Assimilative Model for Ionospheric Dynamics Employing Delay, Doppler, and Direction of Arrival Measurements from Multiple HF Channels

Sergey V. Fridman, L. J. Nickisch, Mark Hausman, Shawn Kraut, and George Zunich*

NorthWest Research Associates
Monterey, California
sergey@nwra.com

The Ionospheric Reconstruction Problem: Tikhonov Method

$$N(\mathbf{r},t) = N_0(\mathbf{r},t)Q(u(\mathbf{r},t)); \qquad \text{i.e.}\, Q(u(\mathbf{r},t)) = e^{u(\mathbf{r},t)}$$

$$U = \{u(\mathbf{r},t)\}$$

$$Y \approx M[U]$$

Y is the set of measured values obtained via various ionospheric measurements (such as TEC data, HF oblique propagation delay)

The solution must fit the data within errors of measurements. → $(Y - M[U])^T S^{-1}(Y - M[U]) / \dim(Y) \leq 1$

There are infinitely many such solutions:

The smoothest solution is selected by minimizing the stabilizing functional → $U^T P^{-1} U \rightarrow \min$

-The pseudo-covariance *P* matrix is defined in such a way that the stabilizing functional tends to take on larger values for unreasonably behaving solutions (“reasonable” ⇔ “smooth”).

-The nonlinear optimization problem is solved iteratively (Newton-Kontorovich).

HF Oblique Propagation Data within GPSII

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-Simulated values of measured data can be obtained for any ionospheric model U via numerical ray tracing (RT).

-This defines the non-linear functional of measurements $M[U]$

Ray Tracing Equations

Hamiltonian Formulation of RT Equations [*Haselgrove*, 1957, *Jones*, 1975]

$$\frac{d\mathbf{R}}{d\tau} = -\frac{\partial H}{\partial \mathbf{k}} \Big/ \frac{\partial H}{\partial \omega} \; ; \qquad \frac{dg}{d\tau} = c \qquad \text{- Group path equation}$$

$$\frac{d\mathbf{k}}{d\tau} = \frac{\partial H}{\partial \mathbf{R}} \Big/ \frac{\partial H}{\partial \omega} \; ; \qquad \frac{d\omega}{d\tau} = -\frac{\partial H}{\partial t} \Big/ \frac{\partial H}{\partial \omega} \qquad \text{- Doppler equation}$$

$$\frac{d\mathbf{X}}{d\tau} = \mathbf{F}(\mathbf{X},[N,\frac{\partial N}{\partial t}]) \qquad \mathbf{X} = [X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8]^T$$

position: X_1, X_2, X_3; wave vector: X_4, X_5, X_6; group path: X_7; Doppler: X_8

HF Oblique Propagation Data within GPSII

-The non-linear inverse problem is solved iteratively as a sequence of linear problems. At the iteration *n* the non-linear functional $M[U]$ is approximated by a linear operator L as follows

$$M[U] = M[U_{n-1}] + L(U - U_{n-1}) + o\left(\left\|U - U_{n-1}\right\|\right) \qquad \Leftrightarrow L = \delta M / \delta U$$

- L is the Ray Path Response (RPR) operator

-The Ray Path Response operator L is estimated using the extended RT equations – the equations augmented with the linearized ray-tracing equations

Extended RT Equations

$$\frac{d\mathbf{X}}{d\tau} = \mathbf{F}(\mathbf{X}, [N, \frac{\partial N}{\partial t}])$$

$$\frac{d\mathbf{A}}{d\tau} = \mathbf{B}(\mathbf{X}, [N, \frac{\partial N}{\partial t}])\mathbf{A} \qquad B_{ij} = \partial F_i / \partial X_j \Big|_{\mathbf{X}=\mathbf{X}(\tau)}$$

$$A_{ij}\Big|_{\tau=0} = \delta_{ij}, \quad i, j \in [1:8]$$

8+8x8=82 equations in the extended system

Evaluation of the Ray Path Response Operator *L*

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The 3-D ionospheric model is specified by U_{n-1}

→ Perform ray-tracing (ray-homing) for the Rx-Tx configuration. Determine ray exit direction

→ Integrate the extended RT equations at the ray exit direction found at ray homing

↓ Produce impulse response function $\boldsymbol{G}^I(\tau,\tau')$ of the linearized RT equations

← Produce the Green function $\boldsymbol{G}(\tau,\tau')$ for the boundary-value problem (subject to the constraint that the ray end points are fixed in space)

← Evaluate linear response $\mathbf{K}(\boldsymbol{\tau}')$ of parameters measured by the RX

↓

Evaluate RPR

$$L^S = \int d\tau' \mathbf{K}(\tau') \left(\frac{\delta \mathbf{F}(\mathbf{X},[N,\dot{N}])}{\delta N} N_0 Q'(u_{n-1}) + \frac{\delta \mathbf{F}(\mathbf{X},[N,\dot{N}])}{\delta \dot{N}} \left(N_0 Q''(u_{n-1}) \dot{u}_{n-1} + \dot{N}_0 Q'(u_{n-1}) \right) \right)$$

$$L^D = \int d\tau' \mathbf{K}(\tau') \frac{\delta \mathbf{F}(\mathbf{X},[N,\dot{N}])}{\delta \dot{N}} N_0 Q'(u_{n-1})$$

Where $N = N_0(\mathbf{r},t) Q(u_{n-1}(\mathbf{r},t))$

$$L\begin{bmatrix} \delta U \\ \delta \dot{U} \end{bmatrix} = L^S \, \delta U\big|_t + L^D \, \delta \dot{U}\big|_t$$

Assimilation of HF Data using RPR

Evaluation of RPR within the 3-D ionospheric inversion problem can be performed with little computational burden because it is reduced to computation of several one-dimensional integrals

For HF data the main computational burden remains associated with the classical ray homing task. This task precedes computation of RPR

Numerical representation of the RPR is a sparse matrix with non-zero elements occupying only nodes of the spatial grid that are adjacent to the ray trajectories that connect receiver and transmitter. Matrix operations with RPR are not a substantial computational burden as we take advantage of the sparse character of RPR.

