#### Playful Math = Engaged Learning



# More Than Counting: Learning to Label Quantities in Preschool

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arly math is as important as early literacy for children's future achievement. In fact, early math skills are more predictive of later school success than early reading skills (Duncan et al. 2007). During preschool many children start to acquire a deeper understanding of number principles, and the quality and quantity of teachers' support of math learning plays a crucial role in the growth of children's math knowledge. The goal of this article is to support teachers of young children by clarifying two key numerical concepts that children acquire during preschool: understanding how to count and how to label quantities. Along the way, educators will learn strategies for teaching these concepts, discover the types of errors many young children make as they gradually acquire the two concepts, and find out how to address the misconceptions and errors.

The primary focus of math instruction in most preschools is on teaching children how to count (Early et al. 2005; Lee & Ginsburg 2009). This is an important skill, but recent research shows that during this developmental period, it is crucial to acquire another skill: attaching number word "labels" (one, two, three) to small sets of objects (Le Corre & Carey 2007; Sarnecka & Lee 2009). Learning to label small set sizes is an essential stepping stone for preschool children on their way to developing an understanding of the cardinality principle-that is, understanding that the last number stated when counting a set of objects represents the numerosity, or quantity, of the whole set. Once children understand cardinality, they can identify how many items are in any countable set. This is crucial for developing more complex numerical knowledge, including how to solve addition and subtraction problems.

# How do preschoolers' numerical skills develop?

Helping children understand cardinality is a lengthy process that involves teachers modeling and supporting both counting and labeling the quantities of sets. Numerical development in preschool includes several skills that are developed simultaneously. Learning one concept or skill (e.g., one-to-one counting, also called one-to-one correspondence) depends on improvements in another (e.g., labeling set sizes), and vice versa. Since learning concepts and skills seldom happens in isolation, they should not be taught in isolation. "Development of Counting and Labeling Set Sizes," a Venn diagram with three overlapping circles, represents the simultaneous development of labeling small set sizes, one-to-one correspondence, and rote counting. These skills typically develop between ages 2 and 3.5 (i.e., between 24 and 42 months). The repeated practice of these skills across many learning experiences leads to children's ultimate understanding of cardinality, which, on average, occurs between ages 3.5 and 4 (but can range from ages 2.9 to 5.6). For purposes of simplicity, in this article we discuss counting and labeling set sizes separately-but we hope teachers will keep in mind that these concepts overlap.



## How does labeling set sizes lead to understanding cardinality?

We begin with children's development in labeling set sizes, because it is based on relatively recent research findings (Le Corre & Carey 2007; Sarnecka & Lee 2009) and therefore may be unfamiliar to some teachers. Many young children—even those who can count up to 10 objects—may not understand cardinality. When they finally reach this insight, it is a major breakthrough in their understanding of numbers and serves as the foundation for learning addition and subtraction—"only children who have mastered the cardinal principle, or are just short of doing so, understand that adding objects to a set means moving forward in the numeral list, whereas subtracting objects means going backward" (Sarnecka & Carey 2008, 662).

Acquiring this understanding takes time. Learning to count and learning to use numbers to label sets of objects—particularly small sets—are critical components of cardinality. In fact, labeling small set sizes may be what most aids conceptual understanding of quantity. Research indicates that the process of learning to label quantities of sets begins with infants and toddlers, who can already tell the difference between small groups of one versus two versus three objects (Cordes & Brannon 2009). Even though preschoolers have been able to see such differences for quite a while, many still have not discovered that the differences among small sets of objects are connected to the concept of quantity. It is critical for teachers to understand that children initially learn to label set sizes by attaching number words *only* to the specific amounts they have been able to discern from infancy (Le Corre & Carey 2007; Sarnecka & Lee 2009). When they can assign the label "two" to a group of two blocks, or the label "three" to a group of three blocks, they are beginning to learn that number words are a unique kind of descriptor that refers to specific quantities.

## Labeling small set sizes may be what most aids conceptual understanding of quantity.

Over time, children develop the ability to use number words to label increasingly larger sets. Typically, children first learn to associate the label "one" with one object; next, they begin to label sets of two, and then three objects before being able to do so for larger sets. Children are able to use number words to label smaller sets first because these quantities can be identified perceptually and sets of these sizes are far more frequently encountered.

