Properties of ULF waves in ionospheric plasma

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Introduction

The propagation of ULF waves in the ionospheric plasma usually is treated in terms of the Shear Alfven (SA) and fast magneto sonic (FNS) waves since the first publication of Greifinger and Greifinger (1968) on the properties of the ionospheric MHD waveguide in frame of magneto hydro dynamics (see f. e. Polyakov and Rapoport (1981), Lysak, at all, (2013), Eliasson, at all, (2012)). We have examined the properties of ULF electromagnetic waves in the multi-component ionospheric plasma in the 1-20 Hz band basing on the magneto ionic theory. Our calculations have showed that the properties of ULF waves differ greatly from those predicted by the magneto-hydro-dynamic approach.

Basic equations

$$-i\omega\vec{\mathrm{v}}_{\mathrm{e}} = \frac{e}{m}\vec{E} + \frac{e}{mc}\left[\vec{\mathrm{v}}_{\mathrm{e}}\vec{H}\right] + \nu_{ei}\left(\vec{\mathrm{v}}_{\mathrm{i}}\text{-}\vec{\mathrm{v}}_{\mathrm{e}}\right) + \nu_{em}\left(\vec{\mathrm{v}}_{\mathrm{m}}\text{-}\vec{\mathrm{v}}_{\mathrm{e}}\right)$$

$$-i\omega\vec{\mathrm{v}}_{\mathrm{i}} = -\frac{e}{M}\vec{E} - \frac{e}{Mc}\left[\vec{\mathrm{v}}_{\mathrm{i}}\vec{H}\right] + \frac{m}{M}\nu_{ei}\left(\vec{\mathrm{v}}_{\mathrm{e}}\text{-}\vec{\mathrm{v}}_{\mathrm{i}}\right) + \nu_{im}\left(\vec{\mathrm{v}}_{\mathrm{m}}\text{-}\vec{\mathrm{v}}_{\mathrm{i}}\right)$$

$$-i\omega\vec{\mathrm{v}}_{\mathrm{m}} = -\frac{mN}{MN_m}\nu_{em}\left(\vec{\mathrm{v}}_{\mathrm{m}}\text{-}\vec{\mathrm{v}}_{\mathrm{e}}\right) + \frac{N}{N_m}\nu_{im}\left(\vec{\mathrm{v}}_{\mathrm{m}}\text{-}\vec{\mathrm{v}}_{\mathrm{i}}\right)$$

Hear $\vec{v}_k$ – velocities of elections, ions, and molecules,
m и M – elections and ions masses,
H – geomagnetic field.

For ionospheric conditions is valid : $\omega >> \nu_{im} N/N_m$ $\nu_{im} << \nu_{em}\sqrt{m/M_i}$

Neglecting the terms of orders $m/M, \sqrt{m/M}$ one can obtain from the this system of equations the permittivity tensor in ULF band which look like in magneto-ionic approach:

Permittivity tensor

$$\varepsilon_{xx} = \varepsilon_{yy} = 1 - \sum_{k=1}^{n} \left[\frac{\omega_{0k}^2}{2\omega} \left(\frac{1}{(\omega - \omega_{Hi}) - i\nu_{km}} + \frac{1}{(\omega + \omega_{Hk}) \ - i\nu_{km}} \right) \right]$$

$$\varepsilon_{xy} = -\varepsilon_{yx} = -i \sum_{k=1}^{n} \left[\frac{\omega_{0k}^2}{2\omega} \left(\frac{1}{(\omega + \omega_{Hk}) \ - i\nu_{km}} - \frac{1}{(\omega - \omega_{Hk}) \ - i\nu_{km}} \right) \right]$$

$$\varepsilon_{zz} = 1 - \sum_{k=1}^{n} \left[\frac{\omega_{0k}^2}{\omega^2 - i\omega\nu_{km}} \right]$$

Here:

k - kind of a charged particle (electrons and ions), $\omega = 2\pi * f$ – wave frequency,

$\omega_{0k} = \left(4\pi e^2 N_k(z) / m_k\right)^{1/2}$ – plasma frequency of particle, m_k – particle mass,

$\omega_{Hk} = \pm eH(z) / m_k c$ – gyrofrequency of electrons (-) and ions (+),

c – light velocity, e – electron charge,

ν_{km} – collision frequency of particles k with molecules of neutral gas.

