Early Childhood Research Quarterly 32 (2015) 204–214

Contents lists available at ScienceDirect

Early Childhood Research Quarterly

# Preschoolers' strategies for solving visual pattern tasks

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ARTICLE INFO

*Article history:*
Received 26 August 2014
Received in revised form 25 March 2015
Accepted 2 April 2015
Available online 23 April 2015

*Keywords:*
Patterning
Relational thinking
Early childhood mathematics
Working memory
Inhibitory control

ABSTRACT

This study examined preschoolers' ($N = 66$) strategies for solving a range of visual repeating pattern tasks. An analysis of the kinds of patterning tasks preschoolers encounter, such as duplicating, extending, and transferring patterns to superficially different materials, suggested the tasks could be solved using either a one-to-one appearance matching strategy or a relational similarity strategy. In the present study, preschoolers completed a series of patterning tasks and their strategies were examined by analyzing (1) accuracy, (2) errors, and (3) the relation of visuospatial short-term memory, working memory, and inhibitory control to accuracy and errors. The pattern of results indicated that preschoolers do use both kinds of strategies, but that the frequency with which they use each strategy depends on task complexity. Preschoolers tended to use an appearance matching strategy on duplicate and extend tasks and a relational similarity strategy on transfer tasks. Implications for understanding what patterning instruction is most likely to support relational reasoning are discussed.

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## Preschoolers' strategies for solving visual pattern tasks

Early experience with patterns is believed to be important for later algebraic reasoning because it provides opportunities for engaging in relational thinking, such as thinking about how elements within a particular problem relate to each other (Fuchs et al., 2012; Lee, Ng, Bull, Pe, & Ho, 2011; Mason, 1996; Mulligan & Mitchelmore, 2009; Orton & Orton, 1999; Warren & Cooper, 2006; Whitin & Whitin, 2011). Indeed, some recent studies have found that elementary school children's ability to solve patterning tasks is positively related to their performance on arithmetic word problems that involve algebraic concepts, such as the concept of equality and variable identification (Fuchs et al., 2012; Lee et al., 2011). Further, these links remain after controlling for a range of domain-general cognitive abilities. Fuchs and colleagues found nonverbal problem solving, including patterning abilities, to be uniquely predictive of arithmetic word problem skills in both first grade and third grade, even after controlling for processing speed, working memory, and language abilities (Fuchs et al., 2005, 2006, 2012).

Despite increasing evidence of a link between early pattern knowledge and later mathematics performance, little is known about the strategies children use to solve basic patterning tasks or the cognitive processes involved. A better understanding of these two aspects could help clarify which patterning tasks are most likely to benefit later mathematics performance and *why*. In the present study, we propose that early patterning tasks vary in the cognitive processes and strategies they require. More specifically, we propose that repeating patterning tasks can be solved using either a one-to-one appearance matching or relational similarity strategy and that only tasks that involve mental representation and manipulation of the repeating unit are likely to elicit use of a relational similarity strategy, which, in turn, might lead to better relational reasoning and support later mathematics learning.

### *Preschoolers' pattern knowledge*

Patterns are sequences with a replicable regularity that can vary along a number of dimensions (Papic, Mulligan, & Mitchelmore, 2011; Warren & Cooper, 2006). Patterns can vary in their *structure*, such as whether they have a repeating unit (e.g., 1, 2 – 1, 2, with a unit of "1, 2"), a repeating rule (e.g., 1, 3, 5, 7, which has the rule of "+2"), or a growing relationship (e.g., 1, 2, 4, 7, in which each successive difference grows by 1). They also can vary in the *content* of the regularity, such as whether they consist of numbers (e.g., 1, 3, 9, 27), shapes, or colors. Finally, patterns can vary in the *complexity* of the regularity, ranging from simple regularities (e.g., AB-AB; 1, 3, 5, 7) to more complex ones, such as with more complicated units (e.g., ABCABB-ABCABB).

