Theory and Initial Results of AENeAS

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Types of Data Assimilation

Sequential Methods:

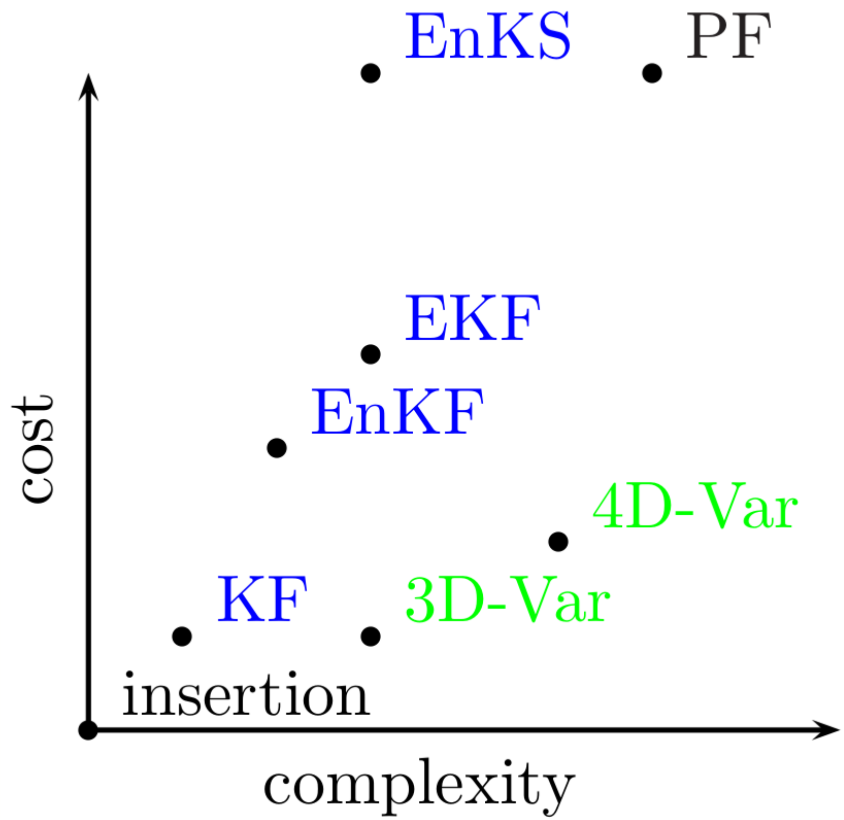
- Kalman filter (KF)
- extended Kalman filter (EKF)
- ensemble Kalman filter (EnKF)
- ensemble Kalman smoother (EnKS)
- Particle filter (PF)

Variational Methods:

- Three dimensional variational method (3D-Var)
- Four dimensional variation method (4D-Var)

Hybrid:

- Evolving covariance matrix in 3D-Var or 4D-Var via ensemble approximation



Ensemble Kalman Filter (EnKF)

- First introduced by Evensen (1994) as a Monte Carlo approximation to Kalman filtering
  - EnKF is a sequential data assimilation method that uses an ensemble of model forecast to approximate the model mean and covariance matrix
  - Ensemble is updated with every analysis to reflect information provided by the observations, and is evolved using the forecast model between analysis

- Example uses of EnKFs:
  - Short-range/global NWP
  - Earth core/middle-atmosphere/Martian GCM
  - Global weather/climate/ocean prediction

Kalman Filters

- Traditional KF Eqns:

$$
\begin{aligned}
B &= MAM^T + Q, \\
K &= BH^T(HBH^T + O)^{-1}, \\
x_a &= x_b + K(y_o - H(x_b)), \\
A &= (I - KH)B.
\end{aligned}
$$

Where: B is the background model covariance matrix
M is a linear model
A is the analysis covariance matrix
Q is the model error covariance matrix
K is the Kalman gain
H is an observation operator
x_a is the analysis state
x_b is the background model state
y_o is an observation vector.

For ‘large’ systems these equations become very computationally expensive.

Ensemble Kalman Filters (EnKF)

- Ensemble KF Eqns:

$$B = \frac{1}{k-1} X_b X_b^T,$$
$$K = BH^T(HBH^T + O)^{-1},$$
$$x_a = x_b + K(y_o - H(x_b)),$$
$$A = (I - KH)B.$$

