Resonance of electromagnetic ion cyclotron waves in diverging magnetic field

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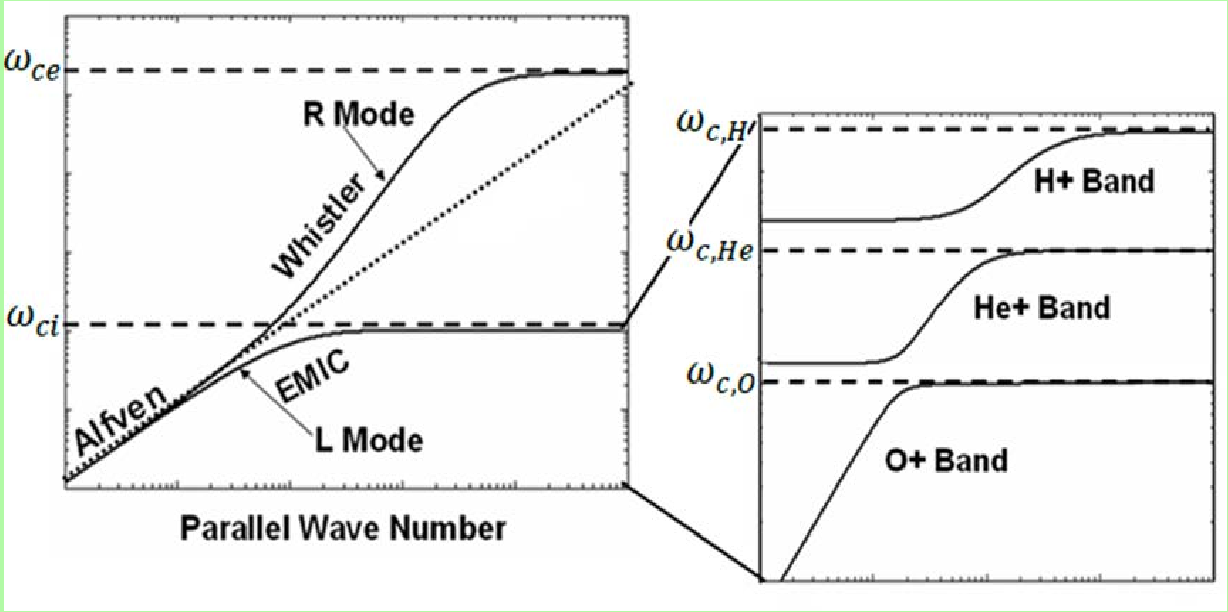
Introduction

Electromagnetic Ion Cyclotron (EMIC) waves are left-hand polarized shear Alfvén waves propagating parallel to the ambient magnetic field with frequency approaching the ion cyclotron frequency. They are often found in the radiation belts and are considered primary candidates for precipitation of MeV electrons trapped in the belts.

The dispersion relation of low frequency waves propagating parallel to the ambient magnetic field B_z in multispecies plasma (O+, He+, H+)

$$\frac{k_z^2 c^2}{\omega^2} = 1 - \sum_j \frac{\omega_{pj}^2}{\omega\left(\omega - \omega_{cj}\right)} - \frac{\omega_{pe}^2}{\omega\left(\omega + \omega_{ce}\right)}$$

Dispersion Relation



EMIC dispersion relation for a multi-ionic plasma including Hydrogen (H), Helium (He) and Oxygen (O) ions.

The pitch angle scattering occurs where electrons with energy $\gamma m_e c^2$ will satisfy the anomalous cyclotron resonance condition

$$\omega - k_z v_z = -\frac{\omega_{ce}}{\gamma}$$

Shao et al. **[2009] proposed that shear Alfvén waves generated by ionospheric heating can create regions of strong EMIC waves when they encounter the O^+ cyclotron resonance at altitude between 1000 and 2000 km.**

Motivations and Objectives

The objective of this work is to motivate a laboratory experiment to study mode conversion of shear Alfvén waves in a mirror geometry such as it occurs in the radiation belts, and study the pitch angle scattering due to anomalous cyclotron resonance interaction with energetic electrons.

