

# Method and Validation for Determining Hooke TID Parameters from GNSS Data

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# Medium Scale TID

Estimate Medium Scale TIDs (MS-TID) using GNSS data

- Multiple types of MS-TIDs

- Acoustic Gravity Wave (AGW) – physical model by Hooke, 1967
- We determine amplitude and phase phenomenologically

$$N'_e = AN_e^0 \sin(I) \exp(k_{zi}(z - z_0)) \omega^{-1} \sqrt{\left(\frac{1}{N_e^0} \partial_z N_e^0 + k_{zi}\right)^2 + \left(\frac{k_{br}}{\sin(I)}\right)^2} \cos \left[ \omega \Delta t - \mathbf{k}_r \cdot \Delta \mathbf{x} + \frac{\pi}{2} - \arctan \left( \frac{k_{br}}{\sin(I) \left( \frac{1}{N_e^0} \partial_z N_e^0 + k_{zi} \right)} \right) + \phi \right]$$

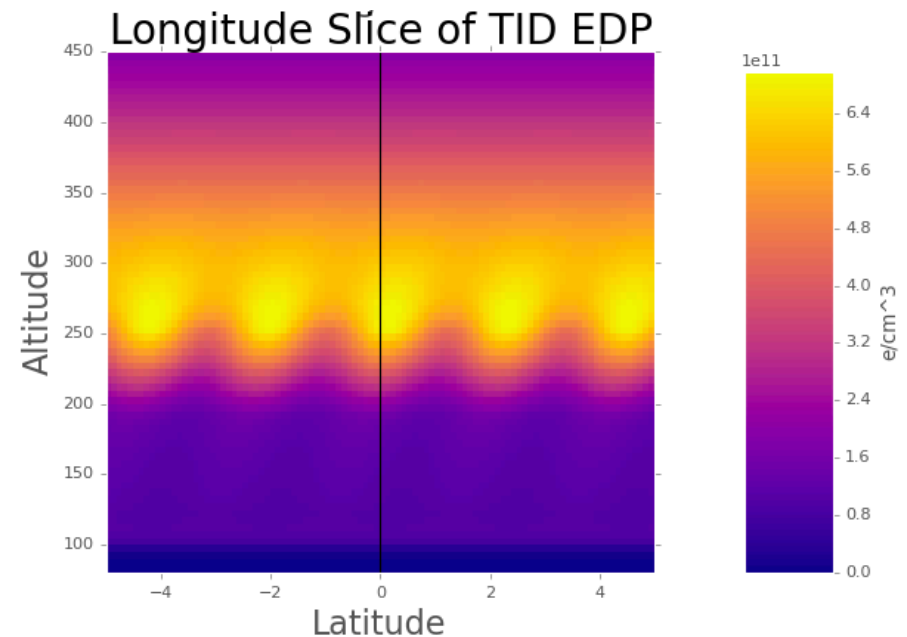
- 'Non-classical' electrodynamics-driven TIDs

- Define MS-TID:

- Period:  $5 < T < 45$  minutes
- Velocity:  $100 < v < 400$  m/s

- TID parameters to estimate

- $k_E$ : wave vector east component
- $k_N$ : wave vector north component
- $k_z$ : wave vector vertical component
- $\omega$ : angular frequency
- A: amplitude
- $t_0$ : phase zero (time, alt, lat, lon)



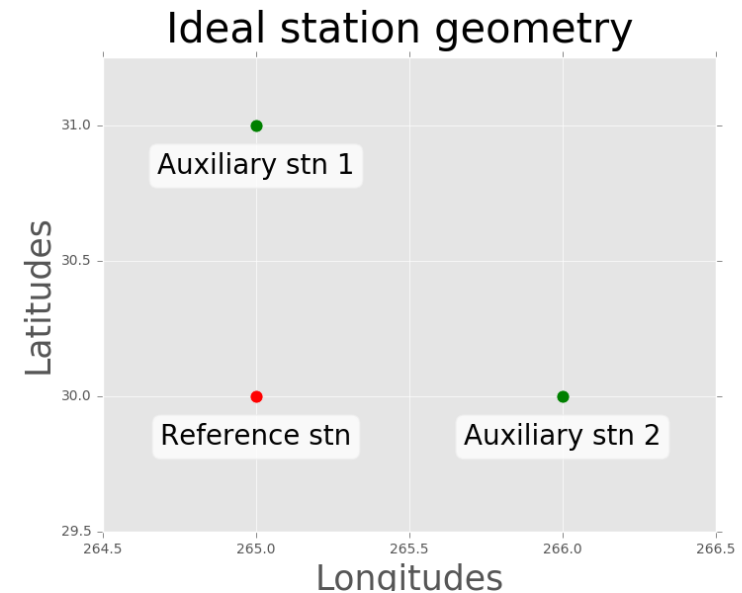
# TID parameter estimation - Input data

## 1. GNSS data

- L1-L2 geometry-free phase or STEC
- Minimum of 3 receivers in a non-collinear arrangement, optimally orthogonal legs
  - Separations 30-75 km
  - Separation ratio max of 2
  - Opening angle 75°-105°
- Minimum pass length 90 min. (Nyquist)
- Satellite elevation angle  $>45^\circ$