Understanding how young children come to label set sizes has important implications for teaching young children about quantity. Recent research shows that the support mothers of 3-year-olds provided by making comments such as "There are three pennies here" or asking "How many Duplos are in this pile?," was more important in predicting first grade math skills than either support of counting or support of written number identification (Casey et al. in press). Why would labeling set sizes be so powerful? When children learn that specific number words can be used to describe certain amounts, they begin to understand that numerosity is a unique and often relevant attribute of the world. (The process is similar to how they come to understand that color is an attribute of the world: children observe others, learn to use color term, and apply specific color words to describe a particular attribute of an object.) Thus, by helping children use number words to label small quantities of objects whose number they can already distinguish, children gain a deeper understanding of numerosity.

#### How do counting skills develop?

Children's ability to count sets larger than three or four develops gradually. This process occurs as they refine their knowledge of the counting sequence and as they learn how to apply the counting sequence to counting numbers of objects. Children's recitation of the counting sequence becomes longer and more stable over time. Initially, children may rote count in random order and may even start the count sequence at a number other than one, saying, "three, four, two, five." With practice, children become able to recite the numbers they know in order. This demonstrates their understanding of the *stable-order principle*—that number words must follow a specific sequence.

While learning to rote count, young children also begin to point to objects when reciting number words. Initially, however, children's pointing does not necessarily match up with the words they are reciting. They may point to objects too quickly or too slowly compared with their verbal recitation of number words. After multiple opportunities to observe counting being modeled, children come to understand that each object should be counted once and only once-which is the one-to-one correspondence principle. They are then able to say the number words in order, matching one number word to each object. A final achievement in learning to count is being able to start counting accurately from any number, often referred to as counting on. For example, when asked to start counting at five, a child who has achieved this milestone will say "six, seven, eight" and so on.

Note that this progression of skills can happen multiple times for different number ranges. Young children may be able to accurately recite number words and count sets of objects up to five while still making errors for sets with six or more objects. Thus, it is important to pay attention to children's skills in different numerical ranges and contexts.

As they practice counting larger sets, children start to notice that counting can be used to determine the quantities of larger sets and that the final number word stated is the correct label for sets of *any* size. This indicates that the children have achieved cardinality they finally understand conceptually why this last number represents something unique.

### **Children's Common Errors and Corresponding Teaching Strategies**

Principle	Type of error	Reason for child's error	Teacher strategy
Stable order	Sequencing error Omission of number(s) Repetition of number(s) Wrong numerical order	<ul> <li>Has a fragile memory of the number words or the order</li> <li>Is distracted</li> </ul>	<ul> <li>Have children practice rote counting</li> <li>Join in children's rote count at the point of error</li> </ul>
One-to-one correspondence	<ul> <li>Tracking error</li> <li>Objects counted more than once</li> <li>Objects skipped with counting</li> <li>Coordination error</li> <li>Failure to count in step with pointing</li> </ul>	<ul> <li>Doesn't understand that each item is counted only once</li> <li>Counts too fast</li> </ul>	<ul> <li>Model pointing and counting (to objects in a row, at first)</li> <li>Have children practice pointing and counting</li> <li>Teach ways to keep track of the count (e.g., mark each counted object with a finger, or move aside counted objects)</li> <li>Vary the speed of the count</li> <li>Have children practice counting up (from a number other than one)</li> </ul>
Cardinality	<ul> <li>Guessing error</li> <li>Failure to produce the correct number of objects for a label</li> <li>Repeat-the-count error</li> <li>Response of a counting string ("One, two, three, four, five") when asked how many in a set of five</li> <li>Implementation error</li> <li>Use of the wrong number word</li> <li>Response of the wrong quantity for a label</li> </ul>	<ul> <li>Miscounts objects</li> <li>Has an incorrect mental representation of the quantity requested</li> <li>Confuses the rote count with the set label</li> <li>Doesn't know the correct numeral</li> </ul>	<ul> <li>Label sets first before counting (for small numbers)</li> <li>After counting, emphasize that the last number counted is the label for the set</li> <li>Have children practice labeling and producing sets</li> <li>Model and encourage labeling sets before and after counting</li> <li>Predict the quantity before counting</li> <li>Use questioning ("Can you give me five blocks?")</li> </ul>

## Children's errors: What they mean, and what to do about them

One of the most useful ways to determine children's knowledge of counting and cardinality is to notice the kinds of errors they make. Targeting instruction specifically to these errors can help children progress to the next phase of understanding. (See "Children's Common Errors and Corresponding Teaching Strategies.")

