpolation schemes during fitting and reconstruction (Figure 4). In contrast, by self-consistently using the same interpolation scheme within the estimation and reconstruction, one achieves much more accurate estimation of the true electron density profile, with much less prominent artefacts (Figure 3, right).

Ionospheric tomography using tetrahedral grids

It is not trivial to choose a 3D grid geometry that is compatible with the Earth's curvature; can handle polar regions; and has an interpolation scheme that allows the algebraic manipulations necessary for high quality tomographic imaging. For example, these criteria are very poorly met by a regular latitude/longitude/altitude grid. If one can restrict one's attention to a 2D vertical slice through the ionosphere, then one can easily form a triangular lattice by subdividing each cell within a rectangular grid [cf. Kunitsyn & Tereshchenko 2010], but this may be problematic unless all one's sensors measurements are also confined to the same plane.

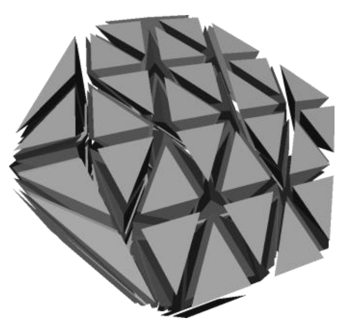

Figure 5 - Illustration of a tomographic grid formed from tetrahedral finite-elements

A grid formed of tetrahedra, and piecewise linear interpolators, has many of the features required for high quality tomographic reconstruction:

- It can be made to conform to the Earth's curvature over an extended area;
- It does not need special treatment of polar regions;
- It allows higher resolution coverage in regions of particular interest;
- Point density and line-integrals of density can be computed exactly over each tetrahedron.

Nevertheless, the process of constructing a grid that does not have gaps or overlaps between tetrahedra, and on which density lookups can be executed efficiently, requires considerable care. Typically, this will involve a process of generating a collection of points within the 3D volume of interest, and then performing a Delaunay triangularization to construct a set of contiguous tetrahedra. This also requires careful bookkeeping to ensure that one can efficiently search for whichever tetrahedron contains a point at which one wishes to calculate a density, or find the set of tetrahedra intersected by the ray between ground-station and satellite for a given TEC measurement.

We have implemented such a finite-element tomographic process for a number of ionospheric measurement scenarios. Below we discuss two forms of tomographic fits: firstly for an exact least-squares fit using “pure” GPS data; and secondly, for a Bayesian finite-element model which combines a background model with real GPS data.

<u>Tomography on synthetic GPS data</u>

A particularly challenging application of the finite-element tomographic process is trying to estimate the 3D electron-density profile exclusively using TEC measurements from dual-band GPS. (This is to be contrasted with assimilative approaches that require a background model such as IRI [Bilitza et al 2011] or models that use a set of empirical basis functions that embody plausible density profiles [cf. Chartier et al 2012].)

In Figure 6 we show results from a simulated scenario in which IRI-2016 is used to generate a set of known 3D density profiles from which synthetic GPS TEC time-series can be derived using true GPS satellite trajectories. The tomographic model uses these TEC time-series from about 18 ground-stations in western Europe to estimate the electron density profile over the region of satellite visibility. The tomographic grid consists of about 22,000 nodes and about 110,000 tetrahedra. In this scenario, the basis functions used in the finite-element model are entirely unbiased by historical trends, or any other form of prior information.

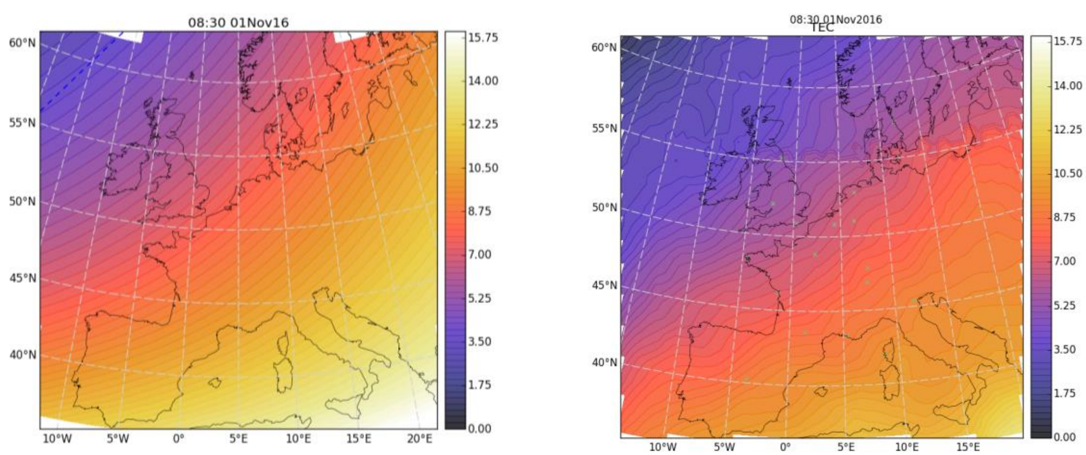

Figure 6 – Illustration of a tomographic reconstruction of (vertical) TEC as produced by our tetrahedral finite-element model for a simulated scenario. The left-hand plot shows (simulated) ground-truth of TEC from IRI-2016, from which our GPS time-series are constructed. The right-hand plot shows the estimated TEC after 3D tomographic fitting of the GPS time-series, with crosses showing the location of the simulated GPS receivers.