Collisional frequencies

Electron collisions with molecules and ions:

$$\nu(e,O_2) = 1.82\cdot 10^{-10} N(O_2)(1+3.6\cdot 10^{-2}\sqrt{T_e})\sqrt{T_e};$$

$$\nu_{ei} = 54 N_e / T_e^{3/2}$$

$$\nu(e,N_2) = 2.33\cdot 10^{-11} N(N_2)(1-1.21\cdot 10^{-4} T_e)T_e;$$

$$\nu(e,O) = 4.5\cdot 10^{-10} N(O)\sqrt{T_e}$$

$$\nu_{em} = \nu(e,O_2) + \nu(e,N_2) + \nu(e,O)$$

Collisions ions O_2^+ and O^+ with molecules:

$$\nu(O_2^+,O_2) = 1.17\cdot 10^{-9}\left(\frac{T_i + T_n}{2000}\right)^{0.28};$$

$$\nu(O_2^+,O,N_2) = (0.75\cdot N(O) + 0.89\cdot N(N_2))\cdot 10^{-9}$$

$$\nu_{O_2^+ m} = \nu(O_2^+,O_2) + \nu(O_2^+,O,N_2)$$

$$\nu_{O^+ m} = 10^{-9}(N(NO) + 1.08 N(N_2))$$

Collisions ions NO^+ with molecules:

$$\nu_{NO^+ m} = 10^{-9}(0{,}83 N(O_2) + 0.76 N(NO) + 0.76 N(N_2))$$

Refractive index if ULF waves in the ionosphere

could be obtained from the magneto ionic theory

$$(n - i\kappa)^2_{1,2} = \frac{L \pm \sqrt{R}}{D};$$

$$L = (\varepsilon_{xx}^2 + \varepsilon_{xy}^2)\sin^2\alpha + \varepsilon_{xx}\varepsilon_{zz}(1+\cos^2\alpha);$$

$$D = 2(\varepsilon_{xx}\sin^2\alpha + \varepsilon_{zz}\cos^2\alpha);$$

$$R = L^2 - 2D(\varepsilon_{xx}^2 + \varepsilon_{xy}^2)\varepsilon_{zz}$$

α – angle between Earth magnetic field and wave vector

$$n_1 = \frac{\omega_{0i}}{\omega_{Hi}}\left(1 + \frac{\omega^2}{\omega_{Hi}^2}\right); \quad n_2 = n_1\cos\alpha$$