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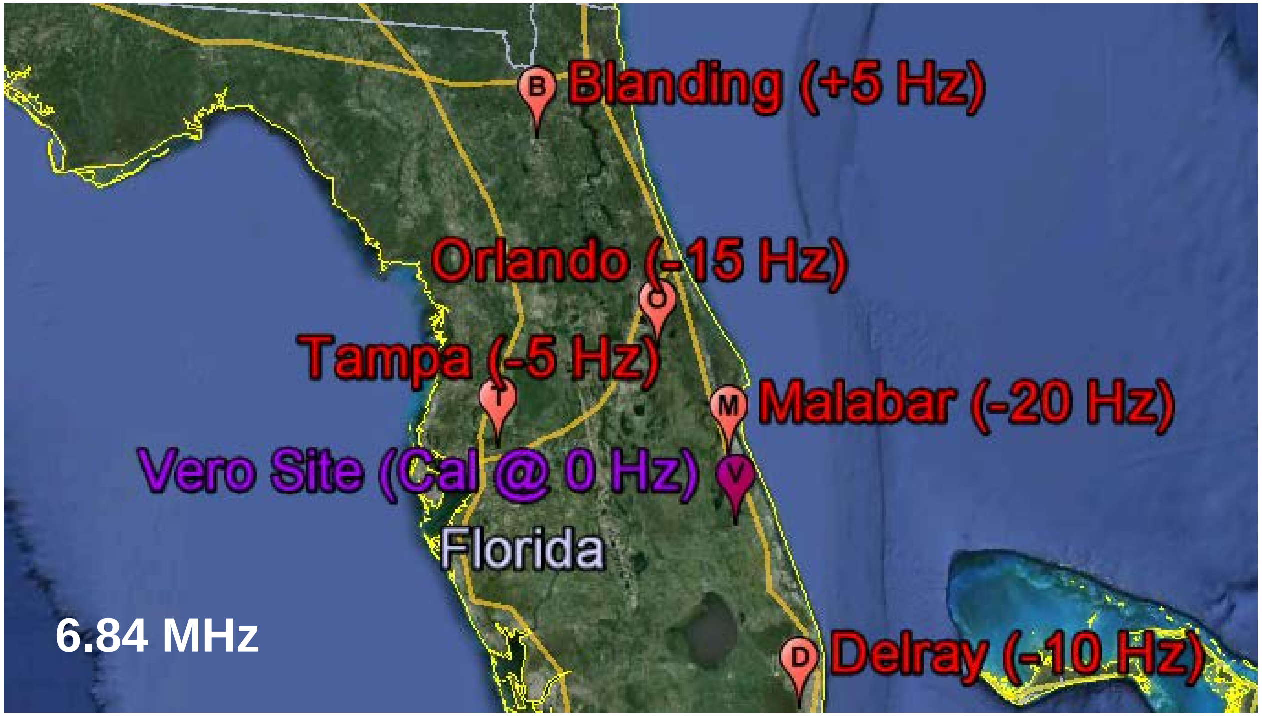
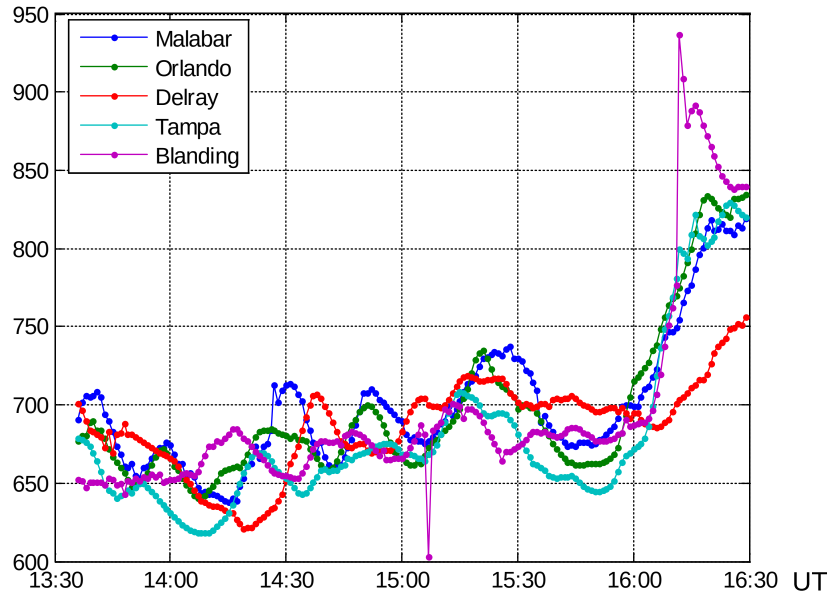
Test with Range-Doppler Data Set



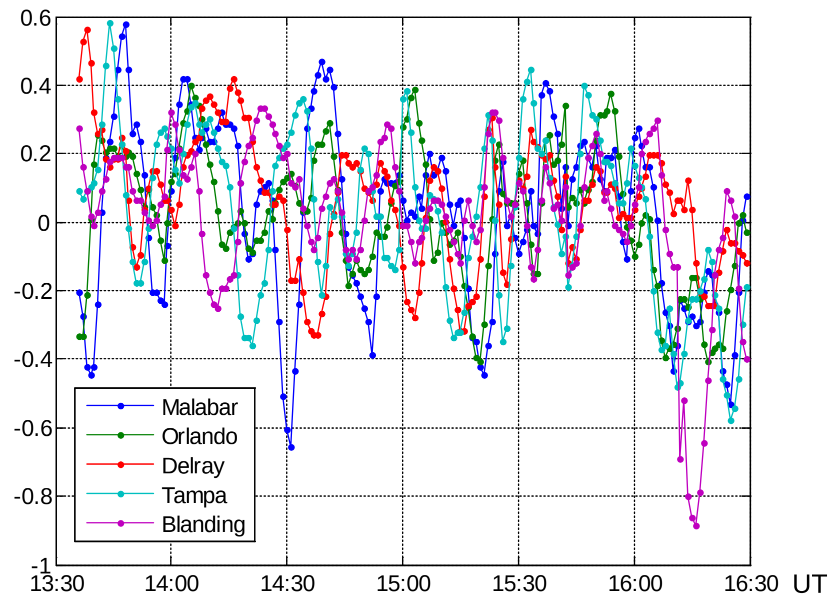
- **Florida Collection (August 13, 2013) Range/Doppler Data**
  - **Receiver at Vero Beach, FL**
  - **Multiple transmitter sites**
  - **Three hour collection of 3 KHz bandwidth FMCW waveform**

