Maximum Likelihood Estimation of Phase Screen Parameters from Ionospheric Scintillation Spectra

Charles S. Carrano, Charles L. Rino, and Keith M. Groves

Boston College, Institute for Scientific Research

INSTITUTE FOR SCIENTIFIC RESEARCH

Ionospheric Effects Symposium   Alexandria, VA • May 2017

Introduction

- At the 2015 IES Meeting, we presented a phase screen solution for the spectrum of intensity scintillations when the refractive index irregularities follow a two-component power law spectrum:

  Carrano, C., and C. Rino (2016), A theory of scintillation for two-component power law irregularity spectra: Overview and numerical results, Radio Sci., 51, 789–813, doi:10.1002/2015RS005903.

- Here, we consider the inverse problem, whereby phase screen parameters are inferred from measured scintillation time series as a means of interpreting them physically.

- We accomplish this by fitting the spectrum of intensity fluctuations with the theoretical model using the Maximum Likelihood (ML) technique.

- We refer to this as Irregularity Parameter Estimation (IPE) since it provides a statistical description of the refractive index irregularities from the scintillations they produce. In this sense, IPE may be thought of as *stochastic* back-propagation.

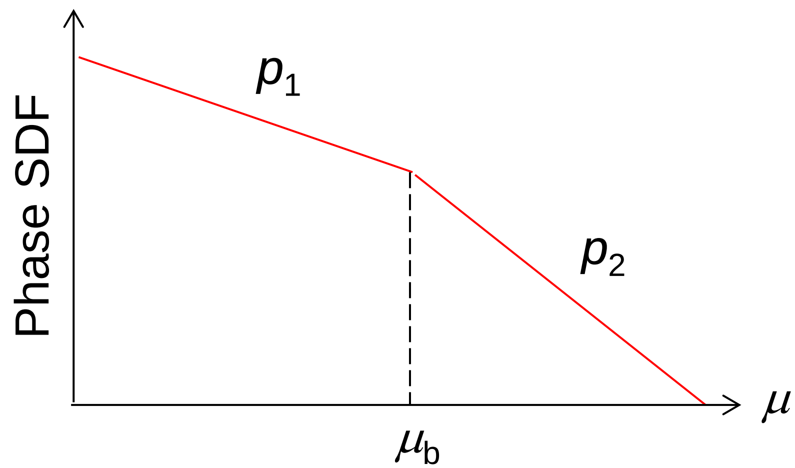
Universal Scaling

- For a piecewise power-law irregularity model, normalizing by the Fresnel scale (ρ_F) casts the problem in dimensionless form.

- Normalized quantities:

  μ – spatial wavenumber
  μ_b, –break scale

  $P(\mu)$ – phase SDF
  $I(\mu)$ – intensity SDF



- The universal scattering strength, U, is defined to be $P(\mu=1)$.
  For weak scatter, U<<1 and for strong scatter U>>1.

- Parameters p_1, p_2, μ_b, and U fully specify all solutions for 2-component spectra (i.e. different combinations of perturbation strength, propagation distance, and frequency produce identical results).

Theoretical Model for Fitting Scintillation Spectra

- Temporal frequencies in the observed scintillation spectra are related to spatial wavenumbers via the so-called effective scan velocity V_{eff} as $\mu = 2\pi f \rho_F / V_{eff}$

- Using this we can express the temporal spectrum of intensity fluctuations as a function of phase screen parameters p_1, p_2, μ_b, and U, and Fresnel frequency $f_F = V_{eff} / \rho_F$:

$$\boxed{I(f;U,p_1,p_2,\mu_b,f_F) = 2\int_0^{\infty} \exp\left[-\gamma\left(\eta, \frac{2\pi f}{f_F}; U, p_1, p_2, \mu_b\right)\right] \cos\left(\frac{2\pi f \eta}{f_F}\right) d\eta}$$

where γ is the so-called structure interaction function (see Carrano et al., 2016).

- This theoretical model lacks a direct dependence on the propagation geometry and even on Fresnel scale! In particular, data can be fit without knowing distance to the screen.

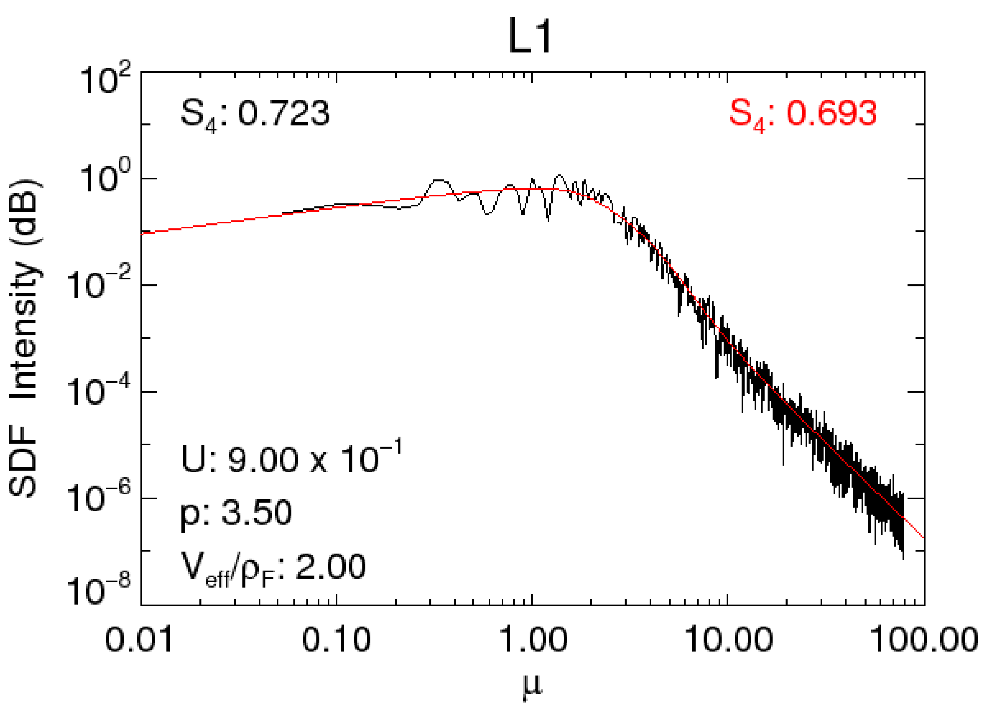
- Model is valid in weak and strong scatter conditions for transverse scans to the extent that a 1D phase screen model adequately describes the propagation physics.