$$n^2 = \frac{\omega_{0e}^2}{\omega_{H_{NO}}\,\omega\cos\alpha}$$

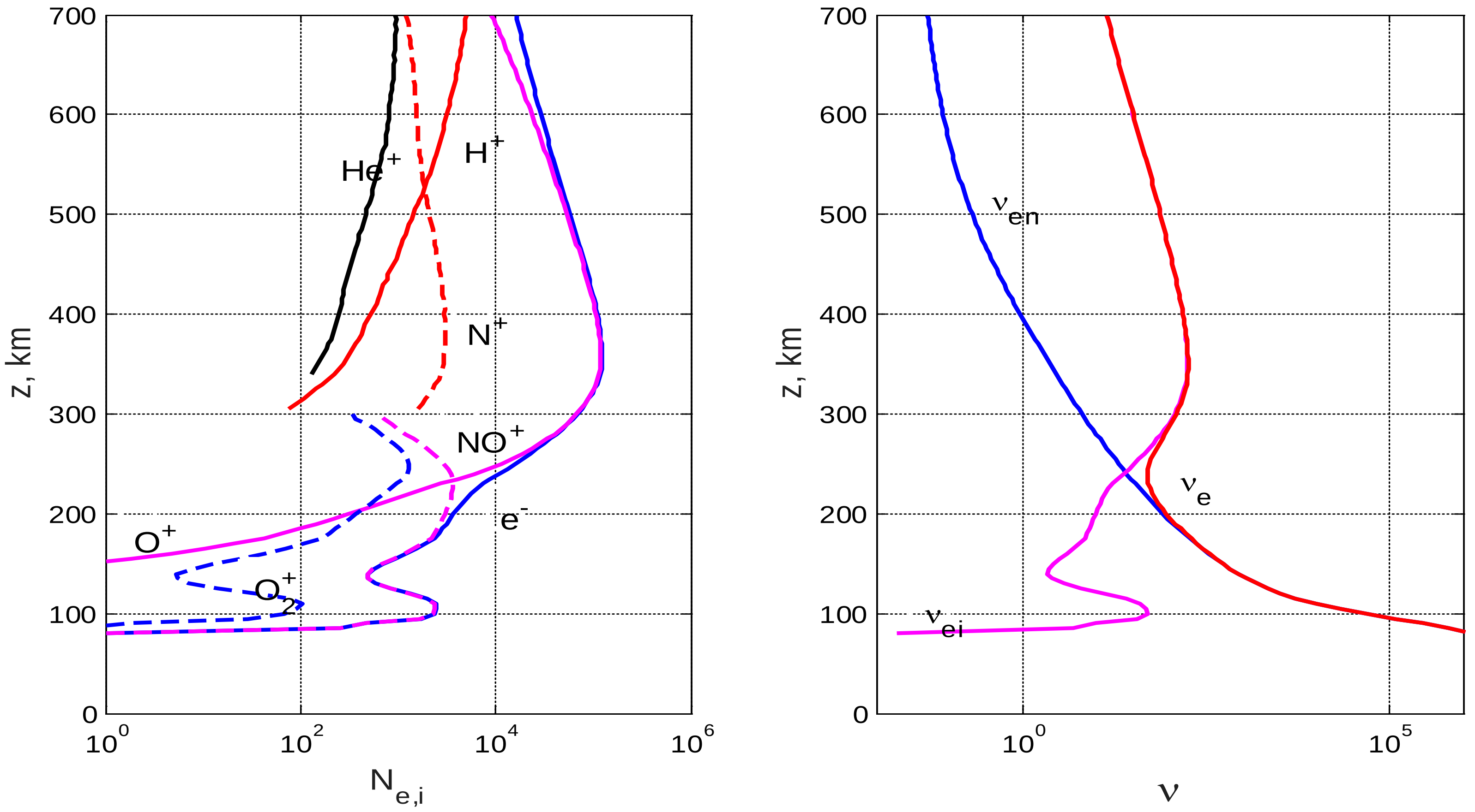
In E layer the refractive index of ULF waves coincides with the index for whistlers