Range-Doppler Data of August 13, 2013 Employed by GPSII

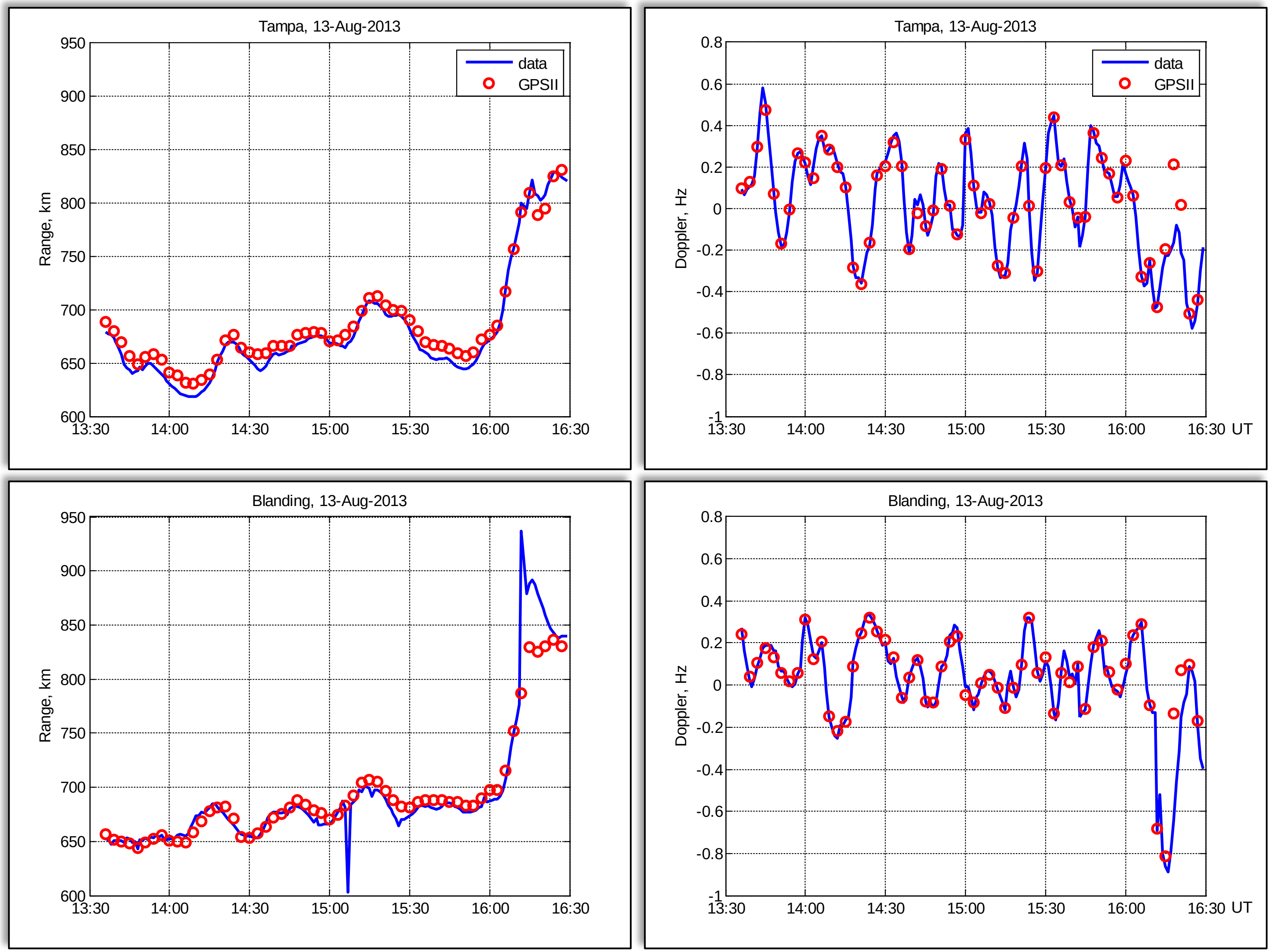
Range data (km)



Doppler data (Hz)