Such an experiment can be conducted at the UCLA LAPD device that has already been the site of studies of kinetic shear Alfvén waves with multi-specie ion plasmas in mirror geometries [*Vincena et al., 2001*, 2010, 2011, 2013; *Farmer and Morales*, 2013].

In this paper we investigate the wave propagation of EMIC waves near cyclotron resonances, including the resultant wave spectra near the singularity and the pitch angle scattering of relativistic electrons.

Mathematical Model

To study the propagation of EMIC waves in spatially varying plasma and the magnetic field, we use the cold Hall-MHD model involving:

- **The linearized ion momentum equation for ion species**
- **The electron momentum equation for the inertialess electrons**
- **Ampere's and Faraday's laws**
- **Quasineutrality condition**

As the EMIC wave approaches a cyclotron resonance, its parallel wave vector component gradually increases. For an L-polarized wave with frequency ω propagating parallel to B_0 the equation for the electric field envelope is

$$\frac{\partial^2 E}{\partial z^2} = \frac{1}{c^2}\left(\sum_j \frac{\omega\omega_{pj}^2}{\omega - \omega_{cj} + i\nu_j} + \frac{\omega\omega_{pe}^2}{\omega_{ce} + i\nu_e}\right)E$$

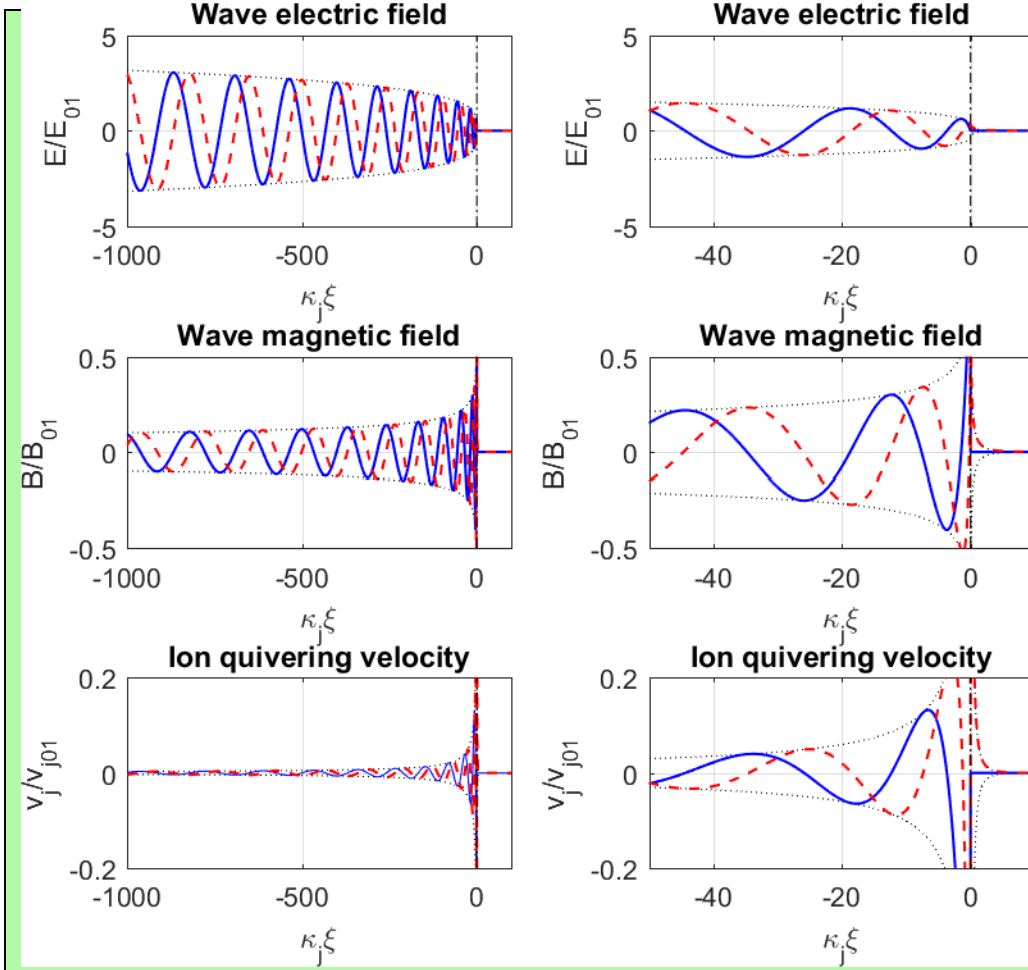
The cyclotron and plasma frequencies vary in space depending on the profile of the number densities and the magnetic field.