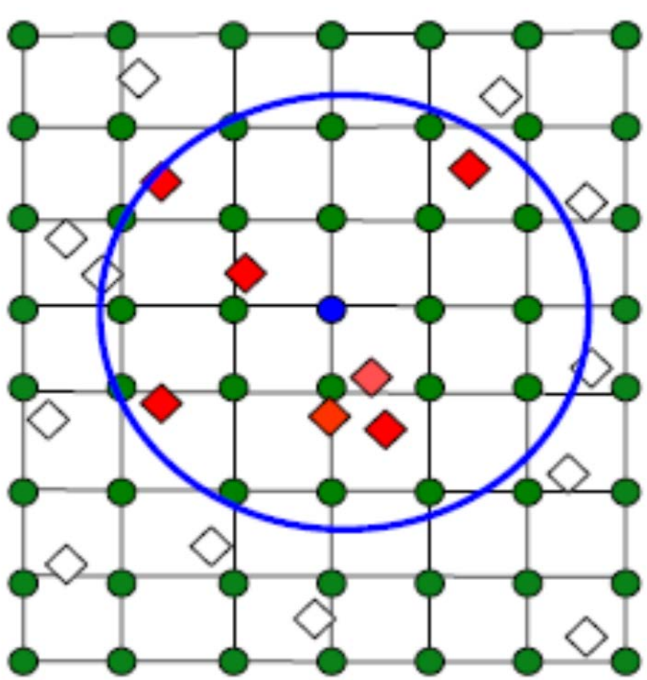
Where: X_b is the ensemble perturbation matrix
k is the number of ensemble members.

EnKF – Assumptions & Considerations

- Assumptions:
  - PDF of errors is Gaussian
  - Model errors are small compared with errors in initial condition and parameters
  - Observations can be represented in the ensemble

- Considerations:
  - $\frac{1}{k-1} X_b X_b^T$ is an approximation to the 'true' background covariance matrix
    - The greater the number of independent ensemble members the better the assumption
    - Errors in sampling decrease proportional to $\frac{1}{\sqrt{k}}$
  - Spurious relationships can be found between variables
    - Possibly due to limited variability in ensemble

Local Ensemble Transform Kalman Filter (LETKF)



Brian R. Hunt, Eric J. Kostelich, Istvan Szunyogh, *Efficient data assimilation for spatiotemporal chaos: A local ensemble transform Kalman filter*, Physica D, 230, 112-126, 2007

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- LETKF assimilates by local volume centred at each grid point variable
    - Area of local volume depends on model dynamics and assumptions of correlations between discrete model grid points
- Each observation may be used more than once for data assimilation
- Smaller region makes ensemble space spanning easier
- The algorithm is highly parallel, all grid points can be assimilated simultaneously

Computational Expense

- The "default" EnKF computational cost is:

$$O(s^3 + s^2k + sk^2 + mk^2)$$

where: s is the number of observations
k is the number of ensemble members
m is the dimension of state vector

- The LETKF computational cost is:

$$O(k^3 + k^2s + k(s + m))$$

- For $s > k$ LETKF is much cheaper than EnKF

Advanced European Electron Density (Ne) Assimilation System (AENeAS)

- A 4D, modular designed, data assimilation thermosphere / ionosphere model with a physics-based background model

- Aims:
  - Good nowcasts
    - Background model dependent
  - Accurate and actionable forecasts up to 6 hour ahead

Background Model(s)

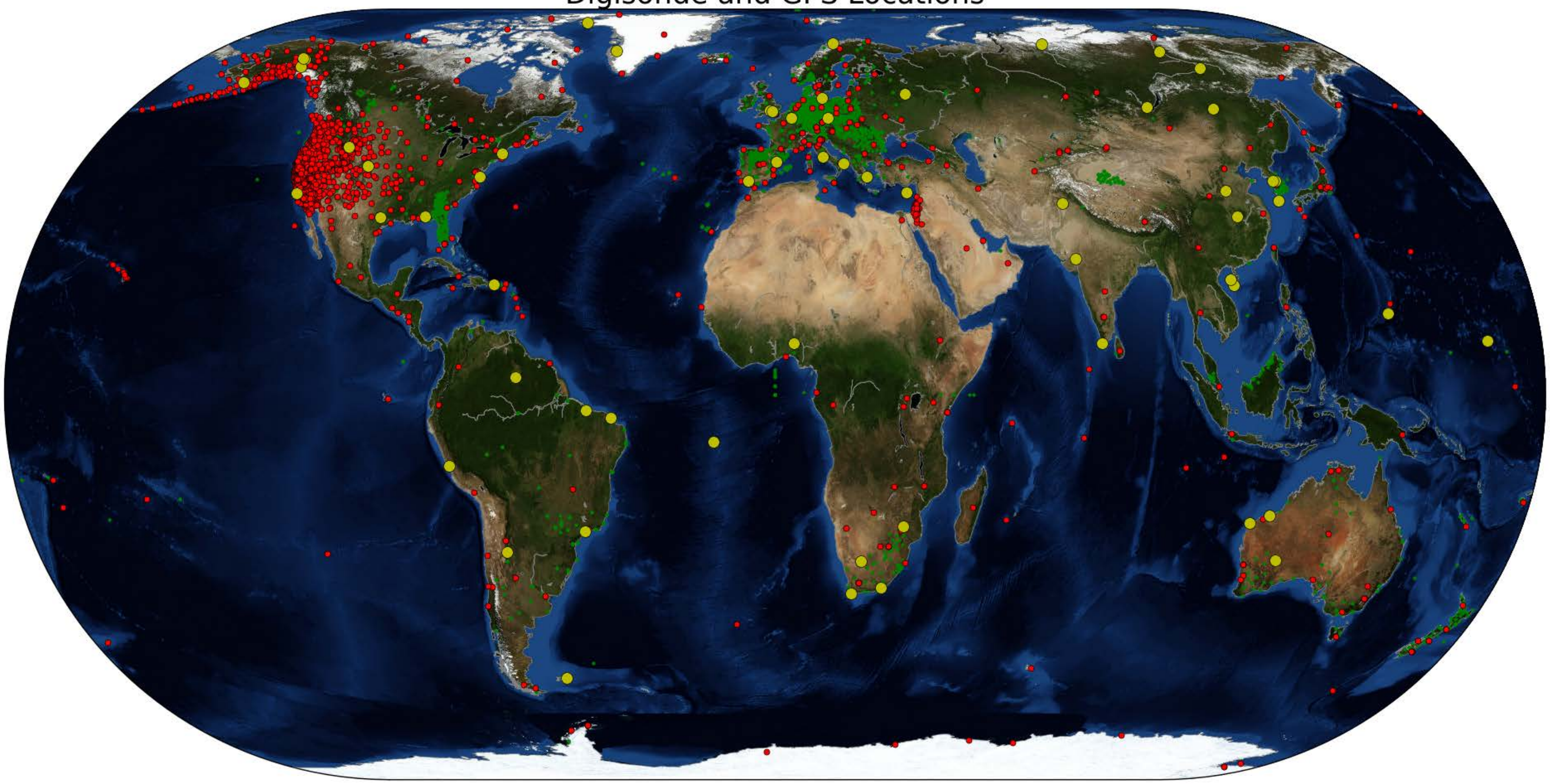
- The NCAR Thermosphere Ionosphere Electrodynamics General Circulation Model (TIE-GCM)
  - Version 2.0
  - 2.5 degree resolution
  - Heights ~90 – ~600 km
  - Variables:
    - Ne, N, NO, O_1, O_2, O^+, O_2^+, He, Ar, Vertical motion, Temperatures, Neutral winds