Range-Doppler Data Compared to GPSII Fit



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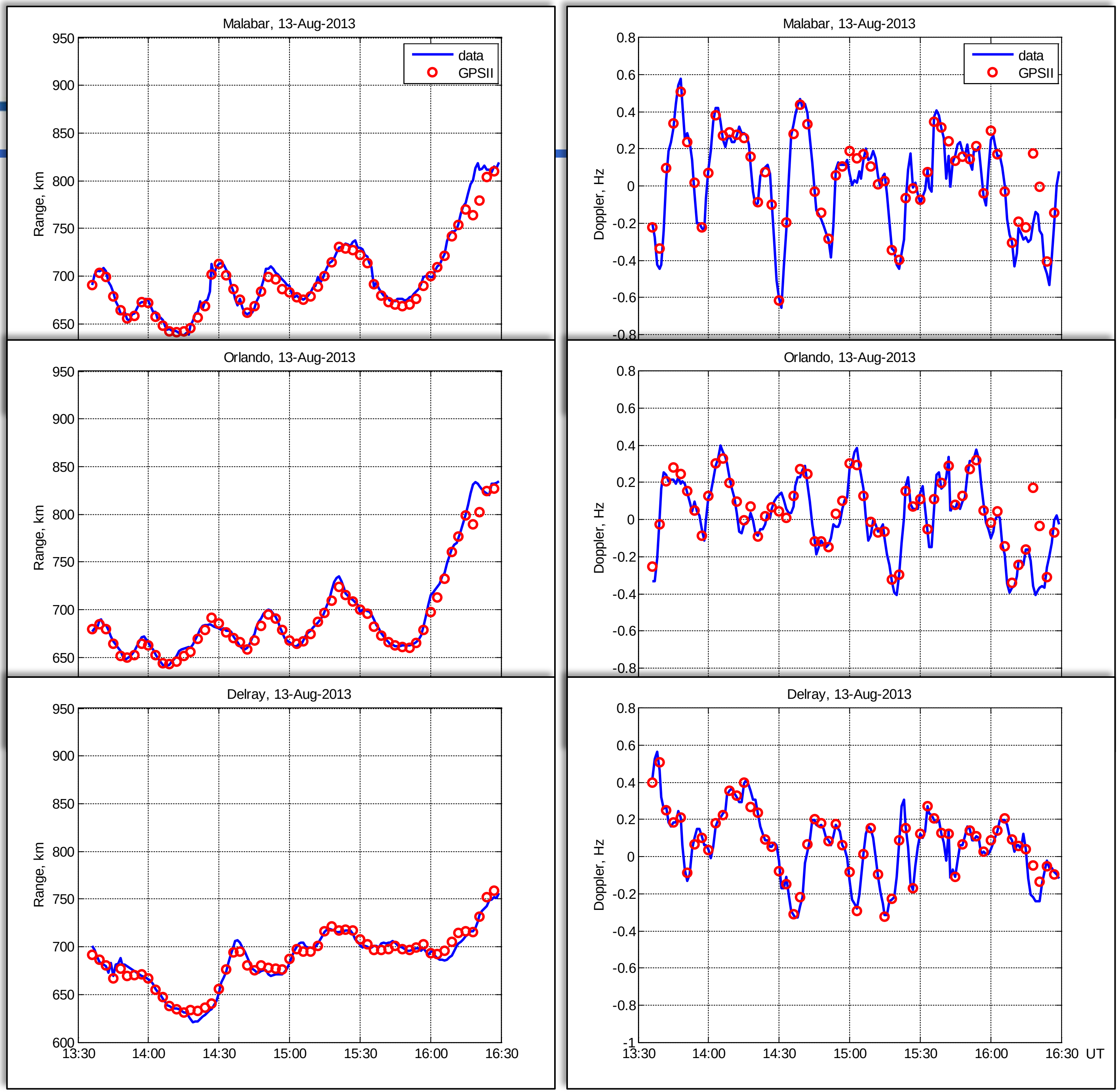
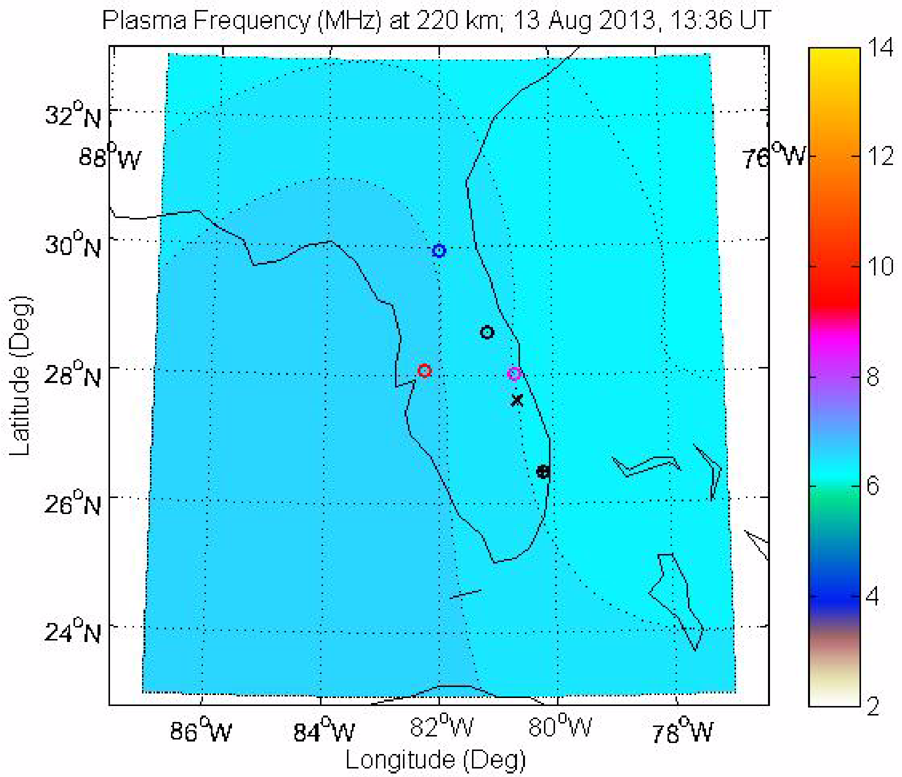
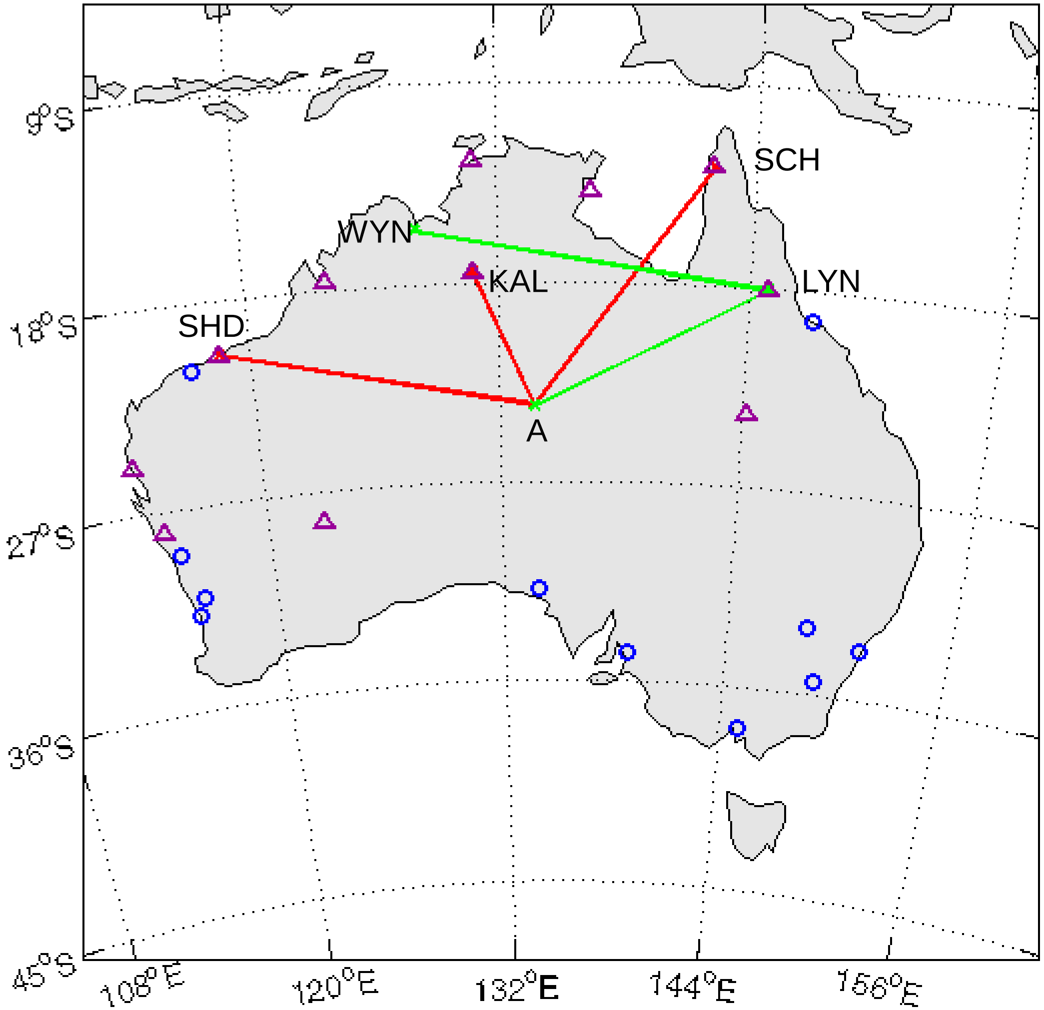