Analytic solutions near resonance, *Stix* [1960]

We assume a diverging ambient magnetic field decreasing with z.
Near the location of the ion cyclotron resonance, $z = z_0$, where $\omega = \omega_{cj}$, we expand $\omega_{cj} \approx \omega(1 - \xi/L)$ for $\xi/L \ll 1$, where $\xi = z - z_0$, and L is the length-scale of the decreasing magnetic field. Neglecting collisions we get

$$\frac{\partial^2 E}{\partial \xi^2} = \frac{\kappa_j}{\xi} E$$

where $\kappa_j = \omega_{pj}^2 L/c^2$. Note that E is oscillatory in space for $\xi < 0$ and evanescent for $\xi > 0$.

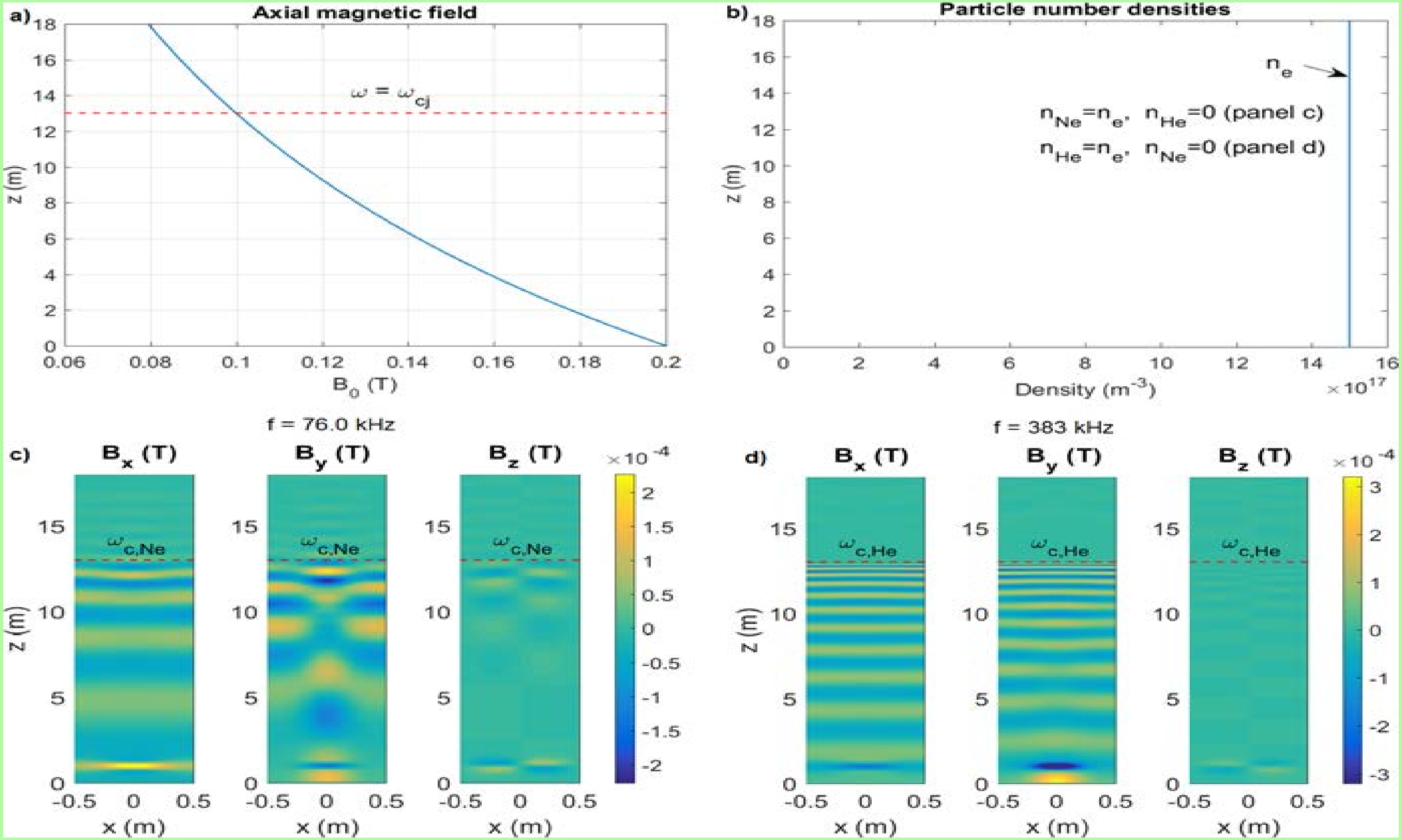


The real parts (blue solid lines) and imaginary parts (red dashed lines) near the resonance layer at $\xi = 0$ for a linearly decreasing magnetic field.

The right-hand column shows a close-up near the resonance.

The EMIC wave is evanescent and the amplitude decreases exponentially with distance for $\xi > 0$.

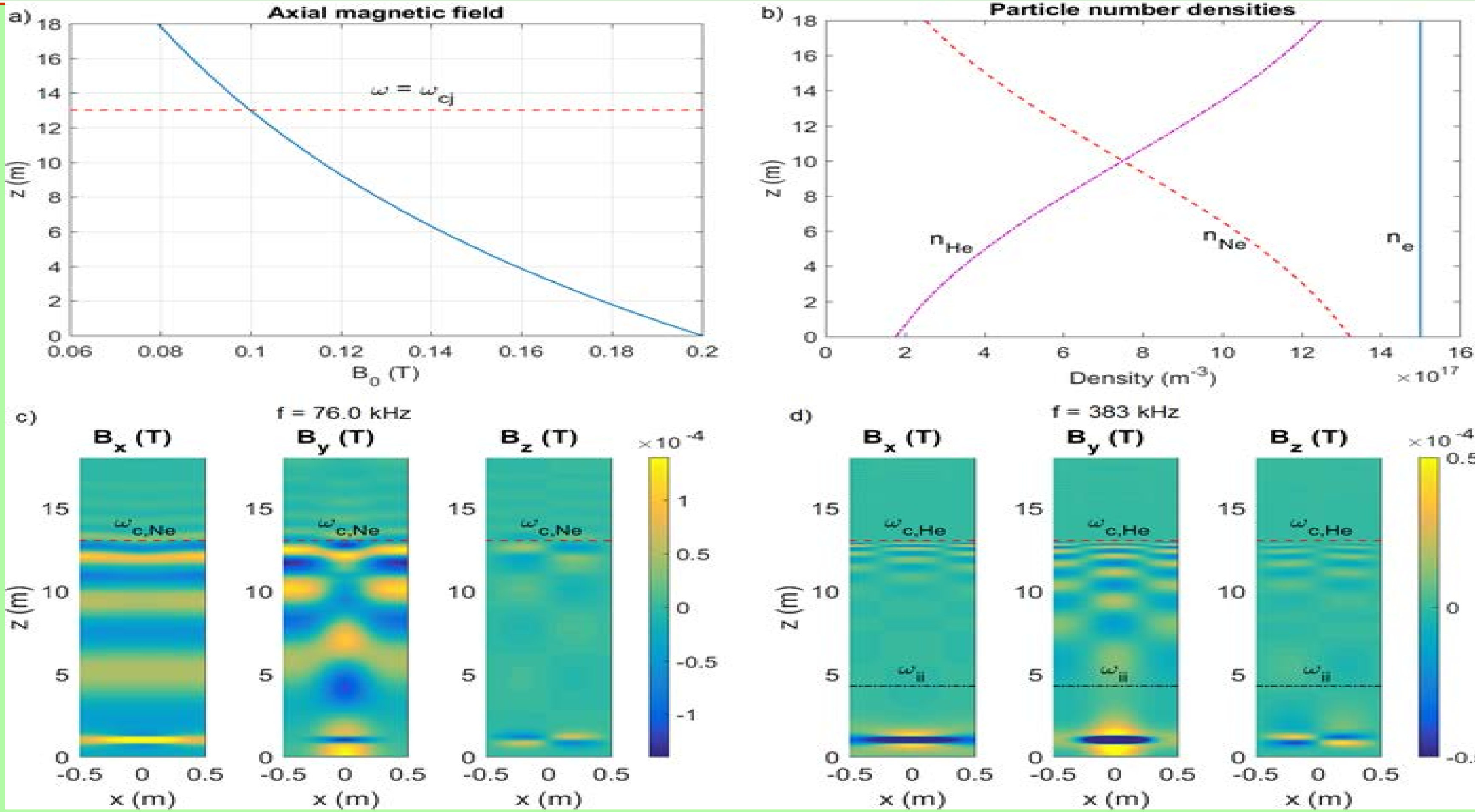
Simulations of EMIC wave propagation (single ion plasma)