Typical ionospheric parameters

IRI-2016 - International Reference Ionosphere

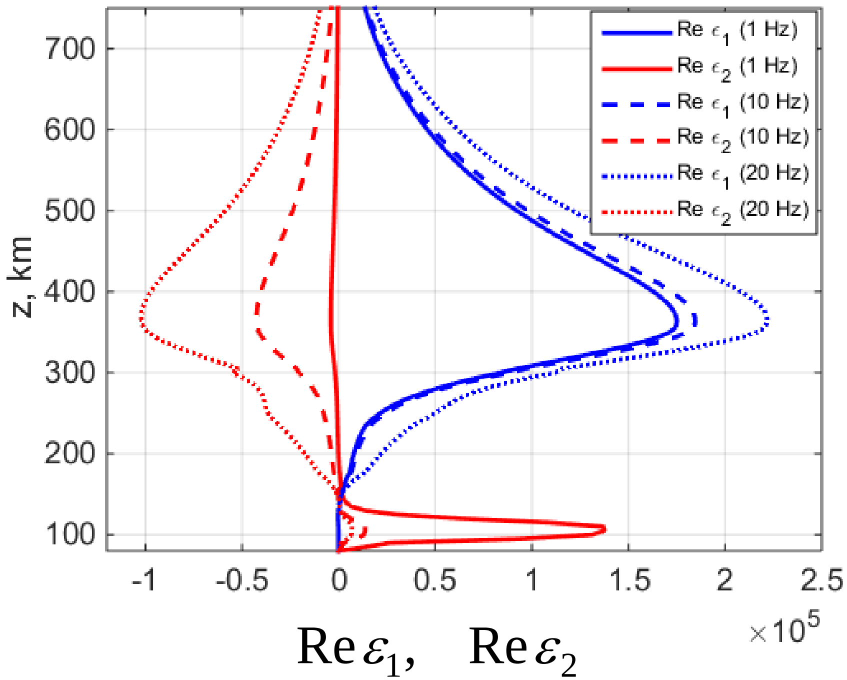
MSIS-E-90 - Atmosphere Model