The estimated electron density profiles in Figure 6 show that it is possible to recover many of the qualitative features of the known density profile (e.g. in terms of typical gradients, variation with solar illumination, etc.). Given the sparsity of observational data relative to the number of parameters that must be estimated (cf. Figure 7), it is unsurprising that the quantitative accuracy of the fit is not perfect. However, the results in Figure 6 show that one can achieve a reconstructed TEC that is within about 1TECu of ground-truth over the footprint covered by the ground-stations, even without any steering from historical ionospheric trends or empirical vertical profiles. Similar results also show that even coarse features of the vertical density profile can be estimated from sparse GPS networks of about a dozen receivers over a 2000km-square region.

Assimilation of GPS data onto a background model

Although it is encouraging that one can obtain sensible 3D electron-density profiles purely from GPS TEC time-series (Figure 6), it is desirable to be able to exploit historical trends from a model such as IRI [Bilitza et al 2011] in order to improve the estimated density in regions where measurements are particularly scarce [cf. Angling & Khattov 2006, Jackson-Booth & Angling 2011]. Indeed, for GPS receivers that are hundreds of kilometres apart, one may not even be able to observe more than a single receiver above the horizon until one has obtained an altitude well into the ionosphere (Figure 7). This severely limits the ability to resolve the vertical profile of the ionosphere at low altitudes using just GPS.

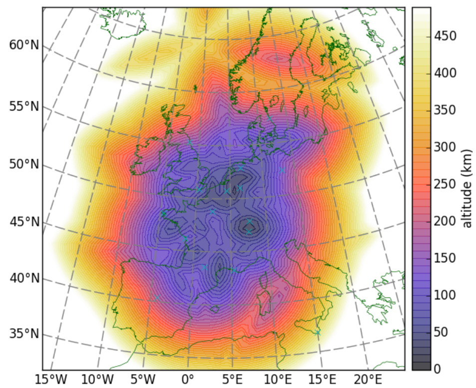

Figure 7 – Illustration of the spatial variation in altitude above which one has visibility of more than one GPS receiver (assuming a 15° elevation cut-off), for a set of 19 GPS receivers (cyan crosses) over western Europe.

We have developed a Bayesian assimilation model which uses IRI-2016 as a statistical "prior" in combination with a model of the GPS TEC measurement process identical to the pure tomographic scenario above. This assimilative model ("EDAM2") uses the same tetrahedral finite-element techniques to represent the 3D electron-density profile, but with IRI-2016 providing a soft constraint on the interpolation weights. Thus, where one has many TEC measurements that intersect a particular region of the ionosphere, they will steer the estimated density towards those implied by the GPS measurements, but in other regions the model will more closely track the trends embodied by IRI-2016.

We have applied this EDAM2 model to a real GPS dataset covering 19 receivers over western Europe, and using a tetrahedral finite-element model consisting of about 42,000 nodes and about 265,000 tetrahedra. One can use vertical cross-sections of the electron density to estimate the maximum density (essentially foF2), and compare this with an independent measurement from an ionosonde (which has not been used in the assimilation).

Comparing the behaviour of the foF2 time-series from IRI-2016, EDAM2 and the ionosonde (Figure 8) shows that the finite-element assimilative model has been able to quite accurately reproduce the variation in foF2 over an entire month, sensibly moving away from the values produced by IRI-2016 and towards values more consistent with the (unused) values from the ionosonde. Given the difficulty in obtaining such bottom-side information from pure GPS measurements, it is encouraging that the model is able to make non-trivial adjustments to the IRI background at altitudes low enough to be able to estimate foF2.

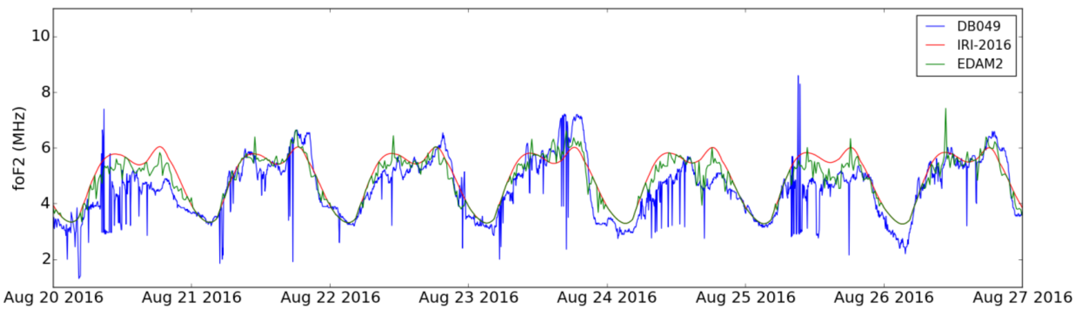

Figure 8 - Illustration of results from a finite-element assimilative model which combines an IRI background with real GPS measurements. The graph compares "true" value of foF2 obtained from the DB049 ionosonde in Belgium (blue, more spiky trace) with that produced by the model's estimate of the vertical profile (green, smoother trace) and the IRI-2016 background (red, smooth trace).

Outlook

We have discussed how the quality of tomographic imaging of the ionosphere is strongly influenced by the type of interpolation scheme used within the fitting process, and how less sophisticated approaches can significantly degrade the quality of the estimated density profiles. We have illustrated the potential benefits of careful choice of interpolation scheme and basis-functions that can be applied self-consistently during both the fitting and final rendering stages. On-going work is investigating how these techniques can improve ionospheric imaging for scenarios that involve combinations of GPS and ionosonde sensors.

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