Propagation of EMIC waves in single ion Ne (c) and He (d) plasmas.

Computed by the code of *Eliasson et al.* (2012)

Simulations of EMIC wave propagation (two ions plasma)



Propagation of EMIC waves in two ion Ne-He plasma using plasma parameters relevant for the LAPD device.

Pitch angle diffusion of relativistic electrons

While the magnetic moment of a relativistic electron [e.g. *Walt* (1994); *Öztürk* (2016)] $\mu = \gamma^2 m_e v_\perp^2/(2B)$ is nearly conserved in a slowly varying magnetic field, hydromagnetic waves can resonantly scatter and break the conservation of the magnetic moment, leading to an increased precipitation of mirror contained electrons.

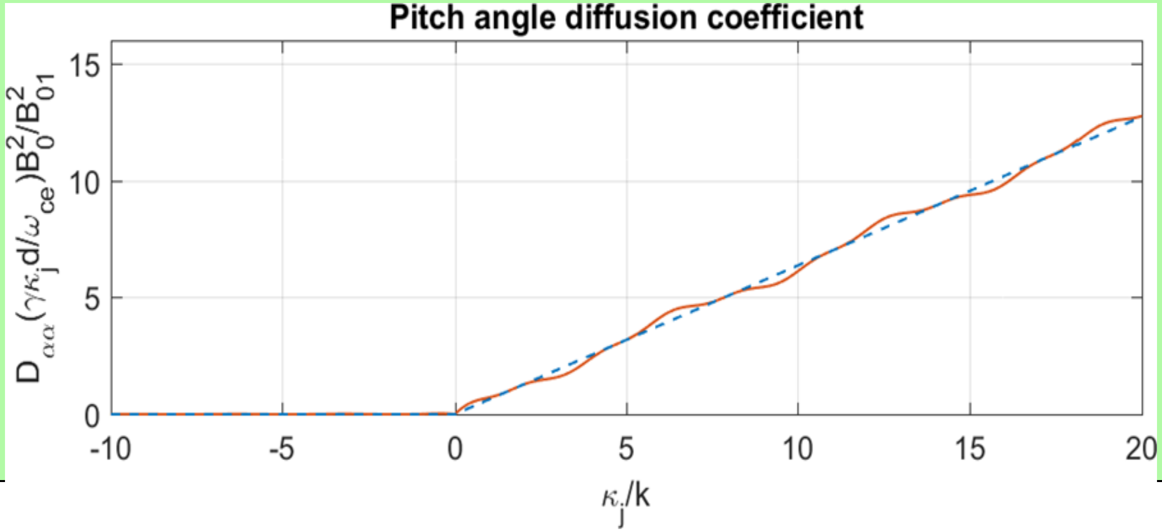
The Doppler resonance condition between a relativistic electron and an EMIC wave propagating parallel to the magnetic field is [e.g. *Summers and Thorne*, 2003] $\omega - v_\parallel k = -\omega_{ce}/\gamma$, $v_\parallel = v_0 \cos\alpha$ is the parallel component of the electron's velocity and $\alpha = \arcsin(v_\perp/v_0)$ is the pitch angle. Under $\omega \ll \omega_{ce}$ the resonance condition becomes