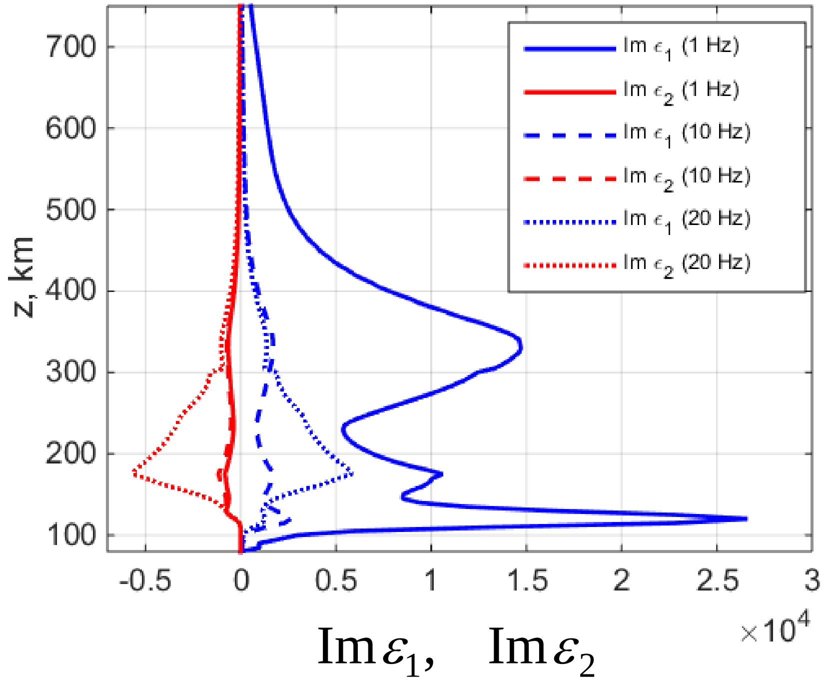
DGRF/IGRF Geomagnetic Field Model



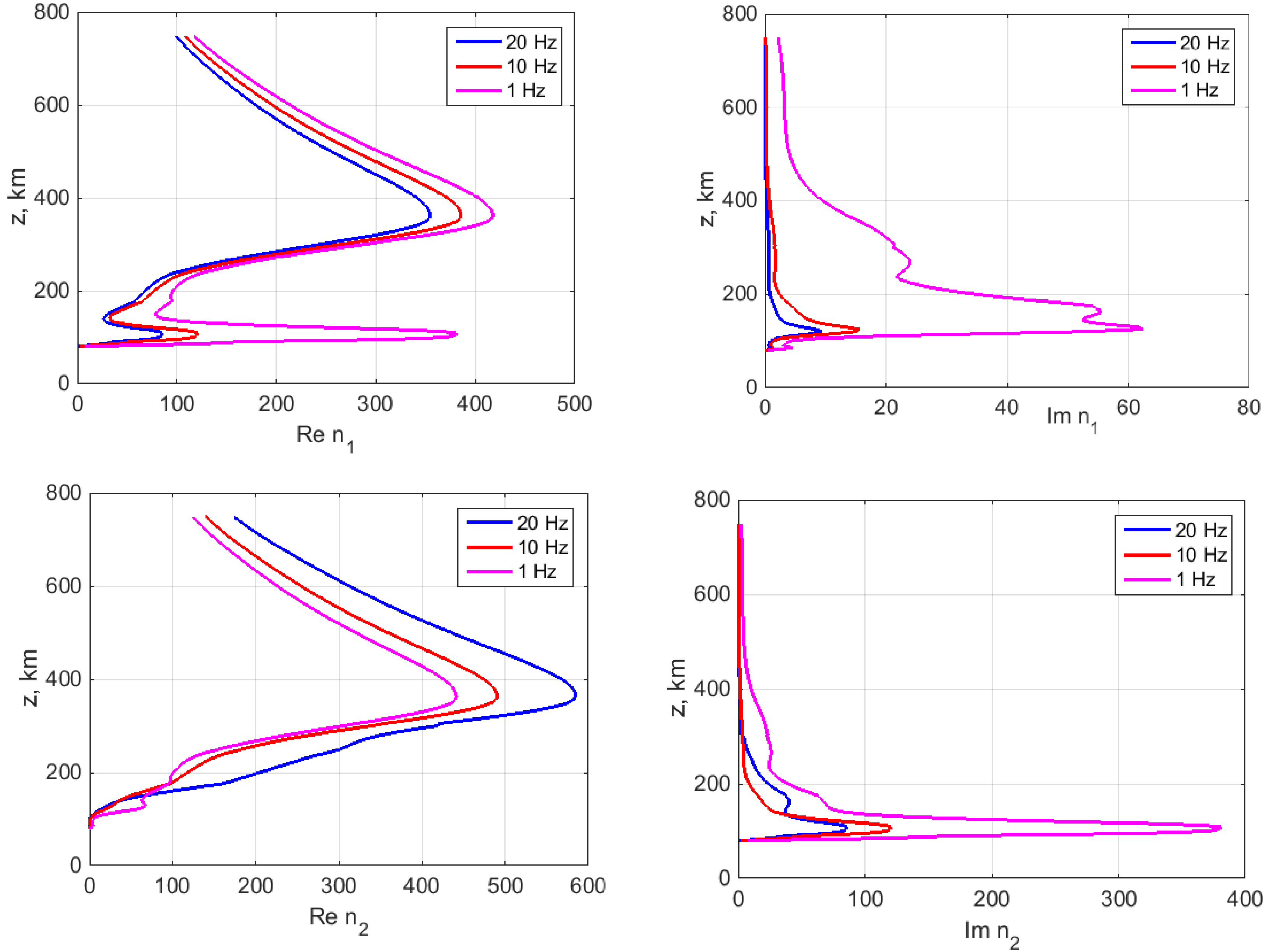
Profiles of charged particles (left) & collision frequencies (right)