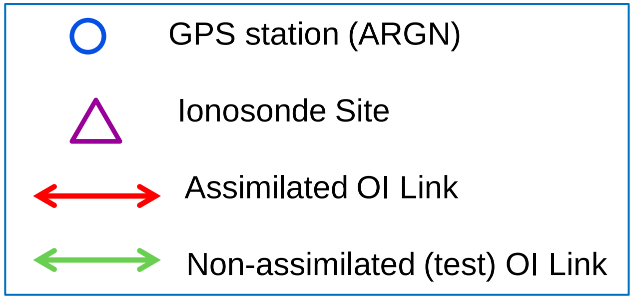
GPSII Solution with Range-Doppler Data



Assimilation of Oblique Ionograms (along with TEC and VI data)

Geography of DSTO data sources employed in this test*



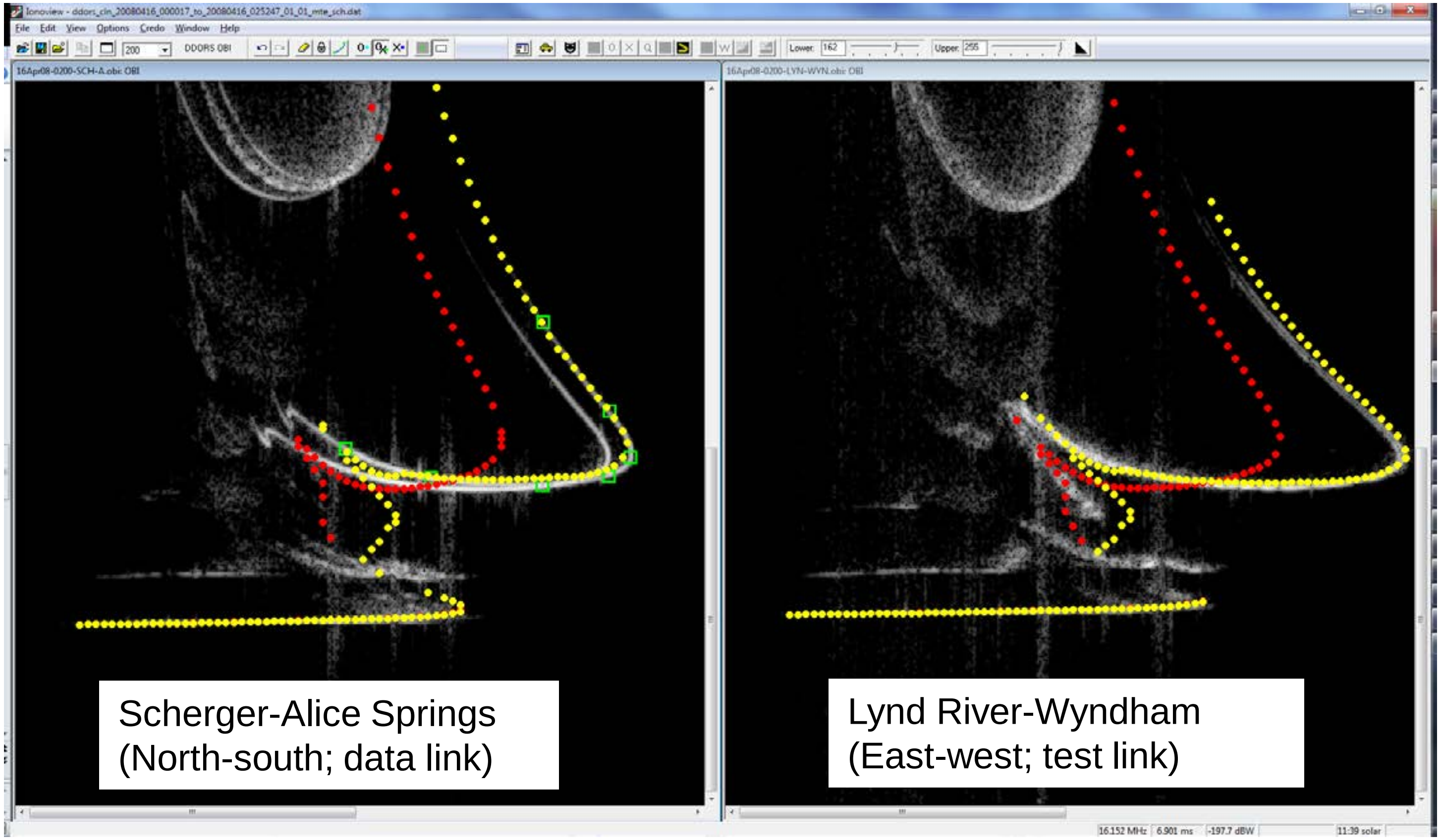


*Only a subset of OI links maintained by DSTO was utilized

Comparison of GPSII results for 2 links

GPSII runs with oblique ionogram reproduce observed OI details (yellow) for both assimilated (left) and test (right) propagation links

Both links shown at 01:45. All synthetic OIs are extraordinary-ray traces.