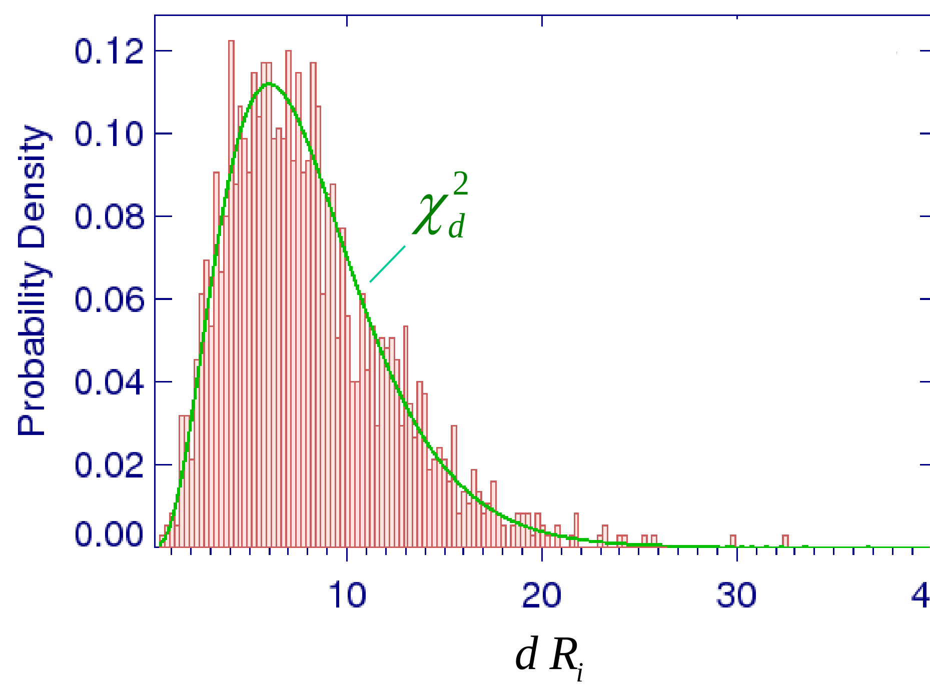
The Spectral Ratio and its Distribution

- Consider $R_i = I^m(f_i) / I(f_i;\theta)$ as a random variable where $\theta = (U, p_1, p_2, \mu_b, f_F)$
- For a perfect model R_i will follow a chi-squared distribution of order d

$$R_i \sim \chi_d^2 / d,$$ where *d* is number of periodograms averaged together

- This assumption looks quite good for simulated scintillation data





Maximum Likelihood Estimation

- The probability of measuring one harmonic $I_i^m = I^m(f_i)$ given the model is

$$p(I_i^m \mid I_i) \sim \frac{d}{I_i} \chi_d^2 \left(d \frac{I_i^m}{I_i} \right)$$

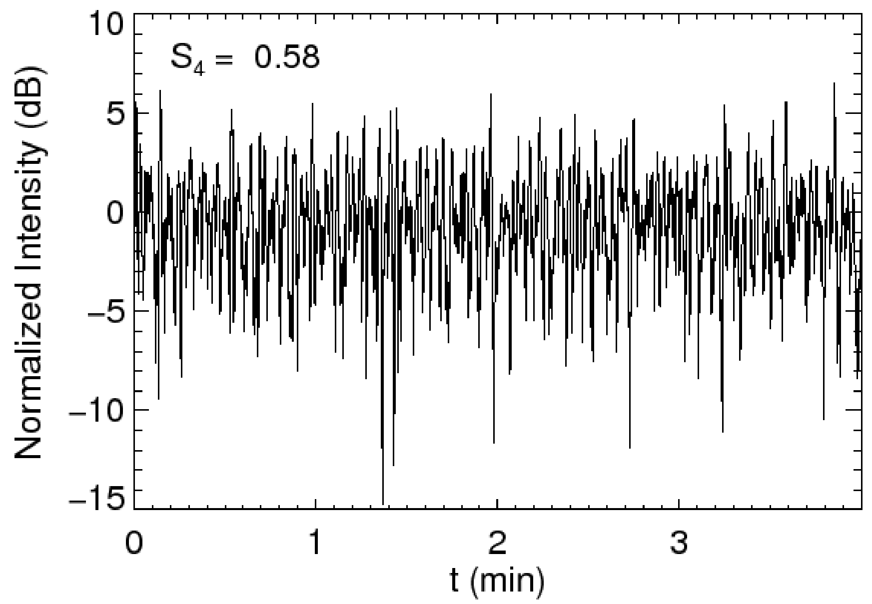
- The probability of measuring the entire spectrum given the model is