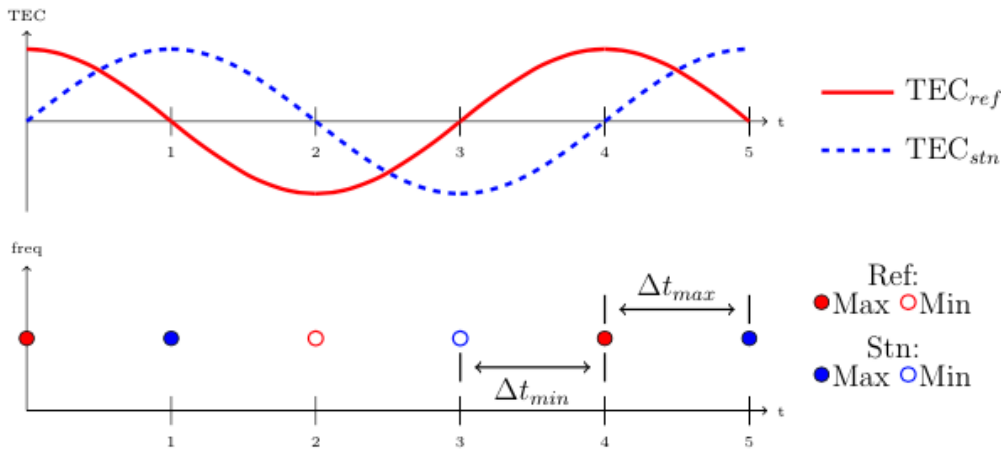
## 2. Ionospheric model

- Full 3D EDP
  - Used to normalize Amplitude
- Need  $yF_2$  and  $h_mF_2$ 
  - $yF_2$  used as  $k_{zi}$  (scale height in Hooke model)
  - $h_mF_2$  used as reference altitude (semi-arbitrary)



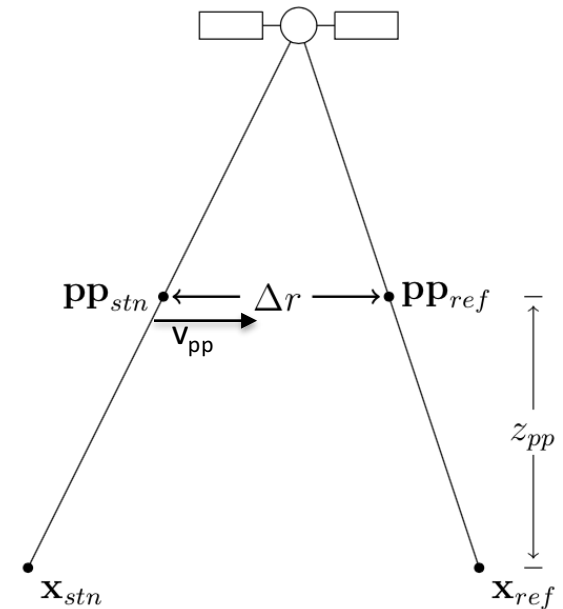
# TID parameter estimation – Pajares defined parameters

- Method based on: “*Medium-scale traveling ionospheric disturbances affecting GPS measurements: Spatial and temporal Analysis*”, M. Hernández-Pajares, J.M. Juan, J. Sanz, JGR **111** A07S11, 2006.
- Defines how to calculate k-vector and frequency
  - Uses STEC measurements from 3 GNSS receivers
  - Correlates bandpass-detrended STEC data to obtain time delays
  - We assume  $k_{zr} = 0$  and use iono scale height  $yF2$  for  $k_{zi}$
- Variables required to solve for k-vector,  $\omega$



$\Delta t_{max}$ : Time of maximum correlation  
 $\Delta t_{min}$ : Time of maximum anti-correlation

$\Delta r_{pp}$ : Ionosphere Pierce Point (IPP) separation  
 $v_{pp}$ : IPP velocity



# TID parameter estimation – Pajares defined parameters

- K-vector linear equations:

$$\Delta t_{max1} = (\Delta \mathbf{r}_{pp1,E} + \Delta t_{max1} \cdot \mathbf{v}_{pp1,E}) \cdot \mathbf{s}_E + (\Delta \mathbf{r}_{pp1,N} + \Delta t_{max1} \cdot \mathbf{v}_{pp1,N}) \cdot \mathbf{s}_N$$

$$\Delta t_{max2} = (\Delta \mathbf{r}_{pp2,E} + \Delta t_{max2} \cdot \mathbf{v}_{pp2,E}) \cdot \mathbf{s}_E + (\Delta \mathbf{r}_{pp2,N} + \Delta t_{max2} \cdot \mathbf{v}_{pp2,N}) \cdot \mathbf{s}_N$$

- Subscript 1 (2) indicates reference to 1<sup>st</sup> (2<sup>nd</sup>) auxiliary station

- Angular frequency equation:

$$\omega = \frac{\pi}{|\Delta t_{min} - \Delta t_{max}| \cdot (1 - \mathbf{s} \cdot \mathbf{v}_{pp})}$$

- We compute omega for velocity of each auxiliary station's IPP
- Final  $\omega$  is the average of the two individual values