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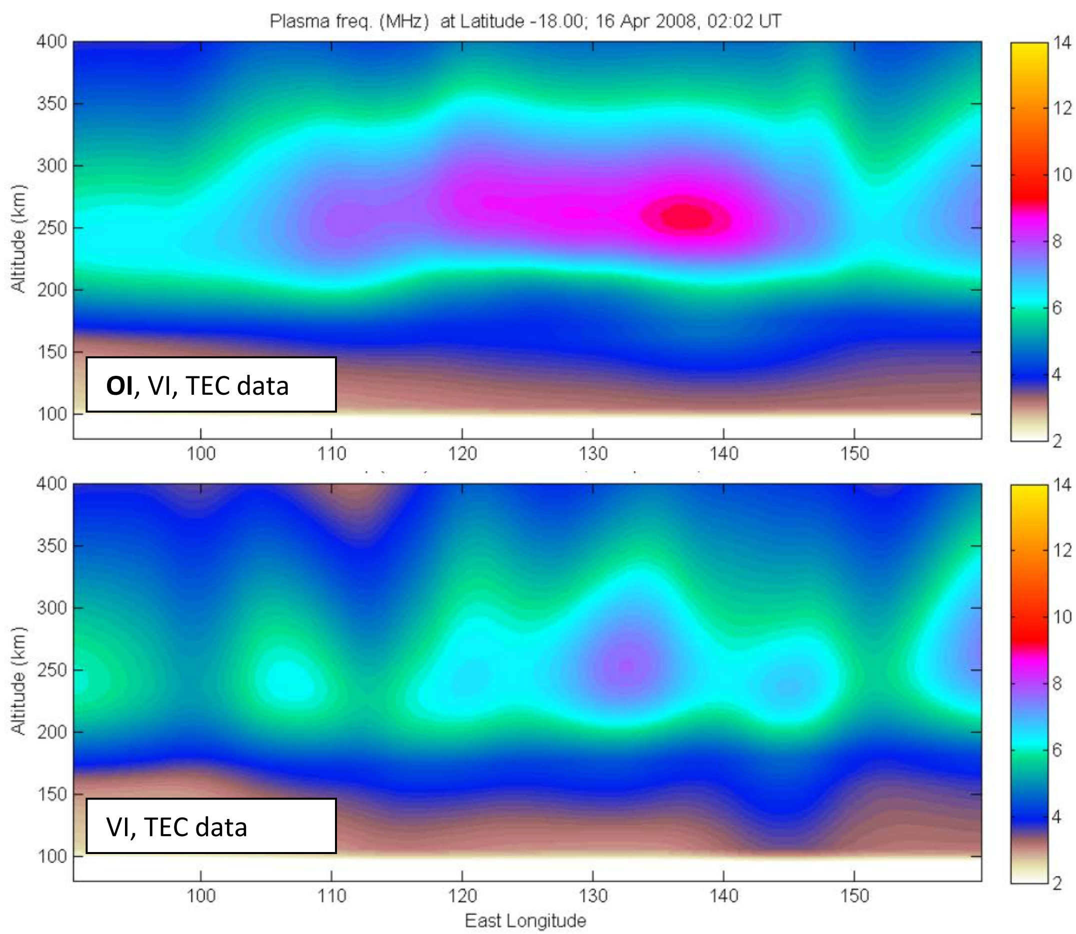
Impact of OI data on GPSII Model

Vertical cut through the model at latitude -18 degrees

Inversion with OI Data



Conclusions

- The theoretical framework for incorporating HF channel probe data (propagation delay, angles of arrival, Doppler shift) into ionospheric inversion algorithms has been developed
- Capabilities to assimilate data from HF channel probes and oblique ionograms have been added to GPSII
- Performance and validation of the algorithm are addressed in the companion paper by L.J. Nickish

Backup

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Integrate the extended RT equations at the ray exit direction found at ray homing

Produce impulse response function $\boldsymbol{G}^I(\tau,\tau')$ of the linearized RT equations

Produce the Green function $\boldsymbol{G}(\tau,\tau')$ for the boundary-value problem (subject to the constraint that the ray end points are fixed in space)

Evaluate linear response $\mathbf{K}(\boldsymbol{\tau}')$ of parameters measured by the RX

Evaluate RPR

$$\mathbf{L} = \int \mathbf{K}(\tau') \frac{\delta \mathbf{F}(\mathbf{X},[N,\dot{N}])}{\delta N} d\tau'$$

DETAILS

τ_L = propagation delay at the landing (Rx) point

$$G^I(\tau,\tau') = \begin{cases} 0, \tau < \tau' \\ \mathbf{A}(\tau)\mathbf{A}^{-1}(\tau'), \tau \geq \tau' \end{cases}$$