- NeQuick
  - Topside (~25,000 km)
  - Use control points from TIE-GCM
  - Smoothly attach profiles

Assimilated Data

- Initially, GPS and ionosonde.
  - AENeAS will handle 1 second to 2 hour latency data
  - e.g. NTRIP, IGS, GIRO, COSMIC2 RO
  - 15 minute assimilation window
- Modular design, more data types can be added in the future.

Digisonde and GPS Locations



Initial Testing

- Assimilated 10 GPS receivers over Europe in December 2008
  - Extremely difficult modelling period
  - More information about this time period in:

    “**A Community Wide Ionospheric Modelling Challenge”**

    Session 5A, Budinich Lecture Hall, Thursday 10:10

- Only used 4 ensemble members
  - No covariance inflation
  - Each member randomly gets value of F10.7, cross-tail potential and hemispheric power (± 20% of observation)

TEC Video

Neutral Meridional Wind Video

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Conclusions

- New 4D physics-based data assimilation model being developed at the University of Birmingham
  - AENeAS
  - TIE-GCM background model
  - Will initially assimilate GNSS and ionosonde observations

- Assimilates data using a Local Ensemble Transform Kalman Filter (LETKF)
  - Computationally efficient
  - Addresses spurious correlation issues from EnKF
  - Grid points processed in parallel
  - Fewer ensemble members required

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LETKF – Theory

- EnKF variants solve the EnKF via differing implementations
- The LETKF approach:
  1. Analysis is performed locally in model grid space
     - Computational efficiency & spurious correlation control
  2. Analysis ensemble perturbations are obtained from background ensemble perturbations using a transform matrix
     - What transform should be used?
       - See next slide
  3. Matrix inverse for Kalman gain is computed in ensemble space
     - Requires use of a known matrix identity
     - $K = X_b \tilde{A} H^T O^{-1}$, where $\tilde{A} = [H^T O^{-1} H + (k-1)I]^{-1}$

LETKF Theory

- Similar to the definition of B, we can define:
  - $A = \frac{1}{k-1} X_a X_a^T$
- X_a is found via a matrix transformation, namely $X_a = X_b T$
  - From classic KF: $A = (I - KH)B$ so:

$$A = \left(I - X_b \tilde{A} H^T O^{-1} H\right) \frac{1}{k-1} X_b X_b^T,$$
$$= X_b \tilde{A} (k-1) I \frac{1}{k-1} X_b^T,$$
$$= X_b \tilde{A} X_b^T.$$

  - Thus:

$$\frac{1}{k-1} X_a X_a^T = X_b \tilde{A} X_b^T,$$
$$X_a = X_b \left[(k-1)\tilde{A}\right]^{\frac{1}{2}}.$$

  - So the transform is given by:

$$T = \left[(k-1)\tilde{A}\right]^{\frac{1}{2}}.$$