- Connect slowness vector and k-vector

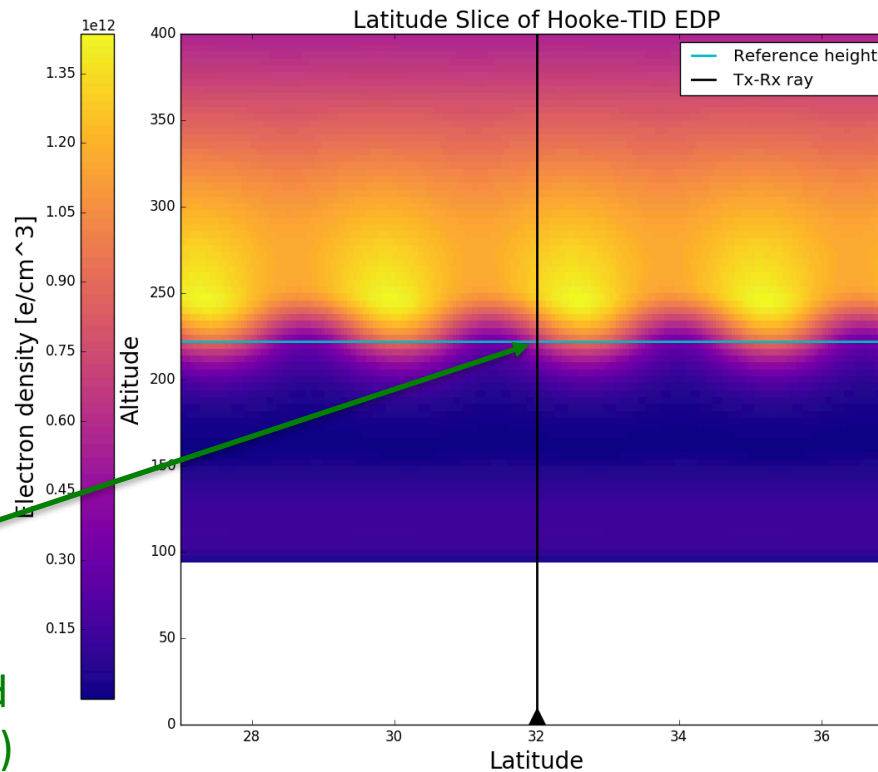
$$\mathbf{k} = \mathbf{s} \cdot \omega$$



# TID parameter estimation – ARL parameters

## ARL:UT method to obtain Phase

- Determined observationally from GFP or STEC data
- Locate null (effective zero crossing) in data



Hooke input:  
Phase zero at time  $t_0$  and  
IPP location ( $z_0, lat_0, lon_0$ )



# TID parameter estimation – ARL parameters

## ARL:UT method to obtain Amplitude

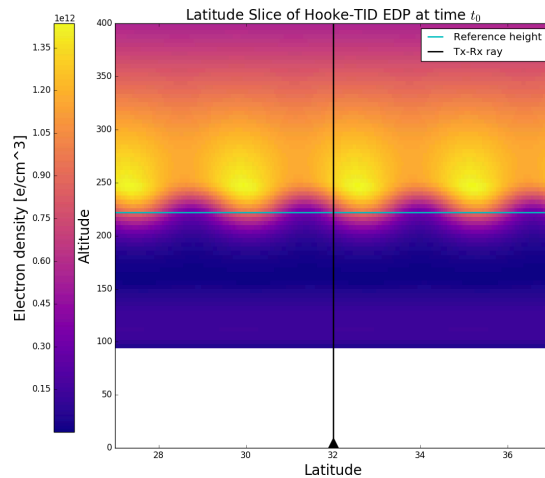
- Dependent on other variables (k-vector,  $\omega$ ,  $\phi_0$ )
- Uses steady-state ionosphere EDP (best available) to normalize TID amplitude
- STEC value definition:  $S(\mathbf{x}, t) = S^0(\mathbf{x}, t) + \beta = \int_s N(\mathbf{x}, t) ds + \beta$
- Hooke electron density perturbation:

$$N(\mathbf{x}, t) = N_e^0(\mathbf{x}, t) + N_e'(\mathbf{x}, t) = N_e^0(\mathbf{x}, t) + A(z_0)N_e''(\mathbf{x}, t)$$

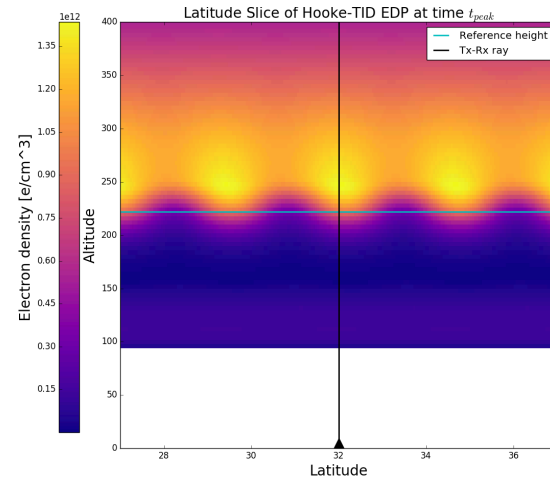
- For two time points, we can solve the matrix

$$\begin{bmatrix} S(\mathbf{x}_1, t_1) - \int_s N_e^0(\mathbf{x}_1, t_1) ds \\ S(\mathbf{x}_2, t_2) - \int_s N_e^0(\mathbf{x}_2, t_2) ds \end{bmatrix} = \begin{bmatrix} \int_s N_e''(\mathbf{x}_1, t_1) ds & 1 \\ \int_s N_e''(\mathbf{x}_2, t_2) ds & 1 \end{bmatrix} \times \begin{bmatrix} A \\ \beta \end{bmatrix}$$