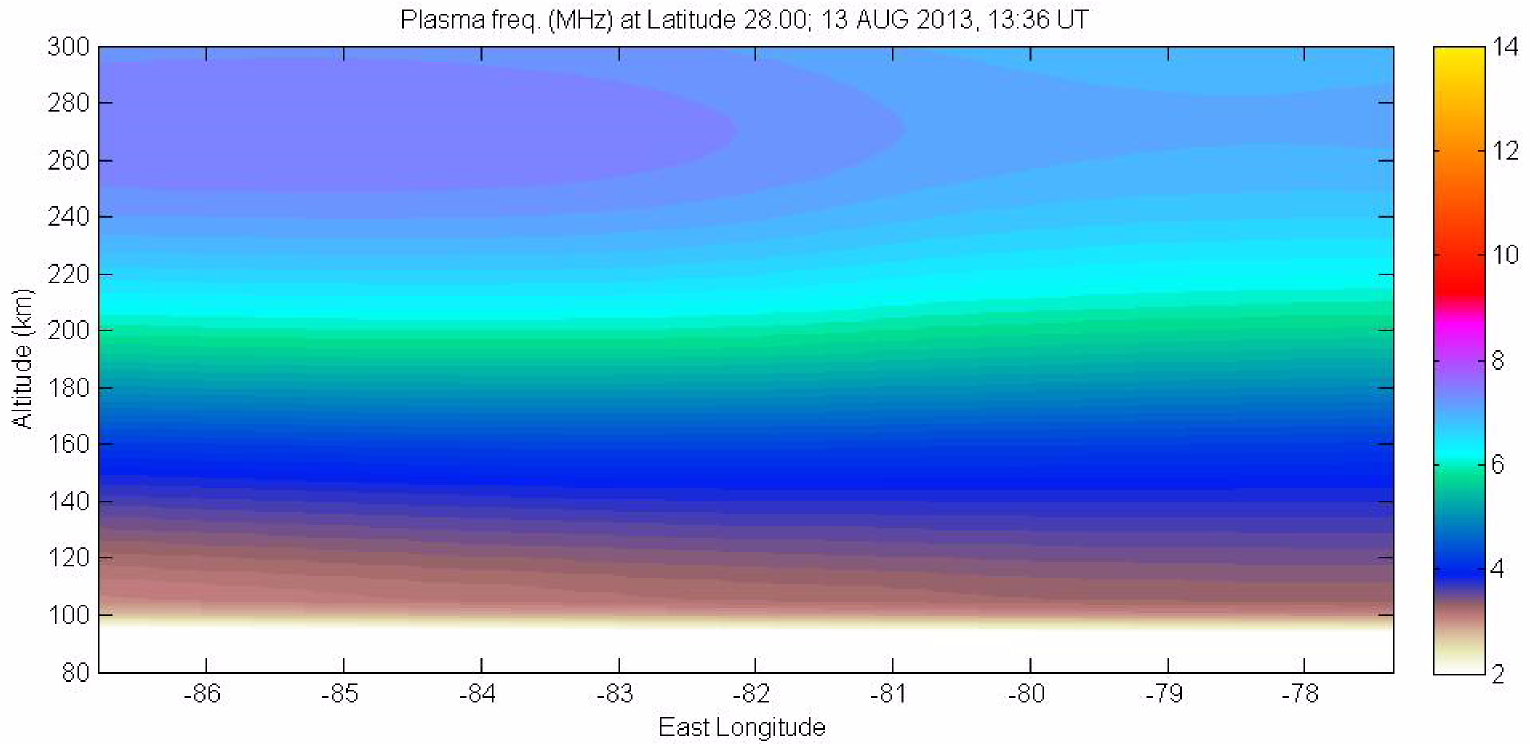
$$\mathbf{G}(\tau,\tau') = G^I(\tau,\tau') + [A_{1:8,4:6}(\tau)][D_{1:3,1:8}(\tau')];$$

$$\mathbf{D}(\tau') = -\begin{bmatrix} A_{1:3,4:6}(\tau_L), F_{1:3}(\tau_L) \\ [F_{1:3}(0)]^T, 0 \end{bmatrix}^{-1} \begin{bmatrix} G^I_{1:3,1:8}(\tau_L,\tau') \\ 0 \ldots\ldots\ldots\ldots 0 \end{bmatrix}$$

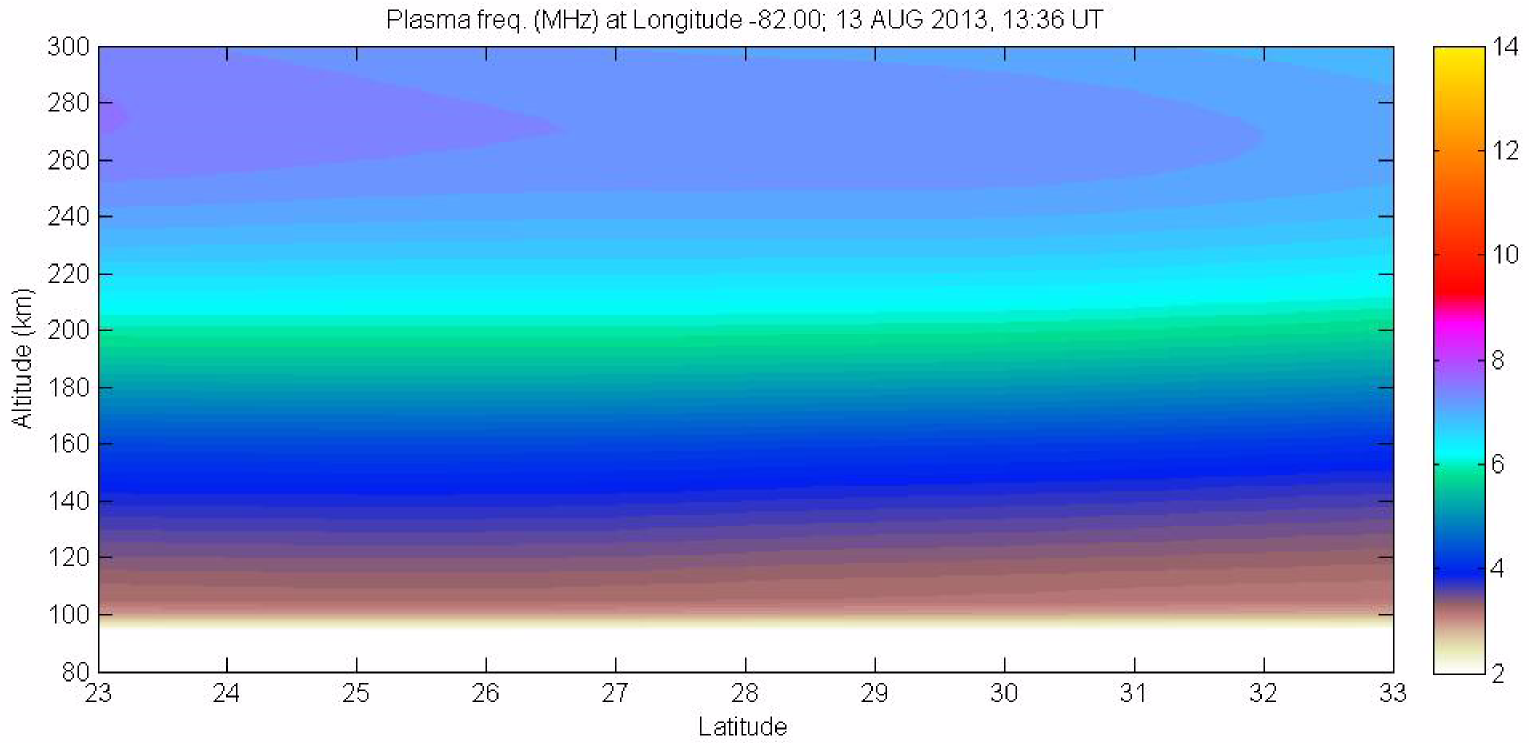
Assuming that RX measures direction of arrival, group path, Doppler

$$\mathbf{K}(\tau') = G_{4:8,1:8}(\tau_L,\tau') + F_{4:8}(\tau_L) D_{4,1:8}(\tau')$$