Components of permittivity tensor

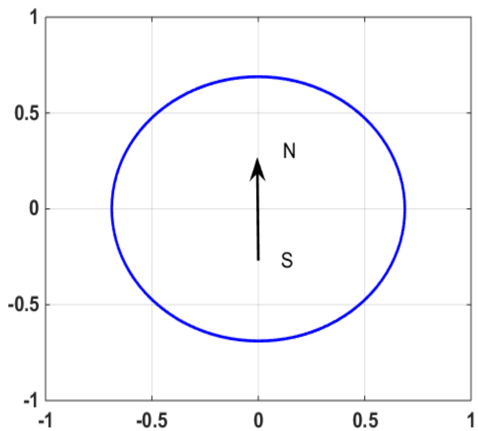
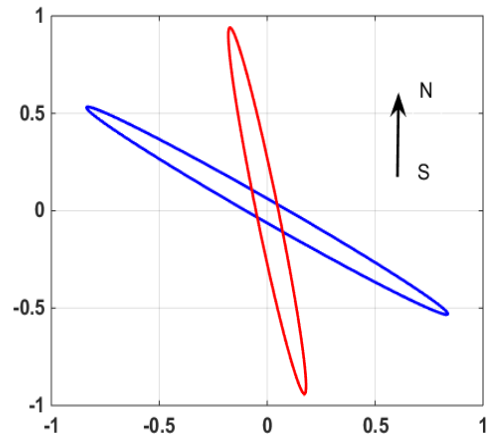
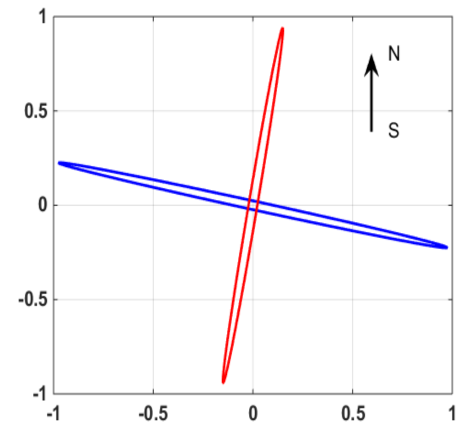
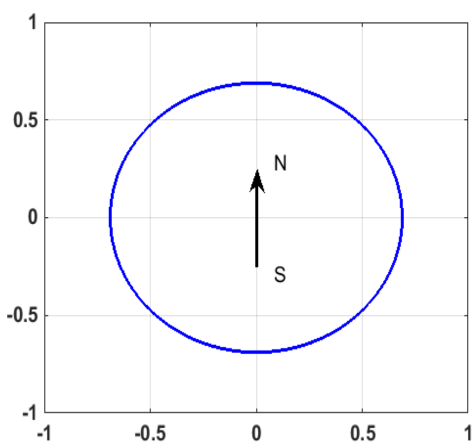
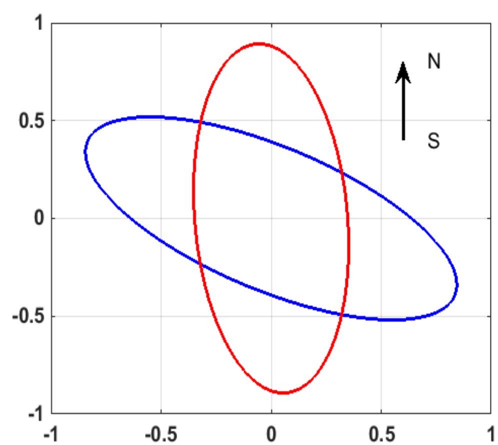
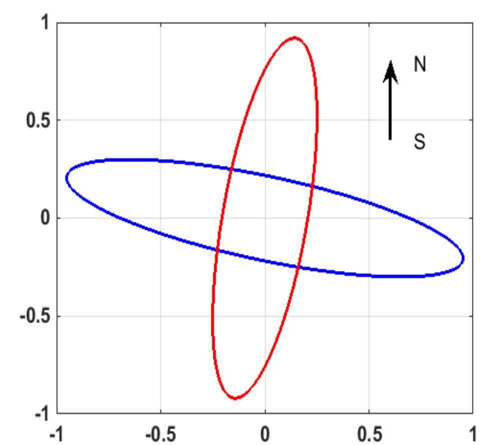




Normal ULF waves modes in the ionoshpere



Polarization`s properties
(blue – FMS wave, red – Alfven wave)