$t_1$  taken  
at  $\phi_0$



$t_2$  taken  
at peak

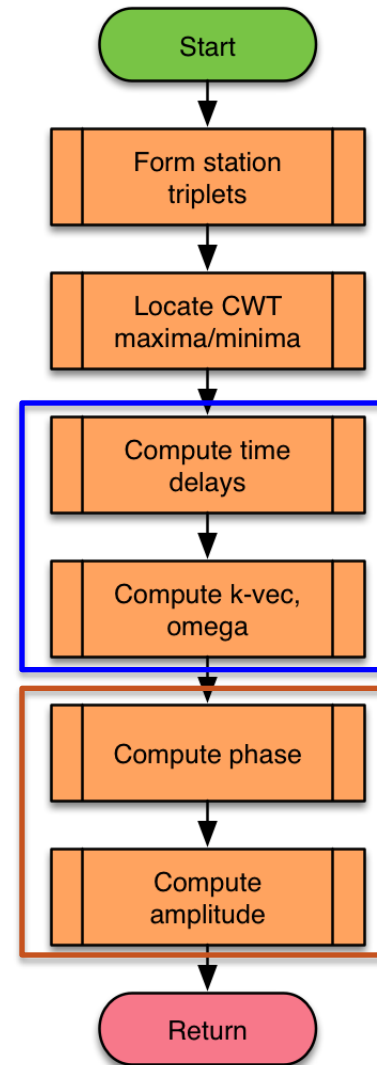


# TID parameter estimation – High-level algorithm

- Typical GNSS data rate: 5s
  - We interpolate to 1s
- Compute continuous wavelet transformation (CWT)

Pajares methods

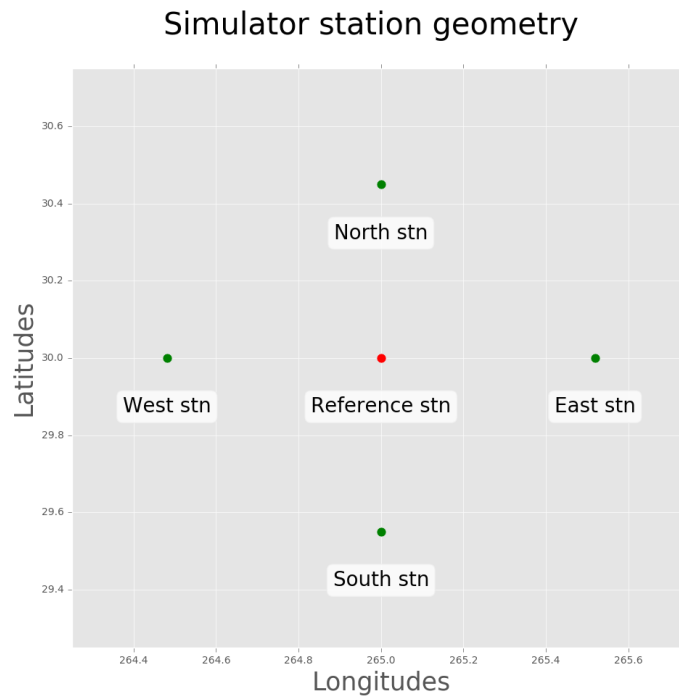
ARL:UT methods





# TID parameter estimation - Simulator

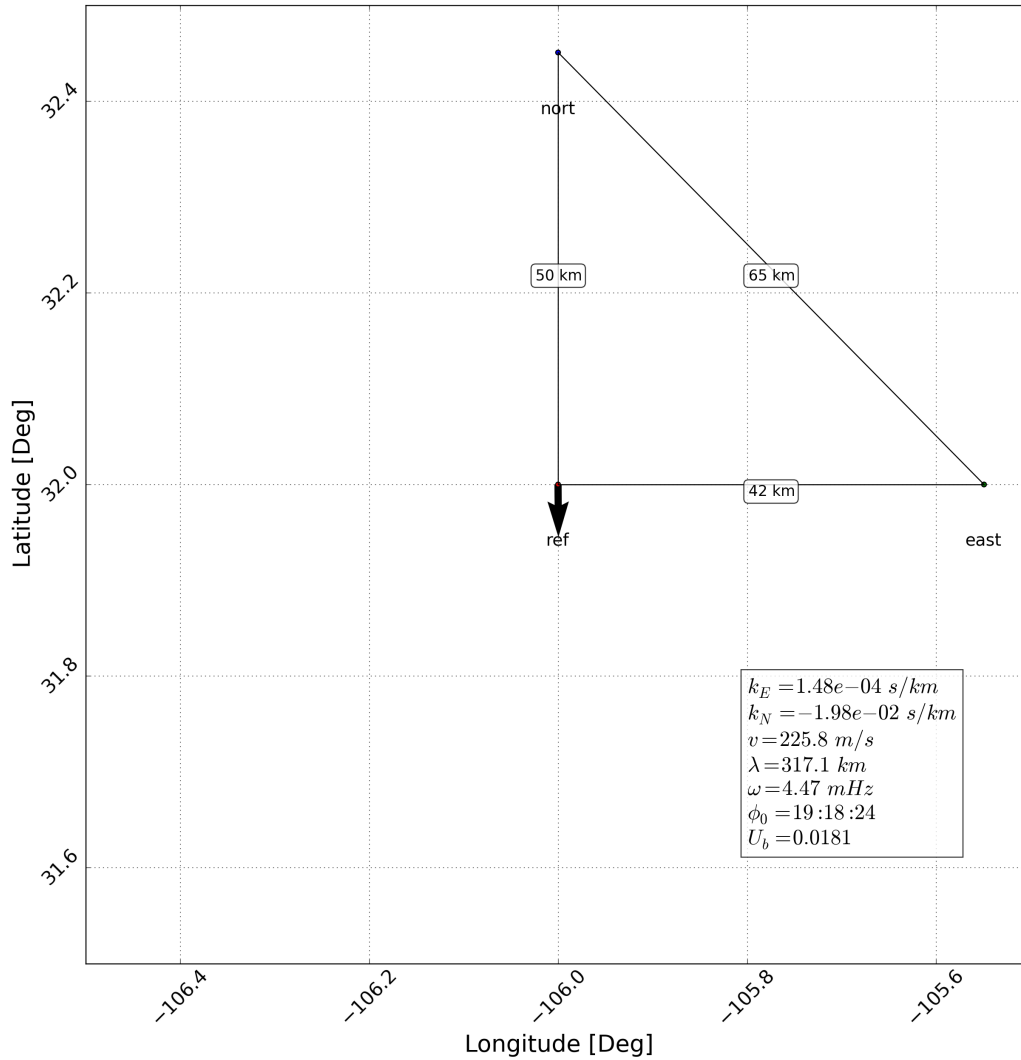
- The GNSS data simulator
  - Hooke TID
  - Ideal station geometry
  - Stationary or moving satellites
  - Strider (ARL:UT Jones-Stephenson type) 3D full-physics ray tracer