## Errors and teaching strategies related to cardinality

Acquiring cardinality, for most young children, takes six months or more. Children's mistakes reveal which

aspects of cardinality are still being developed. There are three kinds of cardinality errors children typically make: attaching labels to quantities unsystematically (guessing errors); simply repeating the count—1, 2, 3, 4—when asked how many objects there are (repeatthe-count errors); and mislabeling the quantity of a set (implementation errors).

**Guessing errors.** Although children may know some number words, such as those for 1 through 10, they may not have learned to assign these number words to a particular quantity. Thus, for numbers that children have not yet learned to label, children may make a guessing error. This involves attaching labels to quantities unsystematically by using an arbitrary number word or by generating a random number of



objects in response to a request to produce a particular number of objects. For example, when a teacher asks for five blocks, a child who knows what quantity two refers to (but has not yet learned three, four, or five) may grab any number of blocks that she knows is more than two.

Teachers can help children at this phase by creating multiple opportunities for children to label ("How many?") and produce sets ("Give me *n* objects."). Teachers can begin demonstrating the concept by attaching a label to sets with quantities less than four, which children can easily discern without counting. By repeatedly hearing number words being used to describe these small sets, children come to understand that these quantities can be differentially labeled one, two, or three, based on how many objects there are in a particular group, and they learn to use these labels. At the same time, children need support to understand that counting can be used to discern the quantity of sets larger than three.

**Repeat-the-count errors.** Some children may confuse counting a set with stating its quantity. In fact, they may have frequently observed adults equate the two: "How many crackers do you have? One, two, three, four, five!" Adults may assume that if a child counts a set, then he must know its quantity. For example, a teacher might ask a child how many crackers he has and accept the production of a counting string—"One, two, three,

four, five"—as the answer. She can uncover whether he has this misconception by asking "How many?" again, after the child has already counted. Children with this misunderstanding are likely to just recount the set rather than indicate the total number of objects.

When having children produce a label, researchers have found that it helps to ask children to predict how many objects there are before the children begin counting (Mix et al. 2012). Making a prediction gives them practice generating a label and brings their attention to the result of the count. It also helps them learn to generate plausible estimates. Further, teachers can explicitly tell children the cardinal principle: "Remember the last number you say, because it tells you how many things were counted."

Thus, as often as possible, teachers should model and encourage children to label sets before and after completing a count. A simple three-step procedure, depicted in "Steps to Support Children's Labeling Set Sizes," is optimal for helping young children acquire this idea.

**Implementation errors.** Even when children understand cardinality, they may sometimes use the wrong number word or produce an incorrect quantity. An implementation error is different from a guessing error in that it is usually less random. The child may attempt to rely on memory, but she makes a mistake in execution because she has not yet developed a strong and reliable association between the size of the set and its label. In this case, practicing rapidly labeling sets can be helpful. A teacher can show the quantities of one to three in different organizations, using a die, picture cards, or an abacus to help make the recognition of these quantities more automatic. correct these mistakes either in the context of counting objects or just rote counting. It's helpful to provide as many opportunities as possible for children to practice rote counting—count while waiting for the bathroom, count while changing activities, count crackers for snack—count, count, count!

Teachers can also anticipate when

individual children are going to make

a mistake and jump in at just that point. For a

3-year-old who counts 1, 2, 3, 6, 7, 10, the teacher can

five" before the mistake occurs. This draws the child's

attention to the portion of the counting string he should

let him count alone until 3, then quickly add "four,

focus on.

For larger sets, implementation errors are generally a result of miscounting. Children who make these errors understand cardinality, but their fledgling counting skills need support. Teachers can encourage children to re-count and provide support for their counting, as described in the next section.

## Errors and teaching strategies related to counting

There are three main kinds of counting errors children make: errors in sequencing the rote counting string (sequencing errors); errors in keeping track of which objects have or have not been counted (tracking errors); and errors in matching their counting string to the objects during the counting process (coordination errors).

