# On the Geometric Dependence of Scintillation and Stochastic Structure Models

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#### **ABSTRACT**

We describe and demonstrate a configuration-space structure model that can accommodate the full range of GNSS and LEO propagation geometries for practical simulation. Configuration-space structure realizations are summations of randomly located physical striations. By using selection rules for the size distribution, the cross-field spectral density function is constrained to be a multi-component inverse power law. The realization at any point in the propagation space is defined by a summation over the striation contributions, whereby the configuration can be computed as encountered in the integration of the forward-propagation equation. Computation and memory management is more efficient and we believe the realizations are more representative of real ionospheric structure. Simulation results will be compared to GPS data from equatorial and high-latitude stations.

**Key words:** Scintillation, Propagation Theory, Ionospheric Structure.

## 1. Introduction

From propagation theory we know that ionospheric scintillation is the cumulative effect of propagation from the signal source through a disturbed ionospheric region and on to a plane of measurement. Additionally, we know that ionospheric structure is highly elongated along the earth's magnetic field. The statistical theory of scintillation has been used extensively for interpreting beacon transmissions from low-earth-orbiting (LEO) satellites. Moreover, with the advent of the GNSS constellation of satellites and LEO GNSS receivers, global ionospheric monitoring has become routine.

In spite of the extensive observational capabilities currently available, scintillation diagnostics rarely go beyond occurrence morphology. Part of the problem has been the lack of a tractable strong-scatter theory that can be used efficiently for data analysis. Additionally, physics-based theories of structure development cannot yet predict structure distributions quantitatively. A tractable strong-scatter theory for one-dimensional phase screens has been developed and applied to the interpretation of strong-scatter scintillation data. [1] The results are encouraging, but the simplifications imply that a structured ionosphere can be characterized by a single equivalent phase screen and that field-aligned anisotropy reduces the dimensionality of the field structure to two-dimensions.

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To explore the geometrical dependence of scintillation more fully, this paper will describe and demonstrate a configuration-space structure model that can accommodate the full range of GNSS and LEO propagation geometries for practical simulation. A standard technique for generating structure realizations imposes the desired spectral intensity distribution on uncorrelated spatial Fourier modes. This is very inefficient for populating a realistic propagation space. We will show that for any multi-component power-law configuration there is a random distribution of physical striations that will reproduce a prescribed two-dimensional power-law structure in any slice plane that intercepts the structure. The slice-plane orientation is normal to the propagation direction, which is unconstrained for real-world applications. Within each slice plane the structure contribution can be computed from the striation distribution parameters as needed.

## 2. Configuration Space Model Summary

A configuration space realization is constructed as follow:

$$N(\mathbf{\rho}, \varsigma) = \sum_{j=1}^{N} N_{j} p_{\parallel} (|\varsigma - \varsigma_{j}|) p_{\perp} (|\mathbf{\rho} - \mathbf{\rho}_{j}| / \sigma_{j})$$
(1.1)

Where  $(\mathbf{p}, \boldsymbol{\varsigma})$  represents coordinates across and along the magnetic field direction,  $N_j$  and  $\sigma_j$  represent the on-axis density and the characteristic size of the  $j^{\text{th}}$  striation. The profile functions  $p_{\parallel}(\boldsymbol{\varsigma})$  and  $p_{\perp}(\boldsymbol{\rho})$  describe the ionization distributions along magnetic field lines radially outward. To calibrate the model, the one-dimensional spectral density function measured across a subset of parallel field lines can be computed as

$$\left\langle \left| \Delta N\left(\kappa_{s}\right) \right|^{2} \right\rangle / N_{0}^{2} = \sum_{j=J_{\min}}^{J_{\max}} N_{j} \sigma_{j}^{2\eta_{1}+1} \left| p^{(1)}\left(\kappa_{s}\right) \right|^{2} / N_{s}$$

$$+ CF \sum_{j=J_{\min}}^{J_{\max}} N_{j} \sigma_{j}^{2\eta_{2}+1} \left| p^{(1)}\left(\kappa_{s}\right) \right|^{2} / N_{s}$$

$$(1.2)$$

The function  $p^{(1)}(\kappa_s)$  is the one-dimensional Fourier transformation of  $p_{\perp}(\rho)$ . The values of  $N_i$  and  $\sigma_i$  are determined by the successive bifurcation rule:

$$\sigma_{j} = \sigma_{\text{max}} 2^{J_{\text{max}}-j}$$

$$N_{j} = 2^{d-j} \quad \text{for } j = J_{\text{min}}, \dots, J_{\text{max}}$$

$$(1.3)$$

The value of  $\eta$  determines the power-law index for the contributing striations. CF is chosen to make the spectral density function continuous at the transition scale.

As an example, a realization is constructed from (1.1) by specifying the profile functions and the parameters in (1.3) that define  $N_j$  and  $\sigma_j$ . Figure 1 shows a cross-field cut of a realization with 7 bifurcation levels spanning the scale range from 625 m to 40 km with 16,256 striations.

### 3. Status and Proposed Simulations

Simulated three-dimensional propagation computations have been successfully performed [2]. The development of the configuration space model was motivated by the difficulty of populating representative striated propagation environment. Here we will investigate realizations representative of both equatorial and high-latitude GPS geometries for which confirming data are

available. The data cover extreme equatorial strong scatter and weak high-latitude scintillation. The HAARP Alaska heating facility is particularly interesting as a controlled man-made source of potentially field aligned structure. The development is evolving, but we believe that configuration-space models can provide connections that tie physical structure configurations to diagnostic measurements.

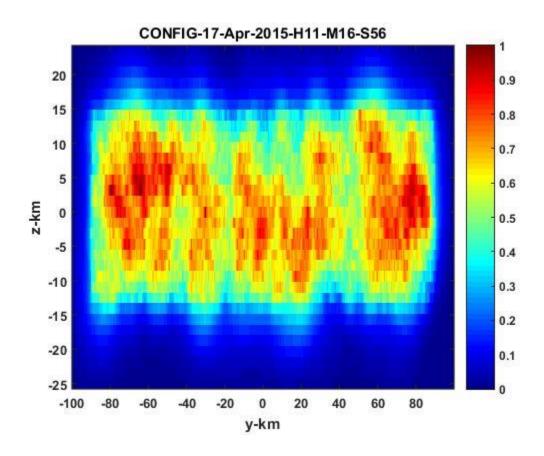


Figure 1. Cross field configuration space realization for single-component power law.

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