Repeating patterns comprised of visual content (e.g., colors/shapes) are a natural entry point for learning about patterns

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http://dx.doi.org/10.1016/j.ecresq.2015.04.004
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because they do not require any additional knowledge, such as skip counting or arithmetic operations, involved in later numerical patterns. By preschool, children are able to identify, create, and explain a variety of visual repeating patterns (Papic, 2007; Papic & Mulligan, 2007; Papic et al., 2011). For example, Rittle-Johnson, McLean, McEldoon, and Fyfe (2013) found the majority of the preschoolers they tested were able to duplicate a visual repeating pattern, even with a complex repeating unit (AABB). This facility with patterning extends beyond instructional interactions. Observational studies of preschool classrooms indicate that children spontaneously engage in pattern identification and pattern making using classroom materials (Fox, 2005; Ginsburg, Inoue, & Seo, 1999; Waters, 2004).

Preschoolers are not equally successful, however, on all tasks involving visual repeating patterns (Papic et al., 2011; Rittle-Johnson et al., 2013). Prior research has found that a majority of children (75%) between the ages of 3 and 5 years accurately complete pattern duplication tasks (i.e., reproduce a pattern using the same materials) and about half accurately complete pattern extension tasks (i.e., continue a pattern). On the other hand, fewer than a third of preschool-aged children accurately complete tasks that require them to isolate a repeating unit (i.e., produce one instance of the repeating unit) or transfer the pattern (i.e., create the same pattern using superficially different materials; Rittle-Johnson et al., 2013). This progression suggests that an analysis of the cognitive demands of different patterning tasks as well as the approaches children may use on them could provide information about how best to support preschoolers' emerging patterning knowledge.

*Strategies for solving visual repeating pattern tasks*

It has been posited that early patterning tasks lay a foundation for later algebraic reasoning because they provide opportunities for children to practice relational thinking and rule deduction (Charles, 2005; Fuchs et al., 2012; Lee et al., 2011; Mason, 1996; Mulligan & Mitchelmore, 2009; Orton & Orton, 1999; Warren & Cooper, 2006; Whitin & Whitin, 2011). Relational thinking is broadly defined as the process of making comparisons and recognizing similarities and differences to discern meaningful structure and patterns underlying information (Dumas, Alexander, & Grossnickle, 2013). It has been posited to be important for mathematics in a number of ways, including understanding the equal sign (McNeil & Alibali, 2005), arithmetic concepts such as commutativity (Farrington-Flint, Canobi, Wood, & Faulkner, 2007), and algebraic problem solving (English & Sharry, 1996). In the early learning context of visual repeating patterns, thinking about relational similarities would include considering how individual pattern components combine to create units (i.e., understanding that in an AB-AB pattern, A and B, together, comprise the unit), how two units within a pattern possess the same underlying structure (i.e., that an AB-AB pattern consists of instances of identical "AB" units), and how the same unit can be represented using superficially different materials (i.e., "circle, square – circle, square" and "blue, red – blue, red" are both AB-AB patterns). While the value of early patterning activities is believed to lie in their promotion of relational thinking, however, it remains to be tested whether the kinds of patterning tasks children are exposed to in early childhood actually lead them to focus on relational similarities.

An analysis of the kinds of patterning tasks preschoolers encounter suggests that while the tasks could be solved using relational strategies as described above, the majority also can be solved successfully, and perhaps more easily, using a one-to-one appearance matching strategy, or matching superficial features without considering underlying structure. Patterning tasks vary in the extent to which they require mental representation and manipulation of the repeating unit. As illustrated in Fig. 1, some patterning tasks ask children to duplicate or extend a pattern, while others ask children to isolate and transfer the underlying structure of a pattern to new materials (Rittle-Johnson et al., 2013; Warren & Cooper, 2007). Duplication and extension tasks may be less difficult for preschool children because they can be completed using appearance matching. For example, a child could duplicate a pattern by matching the color or shape of each item in the pattern, one at a time. On the other hand, unit isolation and transfer tasks may be more difficult because they require children to use relational similarity strategies to mentally represent, abstract, and manipulate the unit of repeat (Rittle-Johnson et al., 2013; Warren & Cooper, 2006).