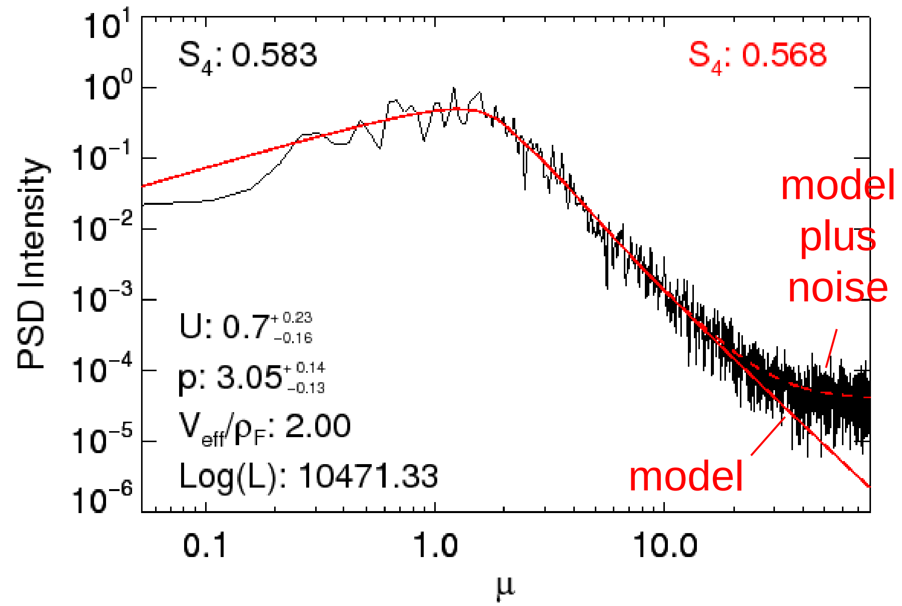
$$p(I^m \mid I) \sim \prod_{i=1}^{N} \frac{d}{I_i} \chi_d^2 \left(d \frac{I_i^m}{I_i} \right)$$

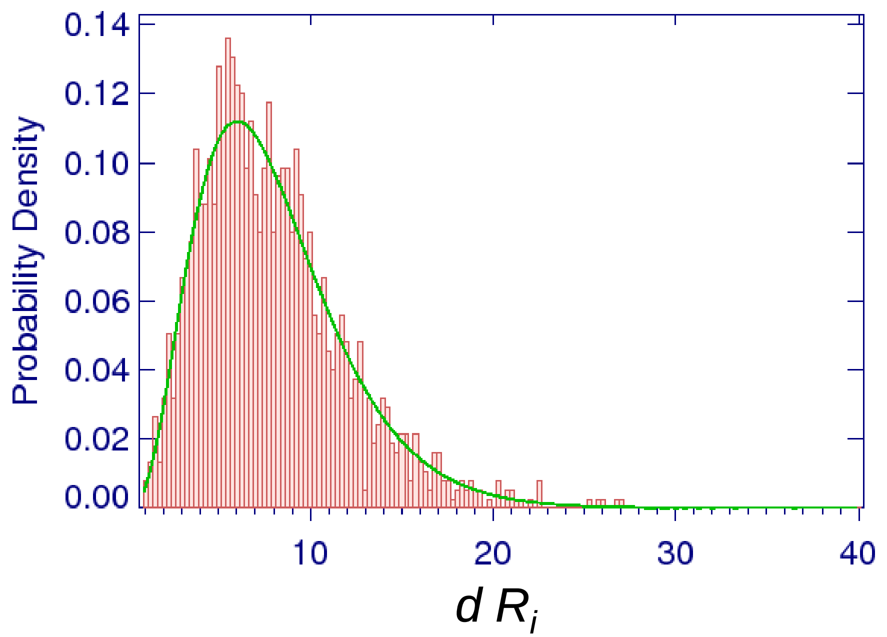
- Likelihood of the parameters given the measurements: $L(\theta \mid I^m) \equiv p(I^m \mid I(\theta))$

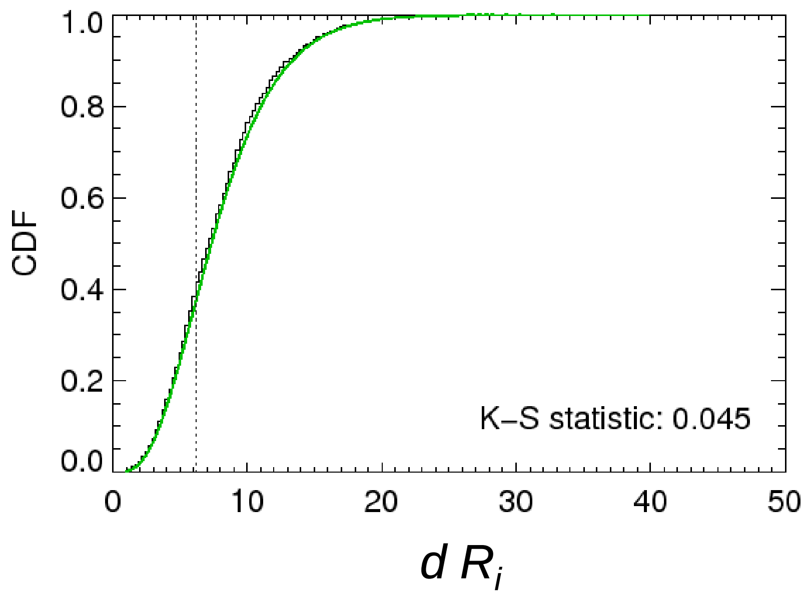
- Goal is to find estimate $\hat{\theta}$ that maximizes $\mathrm{Log}(L) \sim \sum_{i=1}^{N} \frac{d}{I_i} \chi_d^2 \left(d \frac{I_i^m}{I_i} \right)$

Fitting Simulated Intensity Spectrum Plus Noise (p=3.0)