$$k = \frac{\omega_{ce}(z)}{\gamma v_0 \cos\alpha}$$

Pitch angle diffusion of relativistic electrons

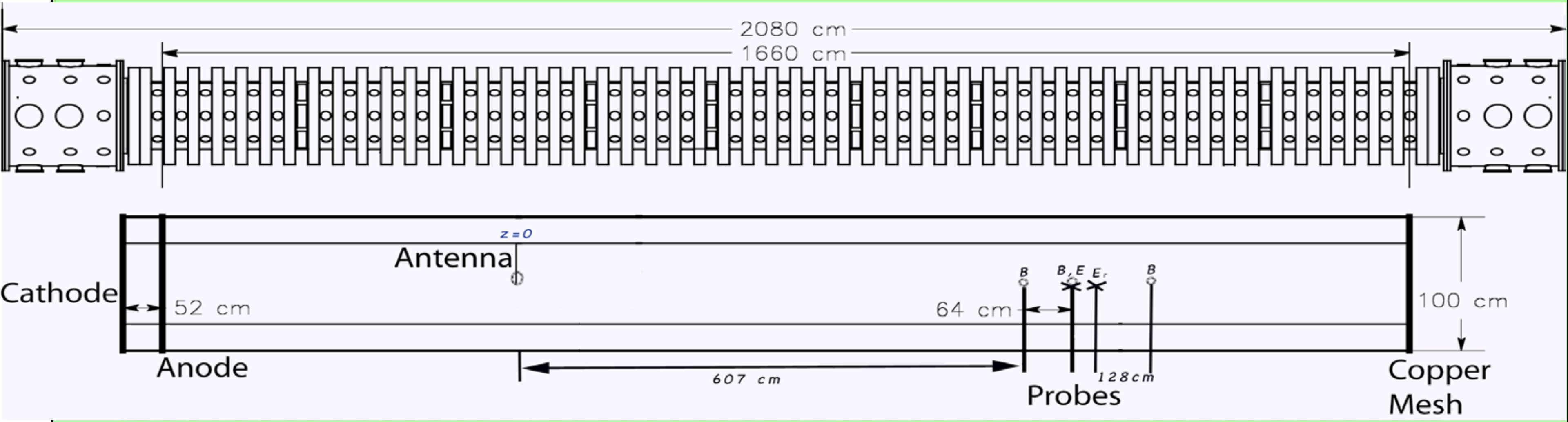
To derive a diffusion coefficient for the pitch angle scattering of relativistic electrons, we first calculate the spatial Fourier transform of the wave magnetic field. Then use *Summers and Thorne* [1992] model

$$D_{\alpha\alpha} = \frac{2\omega_{ce}}{\pi\gamma|k|d}\frac{|B_{01}|^2}{B_0^2} = \frac{2v_0\cos(\alpha)}{\pi d}\frac{|B_{01}|^2}{B_0^2}$$