Children often possess multiple strategies for solving problems and select among them based on task demands as well as their proficiency in executing the strategies available to them (Chen & Siegler, 2000; Siegler, 1996). In the case of visual repeating pattern tasks, we hypothesize that preschoolers are likely to use a one-to-one appearance matching strategy, unless the task complexity requires a relational similarity strategy for accuracy. Even as toddlers, children are able to engage in one-to-one reasoning and superficial appearance matching (Gelman & Meck, 1983; Izard, Streri, & Spelke, 2014; Mix, 2002; Sophian, 1988). On the other hand, reasoning based on relational similarity, particularly when the relations are unknown, develops later between the ages of 3 and 5 years (Gentner, 1989; Goswami, 1991, 2013; Rattermann & Gentner, 1998; Singer-Freeman, 2005). Because children are more capable of one-to-one appearance matching and it can yield a high accuracy on many patterning tasks, it seems likely that this would be preschoolers' predominant approach to solving patterning tasks.

Research on children's relational reasoning provides support for the hypothesis that children may tend to use an appearance matching strategy on patterning tasks. When presented with the opportunity to use either appearance matching or relational similarity, even 5-year-olds are more likely to use appearance matching (Gentner & Rattermann, 1991; Paik & Mix, 2008). Gentner and Rattermann (1991), for example, found that in a task in which children could search for a sticker under objects based on relational similarity (same relative size as in the example search task) or appearance matching (same object type as in the example task), three- and four-year-olds consistently searched based on appearance matching. Further, children over-rely on surface features even when they are irrelevant for solving the task (Kotovsky & Gentner, 1996). For example, children have greater difficulty comparing set sizes when two sets are comprised of dissimilar objects or when objects within sets are heterogeneous than when both sets are homogenous and contain similar objects (Mix, 2008). Thus, unless patterning tasks are carefully structured so that a relational similarity strategy is required or more likely to lead to an accurate response, children may not use it. Only patterning tasks that require children to isolate and transfer the repeating unit, such as asking children to make the "same pattern" using superficially different materials (another color, shape, or both), may push children to attempt a relational similarity strategy. Further, because children are being forced to use a strategy with which they are less accustomed, we would expect lower accuracy on these types of tasks.

*Memory and inhibition in solving patterning tasks*

Given our analysis of the kinds of strategies that could be used on early patterning tasks, we propose that memory and inhibition are involved and that exploring the involvement of these processes could provide insight into children's strategy choice. Some earlier work has implicated verbal working memory in children's performance on patterning tasks (Holzman, Pellegrino, & Glaser, 1983; Rittle-Johnson et al., 2013). For example, Rittle-Johnson et al. (2013) found verbal working memory was related to ability estimates of four-year-olds' patterning performance on a patterning