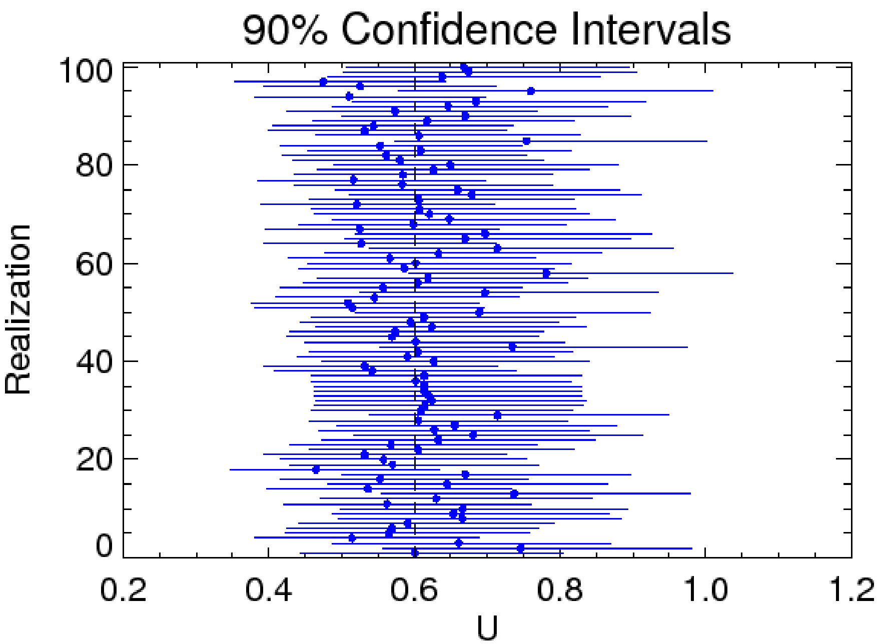


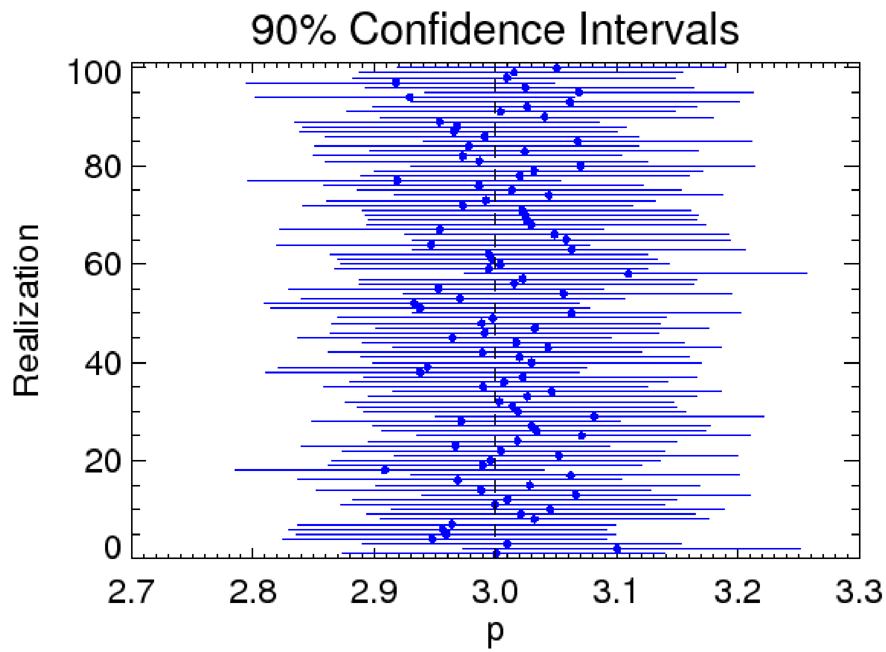


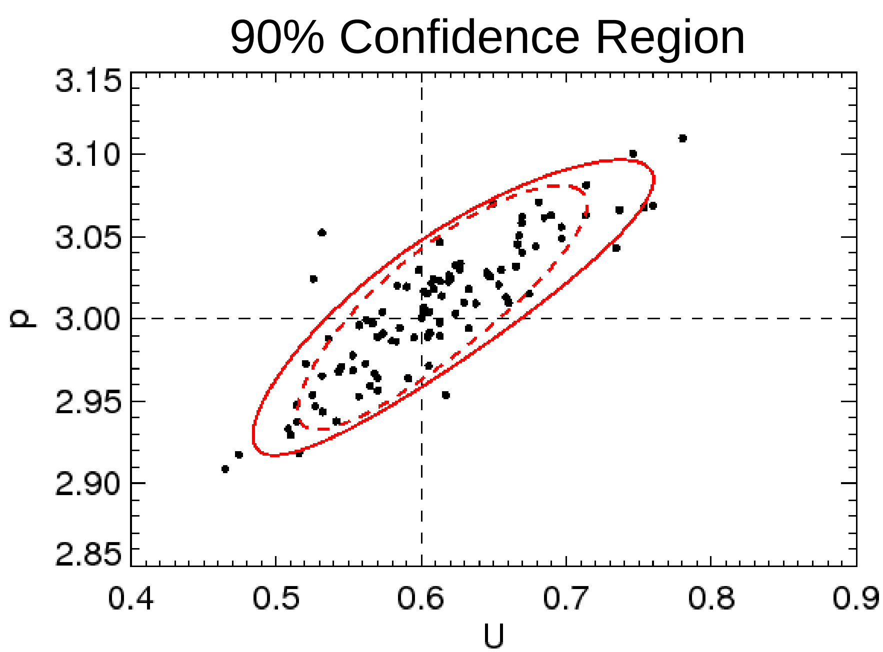


100 Monte Carlo Simulations (p=3.0)

Dashed lines indicate Truth

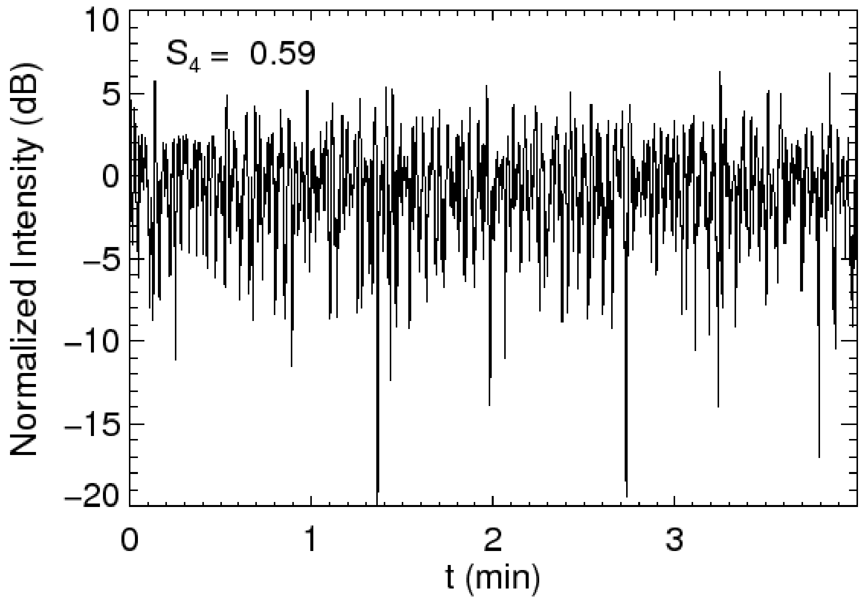


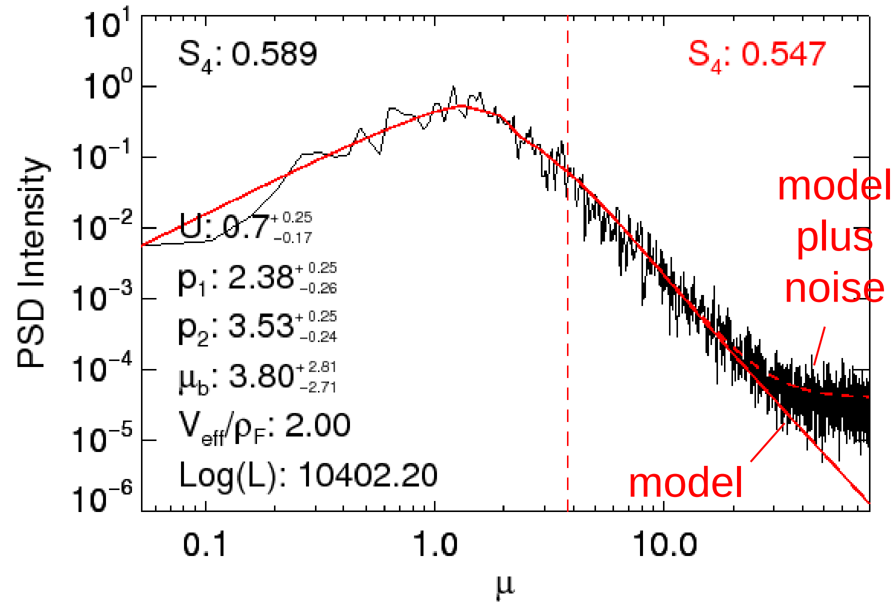


