

Maximum Likelihood Estimation of Phase Screen Parameters from Ionospheric Scintillation Spectra

At the previous Ionospheric Effects Symposium, we presented an extension of the phase screen theory of ionospheric scintillation that accounts for the case where the refractive index irregularities follow a two-component power law spectrum [1]. A specific normalization was invoked to achieve a universal scaling, such that different combinations of perturbation strength, propagation distance, and frequency produce the same results. The theory is equally valid in the weak, intermediate, and strong scatter regimes. In this paper we consider the inverse problem, whereby phase screen parameters are inferred from measured scintillation time series as a means of interpreting them physically [1, 2]. The screen parameters are obtained by fitting the spectral density function (SDF) of intensity fluctuations with the theoretical model using the Maximum Likelihood (ML) technique. We refer to this fitting procedure as Irregularity Parameter Estimation (IPE) since it provides a statistical description of the refractive index irregularities from the scintillations they produce. In this sense, IPE may be thought of as a robust form of back-propagation. In this paper, we introduce an additional rescaling that enables IPE to be applied without a-priori knowledge of the location of the irregularities or their motion.

Previous applications of IPE have been implemented by minimizing the difference between measured and theoretical intensity spectral density functions (SDFs) in a least-squares sense. While accurate estimates of the screen parameters may be obtained in this way, the estimates are biased because the errors are not normally distributed. We show (empirically) that the errors follow a chi-squared distribution of order $2M$, where M is the number of periodograms averaged together to estimate the SDF. In the limit of large M , the errors become normally distributed and the least-squares and ML techniques become equivalent. By using the known distribution of the errors, the ML technique can provide unbiased estimates of the screen parameters even when fitting a single un-averaged periodogram. This is a substantial benefit when fitting scintillation spectra, because averaging removes low-frequency content that can help to discriminate the fit. A parametric model of receiver noise is used to mitigate its influence on the screen parameter estimates. Monte-Carlo simulations are used to characterize the probability distribution of the errors and provide confidence values for the screen parameters in terms of contours of the likelihood function.

The one- and two-component power law irregularity models are nested, since the latter contains the former as a special case. Therefore, the likelihood will necessarily be equal or larger when fitting with the more general model. We show an example where the Akaike Information Criterion (AIC) justifies the need for a two-component irregularity model. We believe this is the first time evidence for a two-component irregularity spectrum has been rigorously substantiated using ground-based ionospheric scintillation measurements. Next, we use IPE to confirm that the theory captures the correct frequency dependence of scintillation by fitting VHF and L-band intensity time series simultaneously. Finally, we

demonstrate how IPE may be used to infer the distance to scintillation-causing irregularities along a radio-occultation ray-path.

References

1. Carrano, C., and C. Rino (2016), A theory of scintillation for two-component power law irregularity spectra: Overview and numerical results, *Radio Sci.*, 51, 789–813, doi:10.1002/2015RS005903.
2. Carrano, C. and C. Rino (2016), Constrained and unconstrained power law irregularity models for interpreting strong scintillation data, *International Beacon Satellite Symposium BSS-2016*, Trieste, Italy, 27 June – 1 July, 2016.