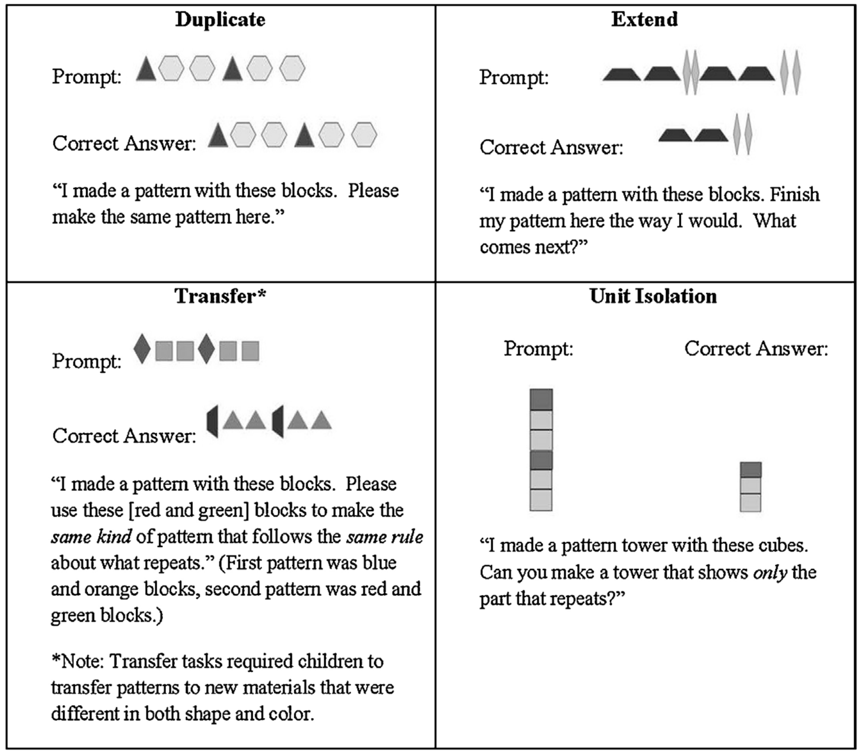


**Fig. 1.** Sample items and correct responses for each pattern task type.

assessment that included a range of patterning tasks. On the other hand, Lee et al. (2012) found no relation between working memory measures and six-year-olds' performance on visual repeating patterning tasks that required children to determine what came next in the sequence. In addition to methodological differences between studies, such as age of participants, it is possible that the inconsistent finding of a relation between memory and patterning performance is because the relation depends on the nature of the patterning task and the predominant strategy being used to solve it.

Memory consists of multiple components (Alloway, Gathercole, & Pickering, 2006; Baddeley, 1986; Baddeley & Hitch, 1974; Shah & Miyake, 1999). Short-term memory refers to an individual's capacity for temporary storage of information, while working memory refers to the ability to manipulate temporarily stored information. In this way, working memory is dependent on short-term memory (Alloway et al., 2006; Baddeley & Hitch, 1974). Additionally, memory is further divided into domain-specific storage areas for verbal and visuospatial input, which have been shown to be distinguishable by four years of age (Alloway et al., 2006; Bull, Espy, & Wiebe, 2008). A common model of the organization of these partially distinguishable but interrelated components of memory proposes three main components: verbal short-term memory, visuospatial short-term memory and a unitary, domain-general working memory (Alloway et al., 2006). The involvement of different components in processing has been found to vary as a function of task demands (Alloway & Passolunghi, 2011; Bull et al., 2008; DeStefano & LeFevre, 2004).

Consider a visually repeating pattern that involves blue and red blocks (blue, red – blue, red). If a child is asked to extend the pattern and uses a one-to-one appearance matching strategy then visuospatial memory should relate to performance. Further, because appearance matching does not require any mental manipulation of stimuli, individual differences in short-term memory should relate to accurate execution of this strategy more than individual differences in working memory. On the other hand, if a child is asked to transfer the pattern into a similar pattern using different materials and does so using a relational similarity strategy, working memory should relate to performance more than short-term memory because the child must actively transform information. Thus, simultaneously examining the relations between children's accuracy on patterning tasks and each of the memory components could provide insight into the strategies children use to solve them.

In addition to considering the relations between different memory components and patterning accuracy, examining the relation between inhibitory control and accuracy could also provide insight into children's strategies. Inhibitory control is defined as the ability to stop a predisposed or ongoing thought or behavior (Logan, 1994). Research in analogical reasoning, a type of relational reasoning, suggests that the ability to inhibit attention to irrelevant information may be critical in identifying relations among objects (Halford, 1993; Richland & Burchinal, 2013; Waltz, Lau, Grewal, & Holyoak, 2000). Inhibitory control may be necessary for relational similarity strategies in early patterning activities in order to allow the child to inhibit attention to superficial features to instead focus on underlying structure and patterns. Thus, if patterning is indeed requiring children to think relationally, then inhibitory control would be related to patterning performance.

### *The present study*

The present study investigated preschoolers' strategies on visual repeating patterns. Preschoolers completed a range of patterning tasks that varied in the extent to which they required mental representation and manipulation of the repeating unit. We hypothesized that children's predominant approach would be a one-to-one appearance matching strategy, rather than a relational similarity strategy, but that this would vary as a function of task complexity. Specifically, we predicted children would be more likely to use a relational similarity strategy when asked to isolate the repeating unit or to transfer the pattern to different materials.