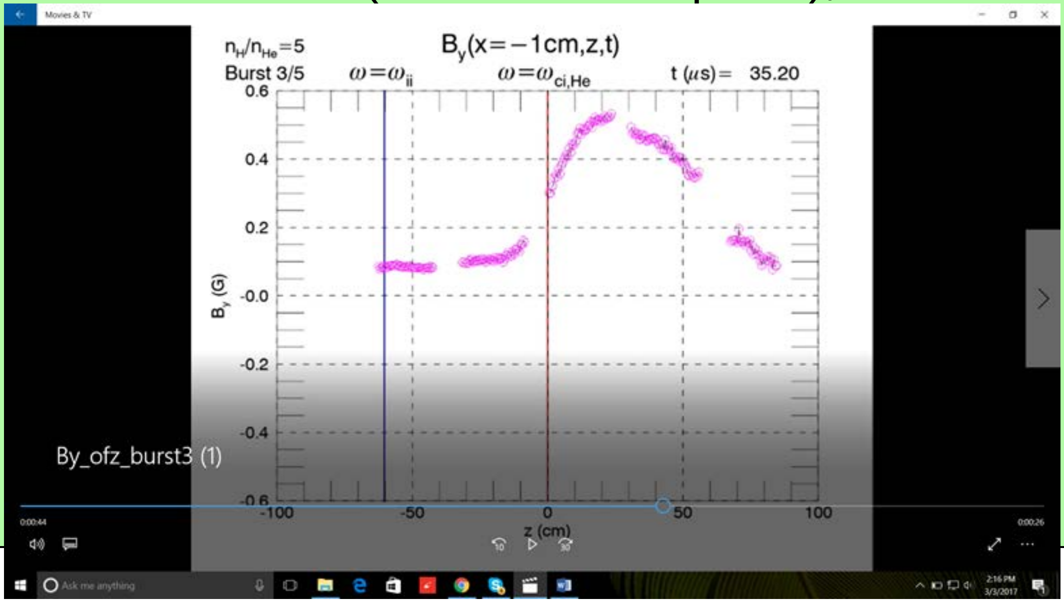


$\kappa_j = \omega_{pj}^2 L/c^2$ and the effective domain length *d*=$v_0\cos(\alpha)\,t_B$. Here t_B is the bounce period.

LAPD Experiment (Preliminary Study)



$B_o(z=0)$=1,000G, $B_o(max)$=1,500G; Helium-hydrogen plasma, n=1.6x10^12 cc.
RMF antenna is at z=500 cm (radiates in x-z plane); f=470 kHz = f_{cHe}



Estimates for the LAPD device

Pitch angle spread

$$\Delta\alpha = \sqrt{\langle D_{\alpha\alpha}\rangle t} \sim \frac{B_{01}}{B_0}$$

For $\frac{B_{01}}{B_0}$**=0.001 it takes 100 back and forth bounces to get a noticeable** $\Delta\alpha \approx 0.1\ \mathrm{rad}.$

A single bounce takes $t_B \approx 2 \times 18/(0.7\ c) = 1.7 \times 10^{-7}\mathrm{s}$**. Hundred bounces require 17 mcsec.**

Conclusions

- **Propagation of EMIC waves near the cyclotron resonances and the pitch angle scattering of relativistic electrons have been studied theoretically.**

- **Cyclotron resonances occur where the wave frequency matches the respective ion cyclotron frequency. Due to rapidly changing wavelength near a cyclotron resonance, a monochromatic EMIC wave gives rise to a broadband spectrum of waves in space, where relativistic electrons are resonantly pitch angle scattered.**

- **Expressions for the pitch angle diffusion coefficient are derived using the spatial profile of the wave magnetic field near an ion cyclotron resonance. Estimates for the life-time of mirror contained relativistic electrons in the LAPD device have been done.**

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