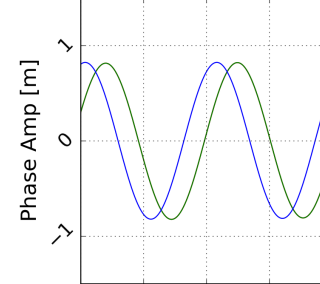
# TID parameter estimation – Sample output

## Simulator results

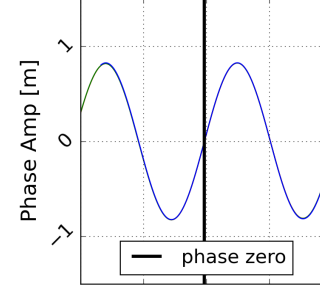
Ref station: ref, PRN 90, 2014-01-26 19:19:00 UTC



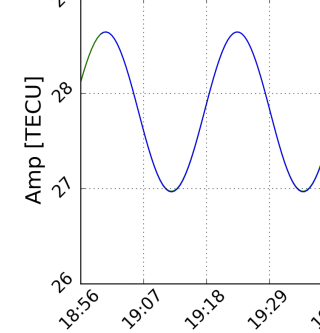
Detrended phase data



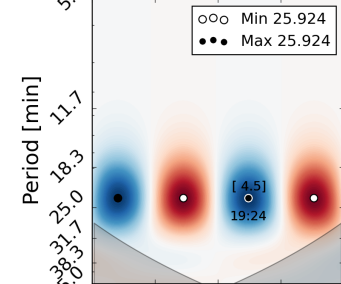
Time shifted phase data



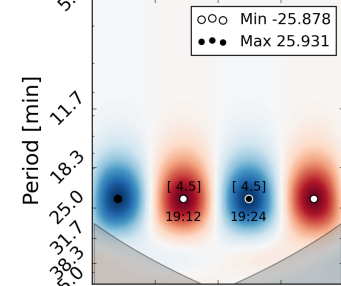
Time shifted Raw TEC



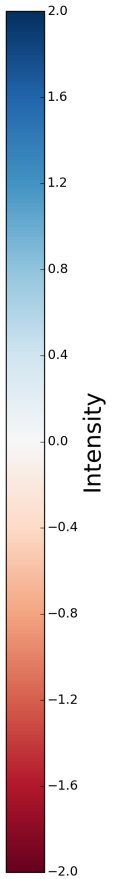
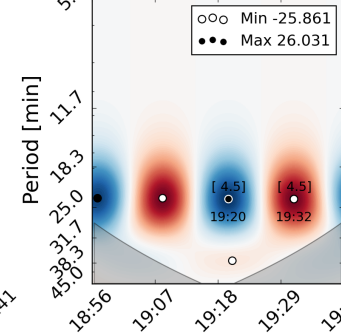
CWT stn: ref



CWT stn: east



CWT stn: nort



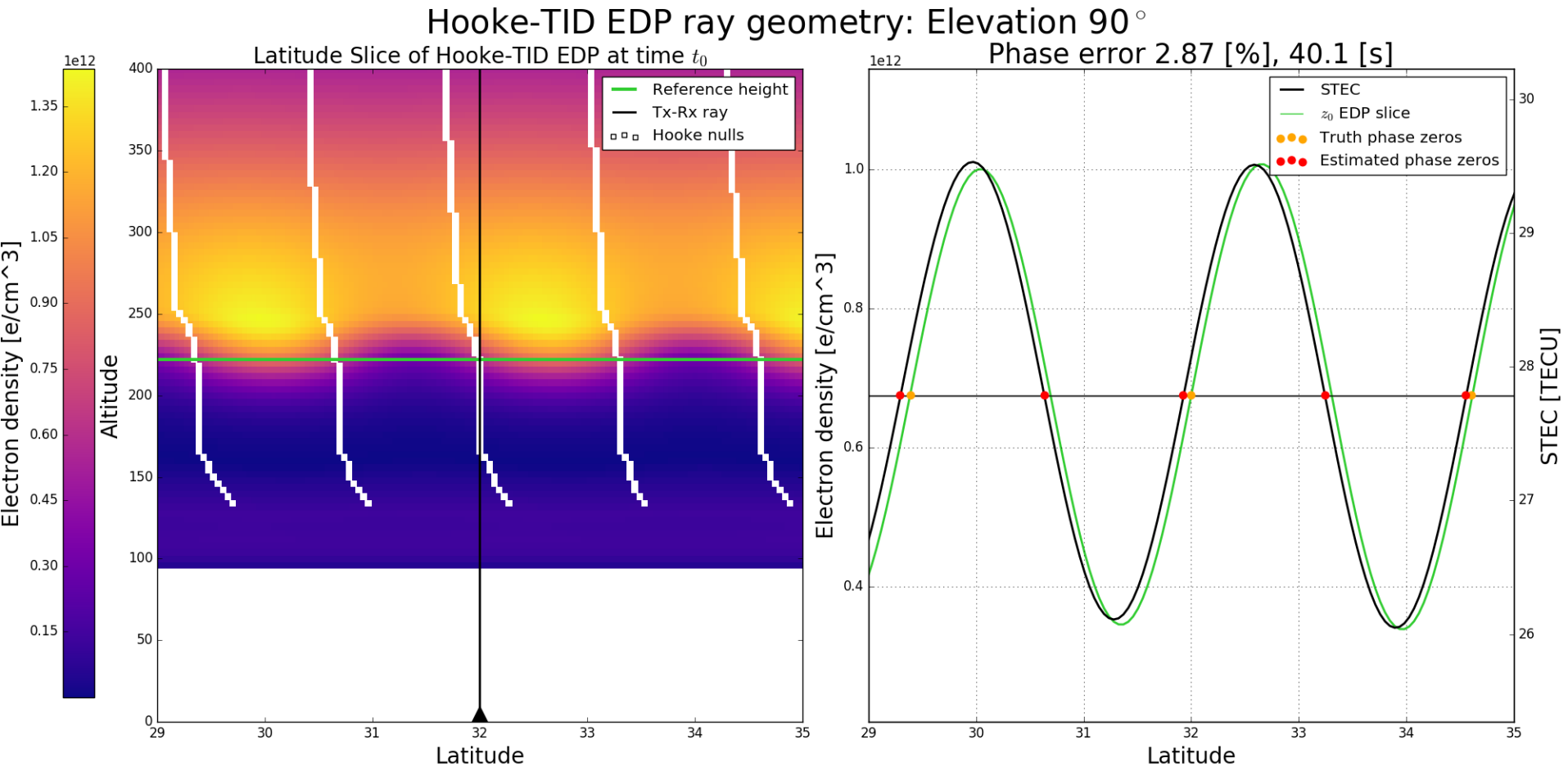
## Algorithmic error due to look angle (El/Az)