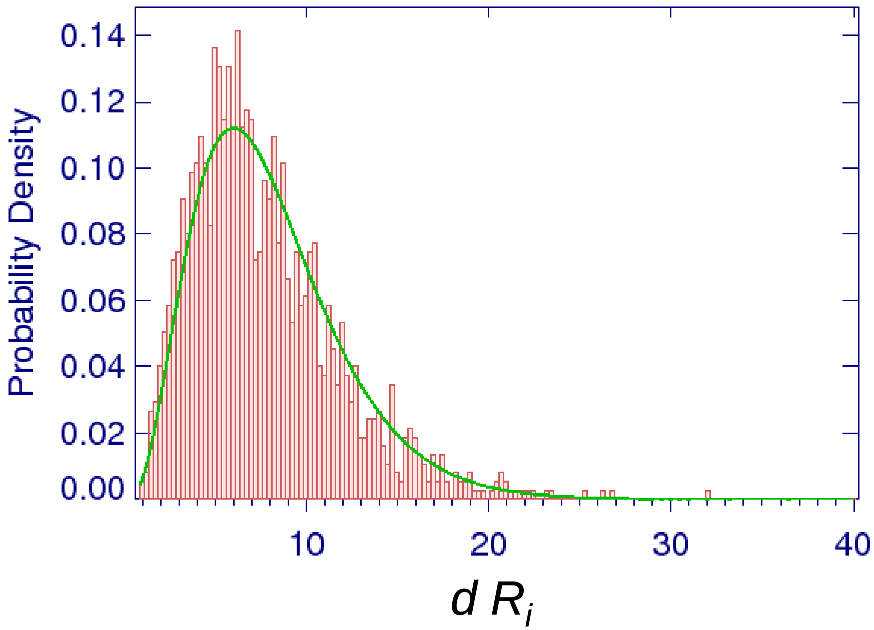


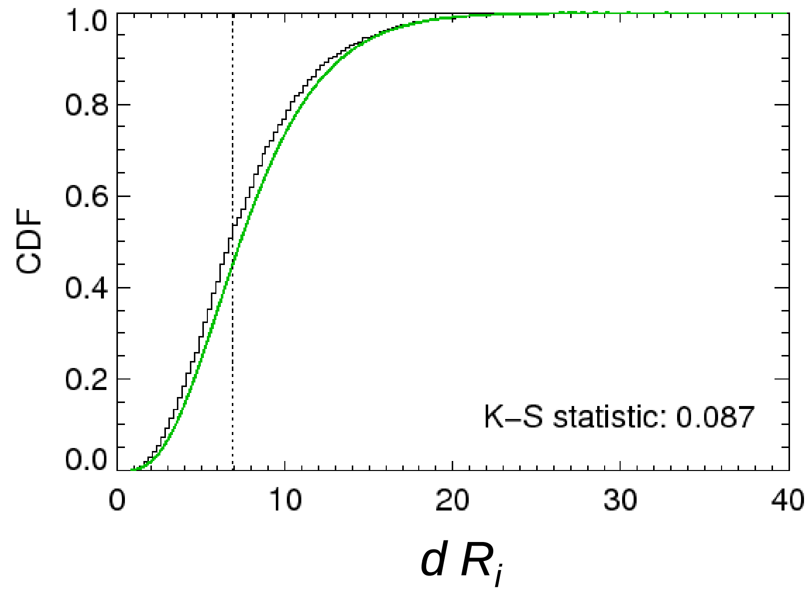
Solid curve is covariance ellipse of Monte Carlo results; dashed is ellipse from MLE

Fitting Simulated Intensity Spectrum Plus Noise
(p_1=2.5, p_2=3.5, μ_b = 5)