Sequencing errors. Because most preschoolers have not had much experience in hearing or practicing the counting string, they often remember only pieces of it. Children master the counting string in segments, starting with the beginning portion (1-3) and slowly extending it over time. The most common mistake young children make is to omit numerals. A 3-year-old commonly rote counts something like this: 1, 2, 3, 6, 7, 10. A 4-year-old may rote count something like this: 1, 2, 3, 4, 5, 6, 7, 10. In addition, children occasionally reorder numbers in the sequence (e.g., 12, 3, 6, 4, 8, 7, 10) or reuse numbers (e.g., 1, 2, 3, 4, 2, 3, 4). These mistakes may happen because children's memory of the counting string is fragile or they become distracted while counting or they are trying to count more objects than they have number words for.

These errors are independent of being able to point to each object being counted. Teachers can help children When having children produce a label, it helps to ask children to predict how many objects there are in a small set.

**Tracking errors.** Children commonly have difficulty keeping track of which objects they have counted. They sometimes count an item twice (double count) or skip an object. Younger children sometimes make these mistakes in counting small sets of objects. They probably do not yet understand the one-to-one principle and would benefit from having many opportunities to see adults count while pointing to objects. Initially, counting objects organized in a row makes it easier for children to notice the one-to-one relationship.

Older children, who understand one-to-one correspondence, sometimes make the mistake when asked to count larger sets, sets organized in a random or circular array, or unmovable objects (like pictures in a book) simply because they don't have a systematic strategy for keeping track of what they have counted. To help children avoid making this mistake, adults can point to objects along with children, keep their finger on the item the children counted first (especially if the objects are arranged in a circle), show children how to indicate which items have been counted, or encourage children to organize objects in rows before counting them. It is important to create opportunities for children to practice these strategies. Teachers might purposefully arrange objects in challenging ways, such as circles and clusters.

Opportunities for counting and labeling sets are everywhere, but children need help to take advantage of them.

**Coordination errors.** Even if a child understands that each item must be counted only once, she must coordinate

reciting the counting string with pointing to the object. Coordination errors often occur when rote counting occurs more rapidly (e.g., in a sing-song fashion) than pointing to the objects. This sometimes leads to extra tags (i.e., saying an extra number word before pointing to the next item). Trouble pacing the count can also lead to extra tags at the end of the sequence as the child finishes the sequence even without any objects remaining to point to.

Children make fewer coordination errors after they gain more control of their recitation of the counting string; they need to learn to recite a series of individual numbers rather than an overall string. In the same way that children sometimes sing "L-M-N-O-P" as one chunk, rather than as individual letters, children can think of the counting string, or parts of it, as an unbreakable chunk. If "one-two-three-four-fivesix" is thought of in this way, then slowing down the recitation or stopping at five can be difficult. To help children overcome coordination errors, teachers can rote count with children at different paces-sometimes slow, sometimes fast-and encourage children to start counting on from anywhere in the sequence ("Let's start counting at four") so they can "break" the string. When children are counting objects, teachers can recite the numeral sequence along with children in order to match the pace of the recitation to the child's pointing, or to emphasize the last number word to indicate to children that they should stop the sequence.



Teachers can also provide different types of objects to count, starting with groups of identical objects (easier) and moving to varied objects within the array (harder). This helps children develop an appreciation of the abstract characteristic of numbers: anything can be counted (Fuson 1988). The arrangement of objects to be labeled and counted can also be varied. This helps children understand conservation of number: though an array may look larger because objects are more widely scattered, it does not mean there are more objects in the set; if the array looks smaller because objects are close together, it does not mean that there are fewer objects in the set (Piaget 1997).

# **Conclusion: Extending numerical development in the classroom**

With an understanding of the development of key numerical principles (most critically, stable order, one-toone correspondence, and cardinality), and with careful observation of a child's successes and patterns of errors, teachers can learn to recognize teachable moments. Teachers can provide optimal support for individual development through ongoing monitoring that reveals when to give appropriate guidance at the edge of each child's competence. Teachers can also encourage the development of numerical concepts and skills in more general ways. Opportunities for counting and explicit labeling of sets are everywhere, but children need help to look for and take advantage of them. It is important for teachers to incorporate a variety of adult-guided activities that deepen understanding of the meaning and usefulness of numbers. They can also scaffold development by providing opportune interventions in children's ongoing and self-initiated activities that extend what the child knows and can do in relation to numbers. Equipped with these ideas, teachers can extend children's understanding of numbers during play, mealtimes, and other daily activities— the possibilities are endless!

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