- Eliminate satellite motion effects by using non-moving GNSS satellites
  - This yields a IPP velocity of zero; simplifies the Pajares k-vector equation
  - Done to disassociate geometric effects from satellite motion effects
- Employ our Hooke TID model to simulate GNSS data
- Spherically symmetric ionosphere formed with single IRI vertical profile
- Use same data rate discretization to mimic binning limitations
  - Generate data at 5s cadence (typical GNSS data rate)
  - Interpolate to 1s



# Geometric error analysis

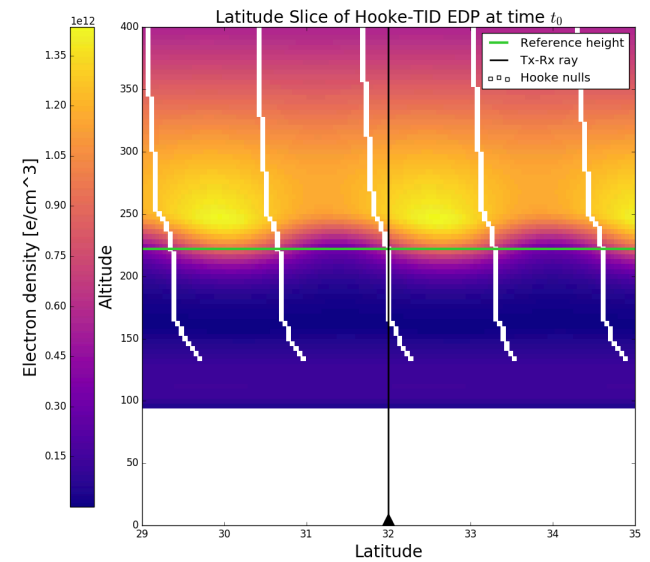
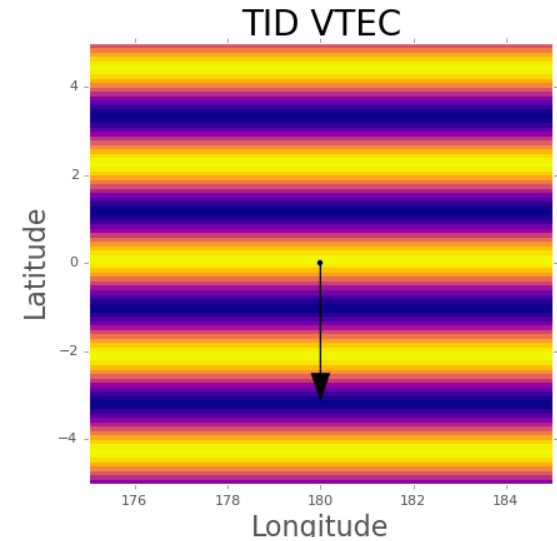
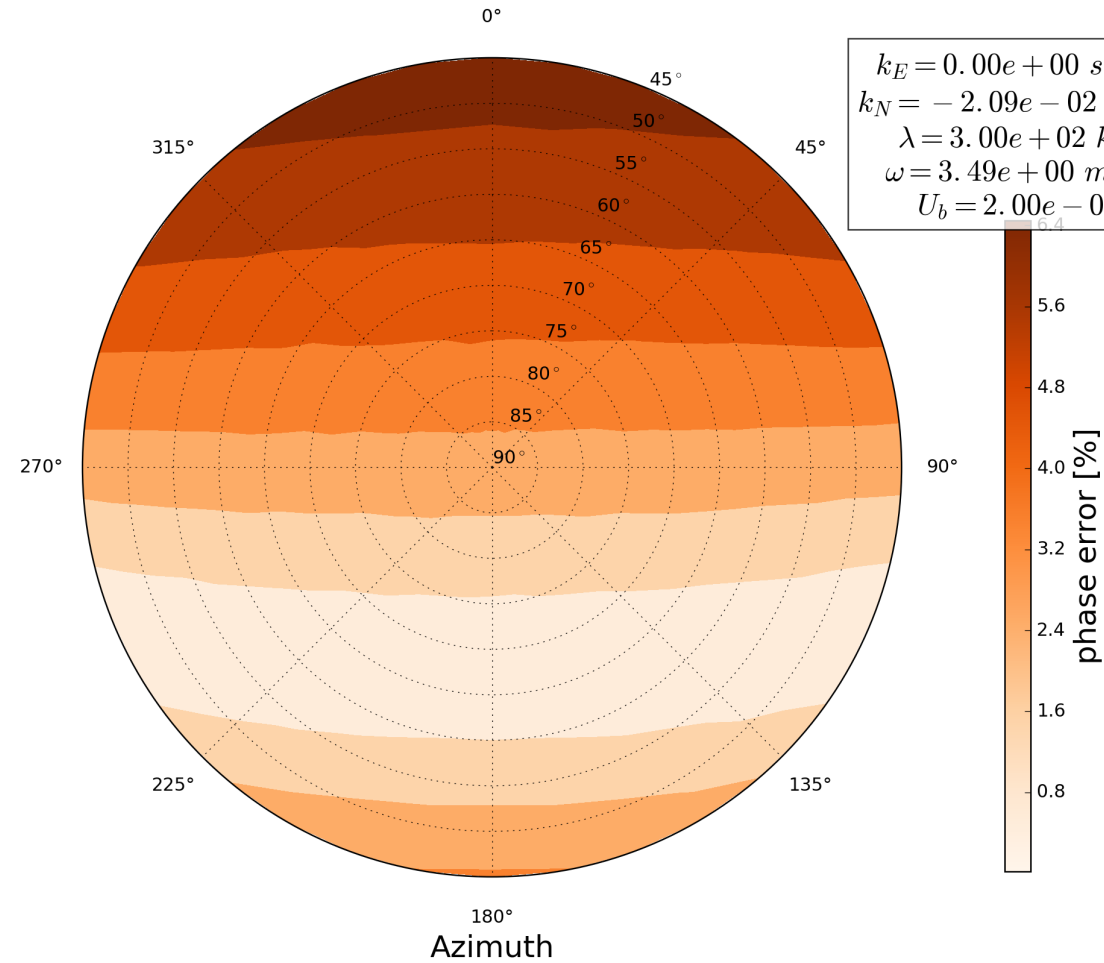
- TID amplitude: 0.04 km/s
- Phase zero is a fn of altitude