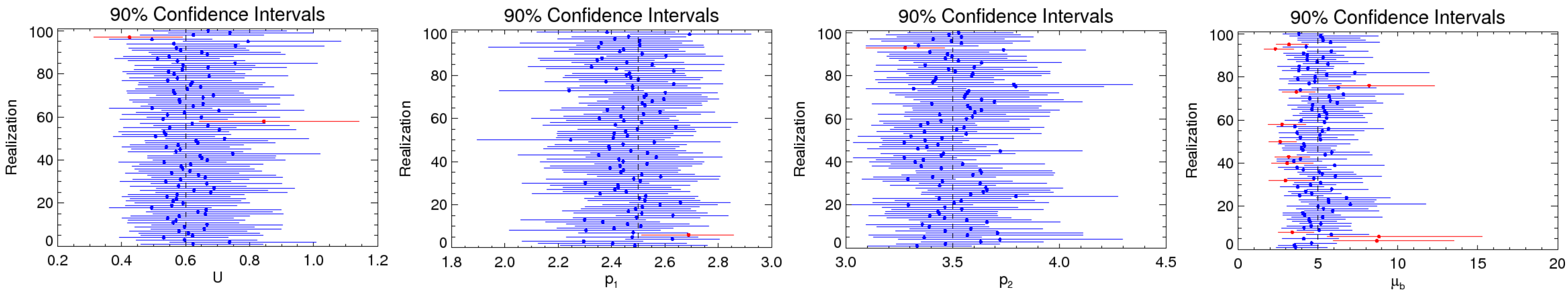




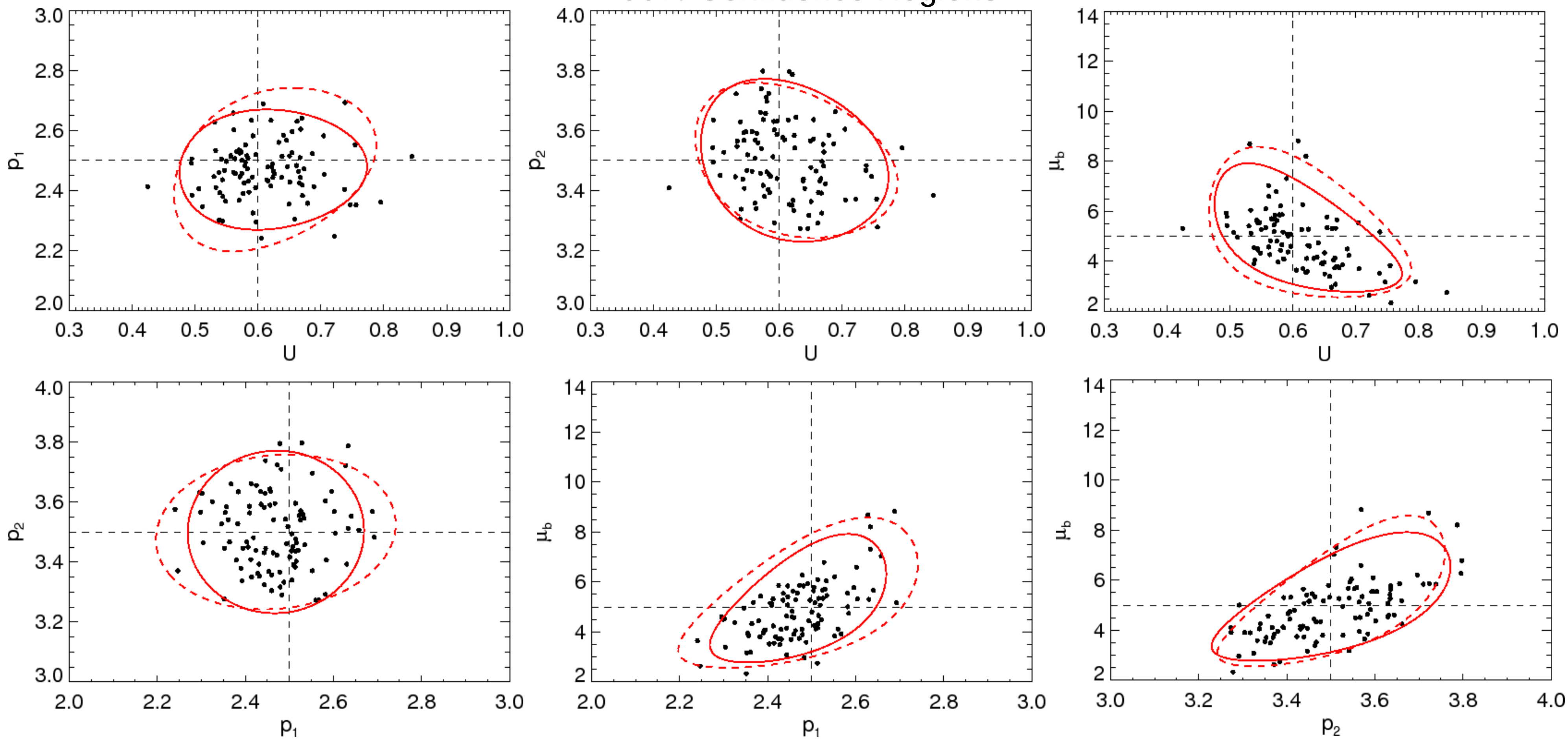


100 Monte Carlo Simulations

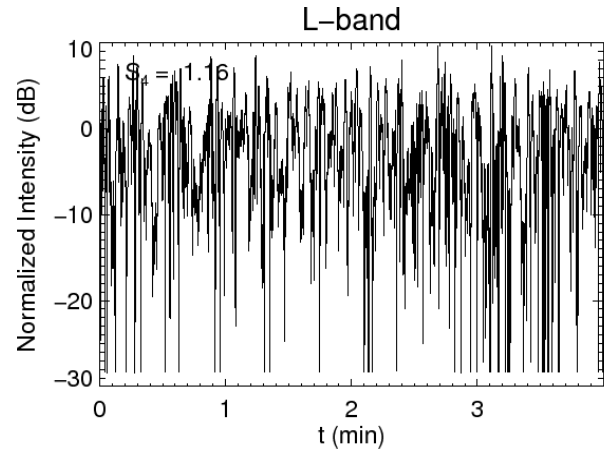
