Constrained And Unconstrained Power Law Irregularity Models for Interpreting Strong Scintillation Data

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ABSTRACT

We compare constrained and unconstrained power law irregularity models for characterizing and interpreting scintillations caused by field-aligned irregularities in the equatorial ionosphere. The unconstrained irregularity model we consider is the two-component power law model described in (Carrano and Rino, Radio Sci., 2016). The constrained irregularity model encapsulates this two-component model between explicit outer and inner scales. For both models, numerical quadrature is employed to solve the fourth moment equation governing intensity variations following propagation through a thin phase changing screen. By comparing the scintillation predictions produced by these models we deduce the influence of outer and inner scales under strong scatter conditions and also how they affect the inverse problem, whereby phase screen parameters are inferred from measured scintillation time series.

Key words: Scintillation, phase screen, two-component spectra, equatorial irregularities.

1. Introduction

In a recent work, Carrano and Rino extended the phase screen power law theory of ionospheric scintillation to account for the case where the refractive index irregularities follow a two-component power law spectrum [1]. A specific normalization was invoked to exploit the self-similar properties of the problem and achieve a universal scaling, such that different combinations of perturbation strength, propagation distance, and frequency produce the same results. Numerical quadrature was employed to obtain solutions of the 4th moment equation governing the intensity fluctuations resulting from propagation through two-dimensional field-aligned ionospheric irregularities. In a companion paper, Rino employed this same two-component structure model to interpret in-situ observations of the disturbed low-latitude ionosphere made by the C/NOFS satellite [2].

This forward-modeling approach may also be applied to the inverse problem, whereby phase screen parameters are inferred from measured scintillation time series as a means of interpreting them. The screen parameters are determined by minimizing the difference between the measured and calculated intensity spectral density functions (SDFs) in a least-squares sense. This technique was used to interpret strong GPS scintillations collected during the 2002 Conjugate Point Equatorial Experiment (COPEX) in Brazil [3]. It was subsequently used to interpret scintillation observations along geostationary satellite links at VHF and L-band at Ascension Island [4].

2. Results and Discussion

Here we share some preliminary results. Figure 1 compares the intensity SDFs predicted using the unconstrained (black) and constrained (red) irregularity models. Shading has been used to delineate the wavenumber range over which departures from power law behavior occur due to the nonlinear effects characteristic of strong scatter conditions. In the wavenumber region of linear response (no shading), the intensity SDF is proportional to phase SDF at each wavenumber. The effects of a spectral break that occurs in this wavenumber region are therefore felt locally. The outer and inner scales shown in panels a) through c) occur within the linear response region and thus the black and red curves are coincident except near the break. The outer and inner scales shown in panels d) through e) occur in the region of nonlinear response because the universal scattering strength U is larger. The effects of the break can be highly nonlocal in this situation, depending on the shape of the phase SDF. In these panels differences between the black and red curves can be seen at wavenumbers quite distant from the break wavenumber.



Figure 1. Comparison of intensity SDFs corresponding to unconstrained (black) and constrained (red) power law irregularity models. The abscissa (μ) is a dimensionless wavenumber with μ =1 corresponding to the Fresnel wavenumber. Black dotted lines depict low and high wavenumber asymptotes for the unconstrained model. Regions of departure from power law behavior are shown with shading. Red vertical lines indicate the wavenumber of each spectral break. The universal scattering strength (U) and phase spectral indicies (p_1 and p_2) are labeled.

<u>Shallow Spectrum Case</u>: In panel d) the inner scale occurs in the wavenumber region of nonlinear response and the phase SDF is shallow ($p_1=p_2=2.3$). The intensity SDF is considerably higher at all wavenumbers when the inner scale is present. Physically, this occurs because the suppression of small scale structure by the inner scale promotes focusing effects by the irregularities which remain. This focusing generates intensity fluctuations approximately uniformly at all scales. As discussed in [1] this occurs for all shallow two-component spectra with $p_1 \le p_2 < 3$, which are sensitive to an inner scale but insensitive to an outer scale. The effect is more pronounced for shallower spectra.

<u>Steep Spectrum Case</u>: In panel e) the outer scale occurs in the wavenumber region of nonlinear response and the phase SDF is steep ($p_1=p_2=3.7$). The intensity SDF is much narrower at high wavenumbers when the outer scale is present. Physically, the suppression of large scale structure by the outer scale mitigates large scale focusing effects. This focusing generates intensity fluctuations primarily at small scales, which is an example of a highly non-local interaction that occurs during strong scatter. As discussed in [1] this occurs for all steep two-component spectra with $p_2 \ge p_1 > 3$, which are sensitive to an outer scale but insensitive to an inner scale. The effect is more pronounced for steeper spectra.

<u>*Mixed-slope Spectrum Case*</u>: In panel f) both outer and inner scales occur in the wavenumber region of nonlinear response and the phase SDF is of mixed slope type ($p_1=2.3$, $p_2=3.7$). It is shown in [1] that mixed slope spectra with $p_1 < and p_2 > 3$ are relatively insensitive to both outer and inner scales.

All real manifestations of turbulence are constrained by inner and outer scales (although the latter may be difficult to characterize due to a departure from statistical stationarity). As such, one might expect the constrained power law model to produce a more realistic characterization of the turbulence. However, it is argued in [1] and also in this paper that the constrained and unconstrained models are effectively equivalent so long as the universal strength of scatter (U) is sufficiently small that outer / inner spectral breaks cannot be "felt." The threshold for U at which a spectral break can be felt depends on the shape of the phase SDF and is predictable based on theoretical considerations [1]. Since U depends on the strength of the ionospheric perturbation, frequency, and propagation distance past the screen, all of these play a role in determining whether or not a spectral break will significantly influence the scintillation.

The presence of a spectral break will also influence the inverse problem if the strength of scatter exceeds this threshold. If it is exceeded, the lack of an outer or inner scale in the irregularity model can lead to inaccurate retrieval of phase screen parameters from the measured scintillation time series. We will demonstrate this by fitting the intensity SDF predicted using both constrained and unconstrained irregularity models to time series of VHF and L-band observations at Ascension Island.

References:

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