# Geometric error analysis – Phase Error

phase error as a fn Az/EI

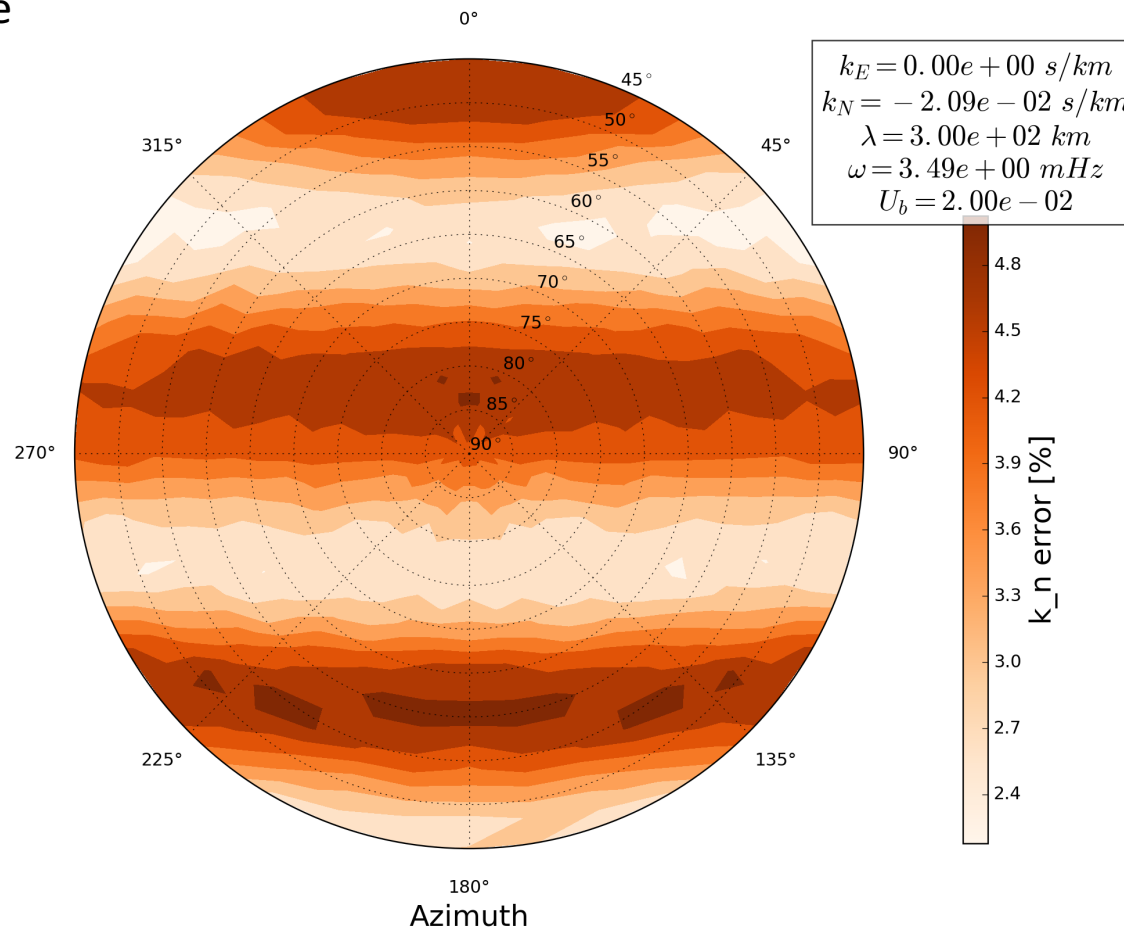
$$\begin{aligned}
 k_E &= 0.00e+00 \text{ s/km} \\
 k_N &= -2.09e-02 \text{ s/km} \\
 \lambda &= 3.00e+02 \text{ km} \\
 \omega &= 3.49e+00 \text{ mHz} \\
 U_b &= 2.00e-02
 \end{aligned}$$