(p_1=2.5, p_2=3.5, μ_b = 5)



90% Confidence Regions



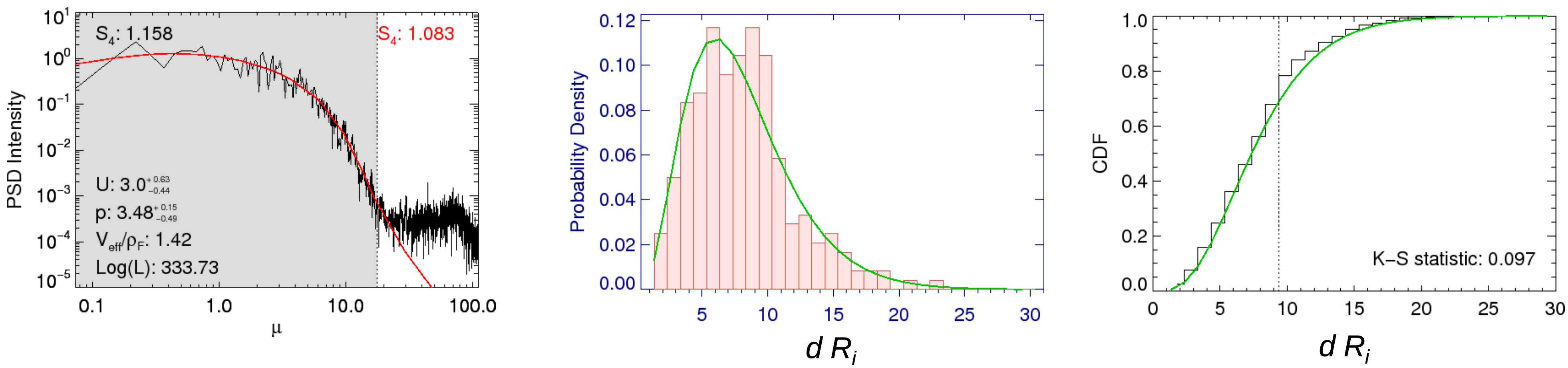
Fitting Actual Scintillation Data (Example 1)