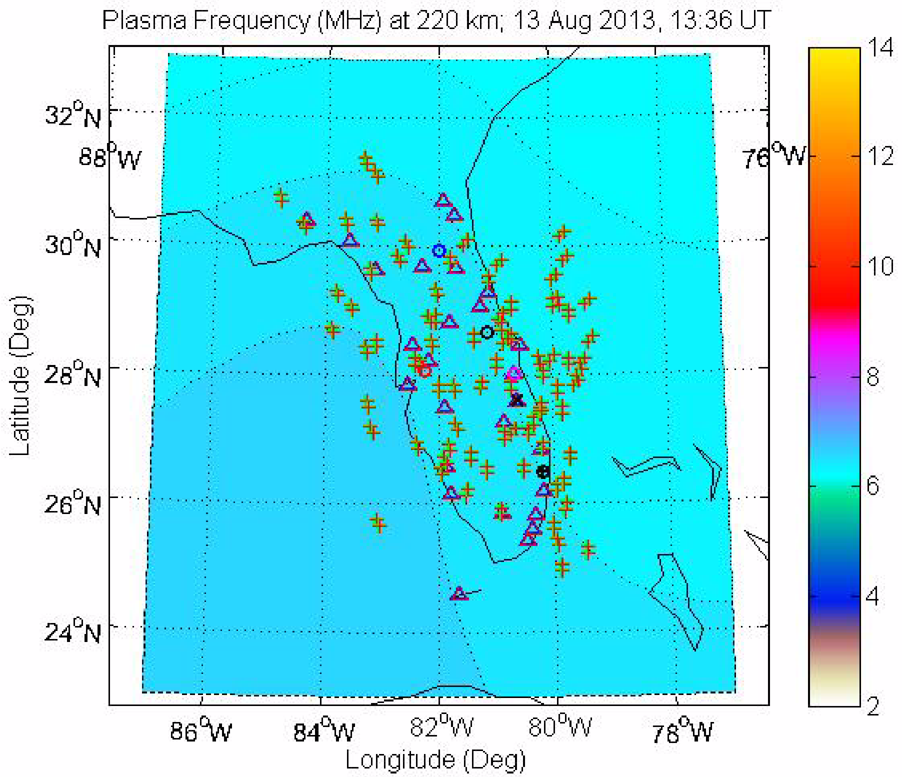
GPSII Solution with Range-Doppler Data



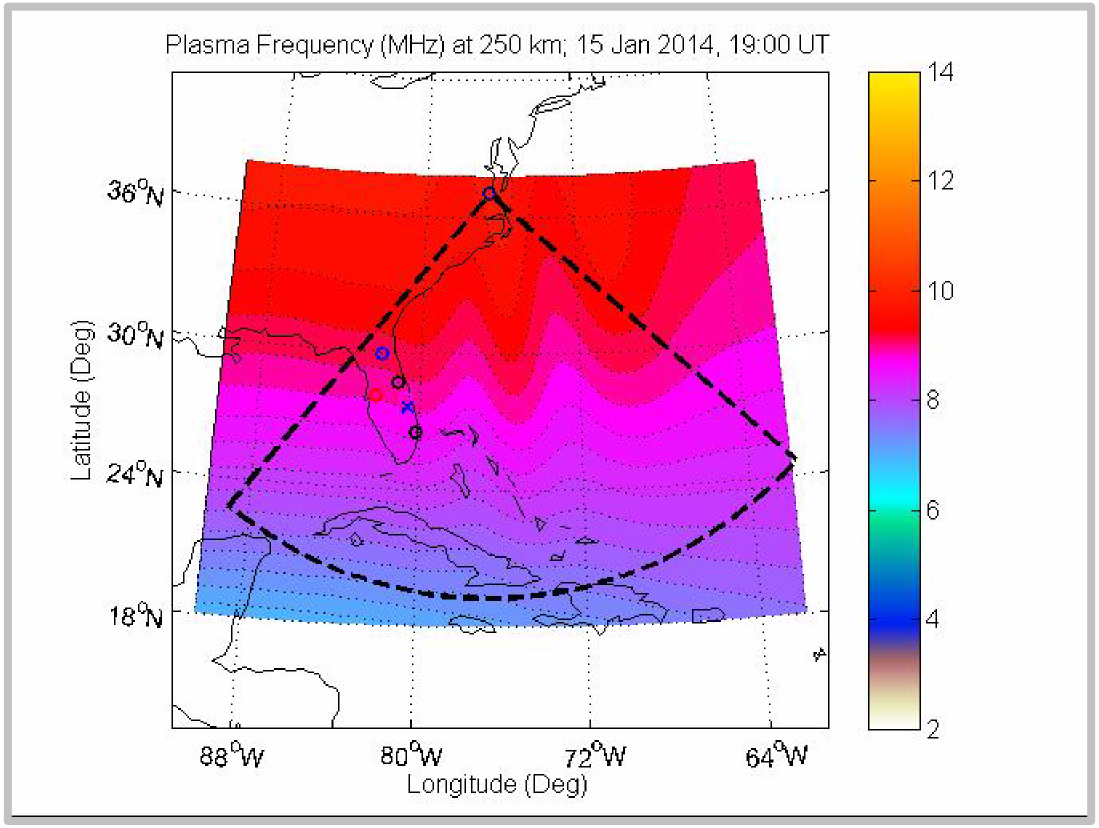
GPSII Solution with Range-Doppler Data



GPSII Solution with Range-Doppler Data and the GPS TEC data



Reconstruction of a TID from Simulated OTHR Data



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$$\delta U\Big|_t = \delta U(t_m)\frac{t_m - t}{t_m - t_{m-1}} + \delta U(t_{m-1})\frac{t - t_{m-1}}{t_m - t_{m-1}}; \quad \delta\dot{U}\Big|_t = \delta U(t_m)\frac{1}{t_m - t_{m-1}} - \delta U(t_{m-1})\frac{1}{t_m - t_{m-1}}$$