# Geometric error analysis – $k_N$ Error

$k_n$  error as a fn Az/EI

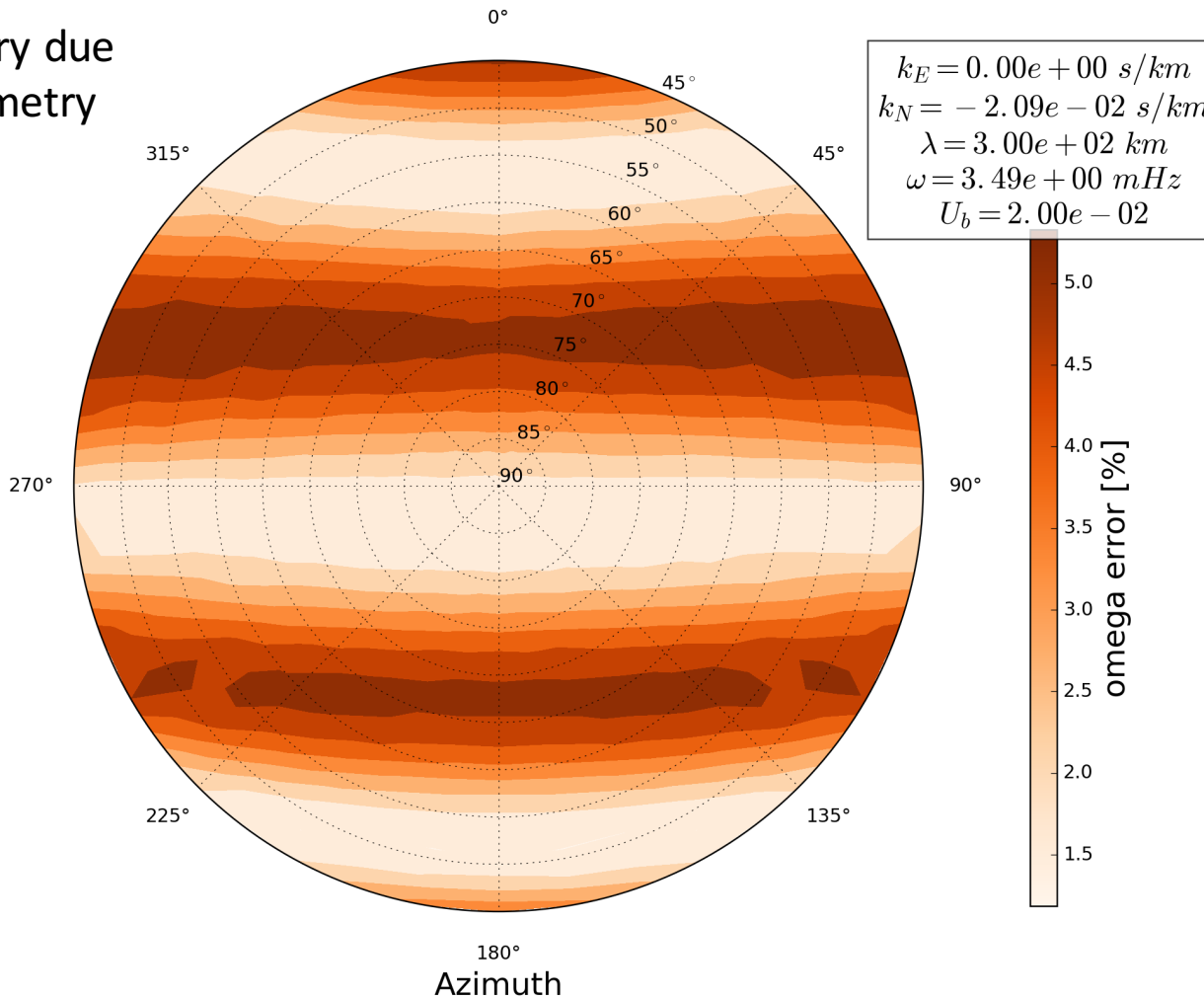
Minor asymmetry due to station asymmetry



# Geometric error analysis – Omega Error

omega error as a fn Az/EI

Minor asymmetry due to station asymmetry



- We establish a new method to determine Hooke parameter error as a function of GNSS ray geometry
- We found significant geometry-induced errors in GNSS data-derived TID parameters
  - Impacts precision HF ray propagation applications
- Next efforts:
  - Determine effects of satellite motion
  - Explore ways to compensate for TID parameter errors, knowing the GNSS ray geometry, as part of processing





## $k_e$ error as a fn Az/EI

Result is zero since  $k_e$  is defined as zero for this scenario