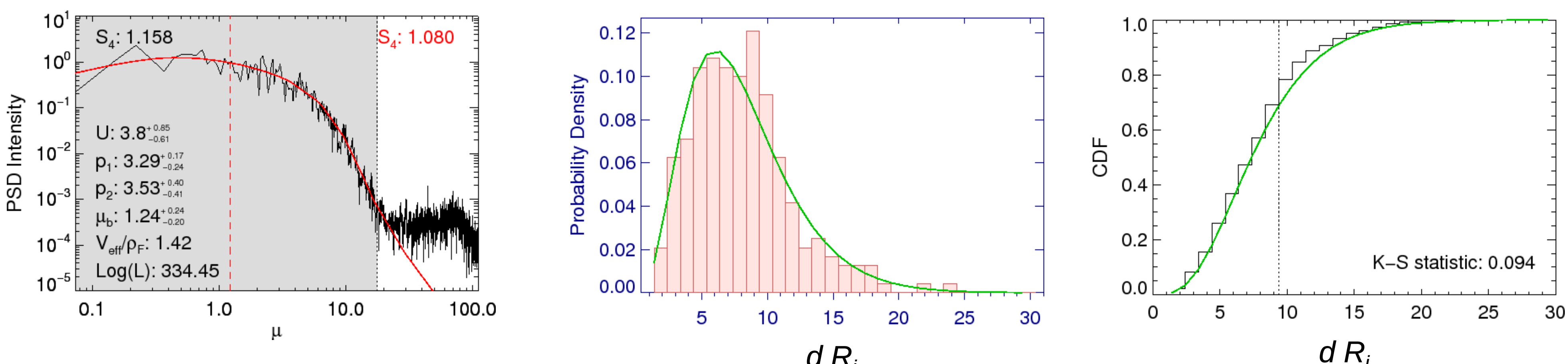


Akaike Information Criterion: $AIC = -2\ \mathrm{Log}(L) + 2k$

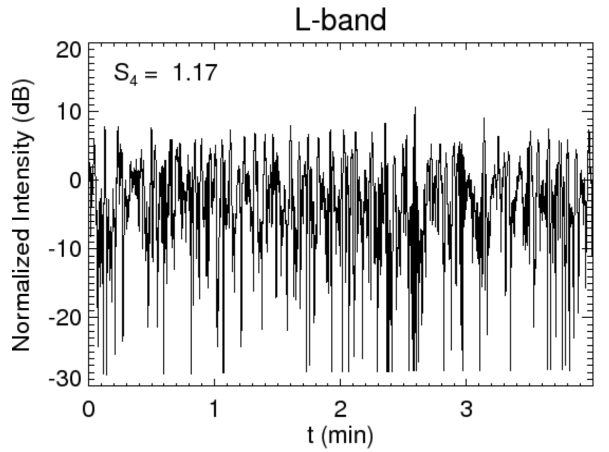
AIC one-component model: -663.45
AIC two-component model : -660.91

Conclusion: this is likely a one-component spectrum





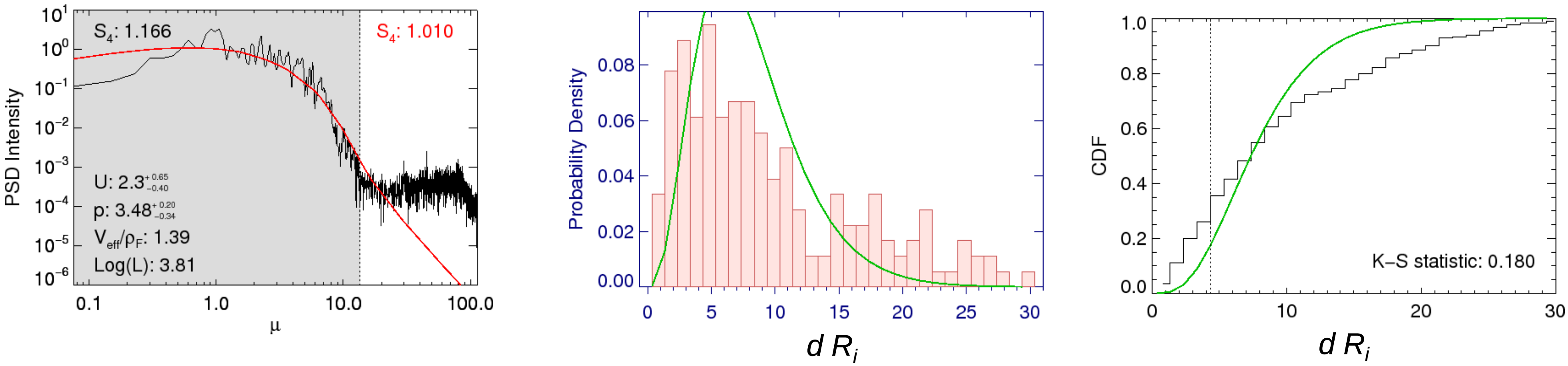
Fitting Actual Scintillation Data (Example 2)

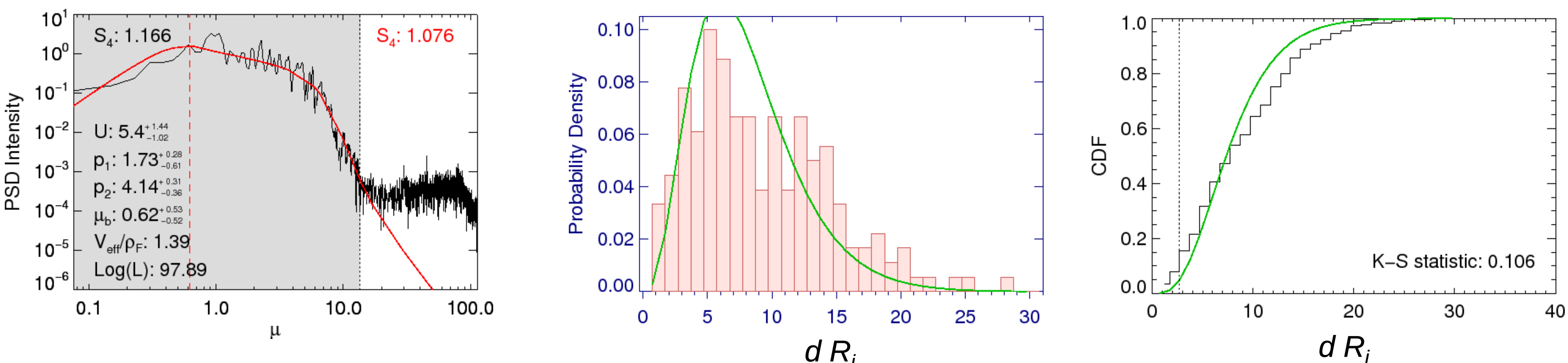


Akaike Information Criterion: $AIC = -2\,\mathrm{Log}(L) + 2k$

AIC one-component model: -3.61
AIC two-component model : -187.78

Conclusion: this is likely a two-component spectrum





Conclusions

- We introduce Irregularity Parameter Estimation (IPE) for inferring phase screen parameters from scintillation observations using the Maximum Likelihood Method.
- We have used Monte-Carlo simulation to demonstrate that the method gives nearly unbiased results with useful confidence intervals (perhaps a little overly-conservative).
- The one- and two-component phase screen models are nested; we use the Akaike Information Criterion (AIC) for hypothesis testing.
- Caveat: MLE estimates and confidences are not meaningful if model is invalid
  - A 1D phase screen model is strictly applicable only for cross-field scans
  - Theoretical model should depend uniquely on the parameters *over measurement range*. As scattering strength increases, spectrum becomes flat with Gaussian-like high frequency roll off—similar shape for different screen parameters