	100 km	200 km	300 Km
0,1 Hz			
1 Hz			

Polarization`s properties

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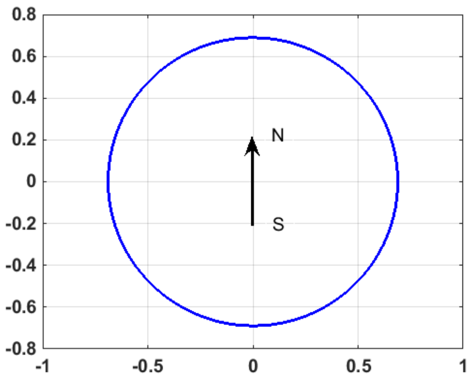
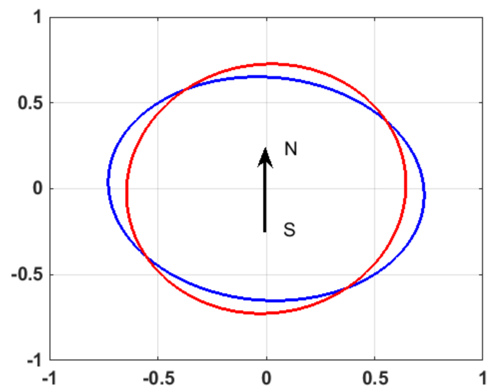
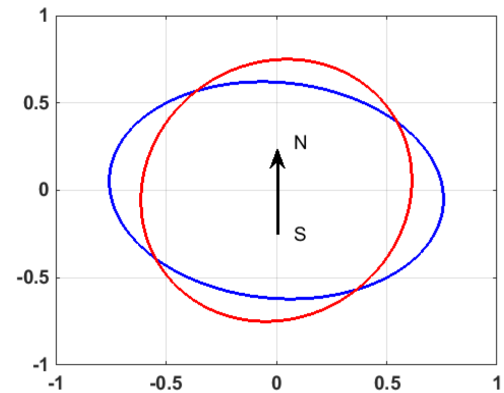
<tr><td>10 Hz</td><td>
</td><td>
</td><td>
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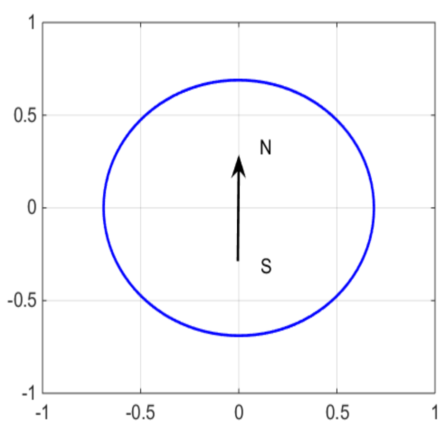
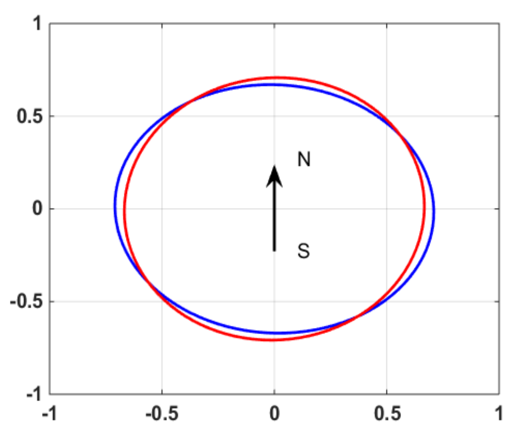
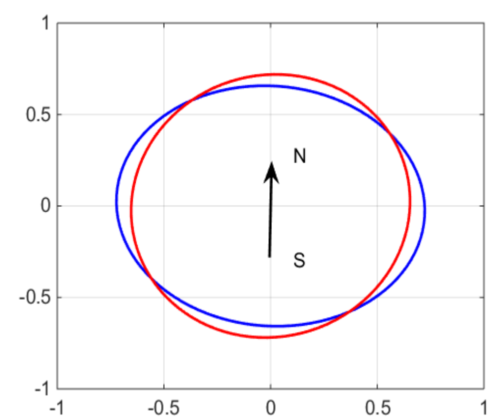
<tr><td>20 Hz</td><td>
</td><td>
</td><td>
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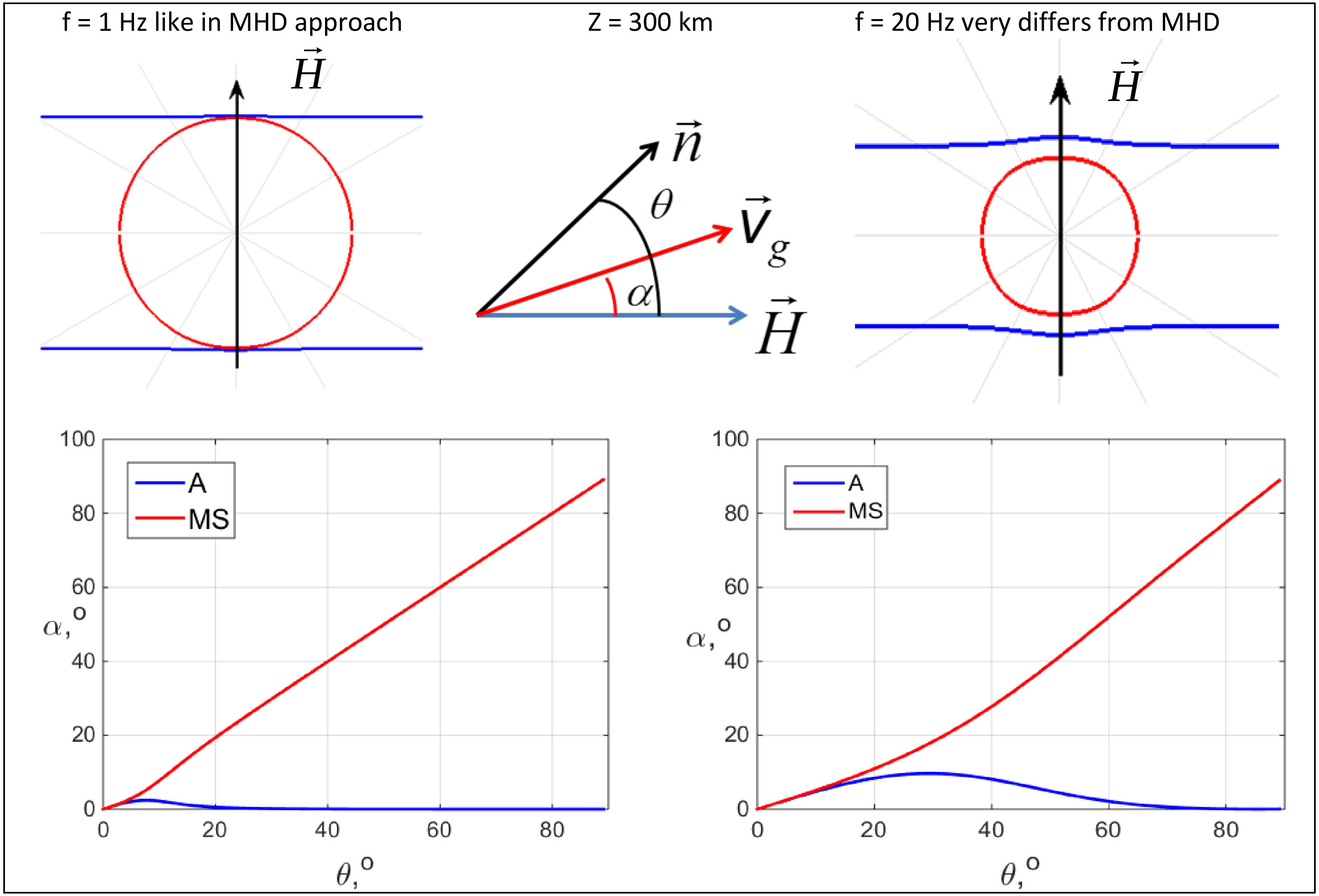
Surfaces of wave vectors and dependence of the angle α between magnetic field direction and group velocity vector and angle θ between magnetic field and wave normal.



Summary

The results of calculation of the properties of ULF waves in ionospheric plasma are shown:

- **The refractive indices of two normal waves (ordinary and extraordinary) are highly dependent on the frequency and height.**
- **The polarization of the two waves is elliptical in the whole range of investigated frequencies.**
- **The dependence of the group velocity of the “Alfven wave" on the angle between the wave vector and the Earth magnetic field is also differs from the MHD approximation.**
- **The refractive indexes and the polarization of normal waves tend to the values obtained in the magneto hydrodynamic approximation only at frequencies much lower than 1 Hz. Therefore, it is possible only conventionally to name one of them the Shear Alfven (SA) wave, and the second fast magneto sonic (FMS) wave. The "Alfven" wave in the lower ionosphere becomes strongly damped and the refractive index of FMS wave takes the form like for whistler mode and it is weakly damped in the lower ionosphere.**
- **The vector of the group velocity of “SA” wavesin upper ionosphere is not directed along the magnetic field, but it is inside a cone within ± (20-25) degrees, depending on the frequency.**
- **The group velocity vector of the second wave corresponding to the FMS is practically independent of the angle with the magnetic field, as in the case of MHD approximation**

Thank you

for attention