Three approaches were used to infer strategy use. First, we examined children's accuracy on different patterning tasks. Based on previous literature demonstrating preschoolers are more accurate on appearance matching tasks than on relational similarity ones, we predicted that accuracy would be greater on tasks that could be solved using one-to-one appearance matching (e.g., duplication and extension) and lower on those that would be more likely to involve a relational similarity strategy (e.g., isolation and transfer).

Second, we conducted an item-level error analysis. Error analysis is a common approach for identifying strategies involved in problem-solving (Brown & Burton, 1978; De Corte & Verschaffel, 1981; Radatz, 1979; Shulman, 1970). Previous descriptions of preschoolers' errors on patterning tasks suggested that preschoolers commit some errors that would result from using an appearance matching strategy, such as skipping a single block within a pattern, and other errors that would result from incorrectly executing a relational similarity strategy, such as creating an incorrect pattern which captures part, but not all, of the core unit (e.g., making an AB pattern when asked to transfer a ABB pattern; Rittle-Johnson et al., 2013). Thus, we predicted that children's errors would vary systematically by task demands: greater one-to-one errors on tasks that could be solved easily using a one-to-one appearance matching strategy and greater relational errors on more complex tasks on which a relational similarity strategy was more likely to be used. While error analysis is sometimes used concurrently with retrospective strategy reports, we chose not to take this approach. It seemed unlikely that preschoolers would be able to describe a relational similarity strategy and we were concerned it would create fatigue that would affect performance.

Last, the relations among children's accuracy and errors on the patterning tasks and their memory and inhibitory control were examined. We predicted that visuospatial short-term memory, but not working memory, would be positively related to accuracy on tasks that can be easily solved using one-to-one appearance matching. Inversely, we predicted visuospatial short-term memory would be negatively related to one-to-one errors. The logic was that a one-to-one appearance matching strategy involves visually mapping elements, but no mental representation or transformation of the information. In contrast, we predicted that working memory would be involved in more complex tasks, such as isolation and transfer tasks, which require mental representation and manipulation of the core unit and, thus, on which a relational similarity strategy is more likely to be used. Specifically, we predicted working memory would be positively related to accuracy on these tasks and negatively related to relational errors. In addition, based on previous research implicating inhibitory control in relational reasoning, we predicted inhibitory control would be positively related to accuracy on tasks evoking relational similarity strategies and negatively related to relational errors.

## Method

### *Participants*

Sixty-six children (52% female) between the ages of 3 years, 11 months and 5 years, 7 months ($M$ = 4 years, 10 months, $SD$ = 5.2 months) were recruited from five preschools serving children from a range of SES backgrounds in a major northeastern city. Between 10 and 18 students participated from each school. Though information on the participating children's family's socioeconomic statuses was unavailable, almost half (45%) of the children attended schools serving low SES neighborhoods; at these schools, at least 70% of all attending children qualified for free or reduced lunch. Children were diverse in race and ethnicity: 44% White/Caucasian, 29% Hispanic/Latino, 15% Black/African American, and 12% Asian/Southeast Asian.

### *Procedure*

Children were tested one-on-one by one of three experimenters during the spring of the academic year over two sessions, which were approximately a week apart and lasted approximately 20 min each. In the first session, children completed two short-term/working memory measures, and three patterning tasks (Duplicate, Extend, Transfer). In the second session, children completed another patterning task (Isolate) and an inhibitory control task. The tasks were administered in this order for all children. The two memory measures were counterbalanced across children to control for potential order effects. All three experimenters had extensive experience working with the preschool population and administering empirical assessments. Thorough training sessions were conducted prior to the start of the study. These sessions included an instructional review of the scripts, a demonstration of administration, and opportunities to practice and receive feedback from the principal investigator. Weekly check-ins provided opportunities for the principal investigator and experimenters to discuss any issues that arose.

Sessions were videotaped to allow for follow-up analyses and for the principal investigator to ensure that proper procedures were followed.

### *Measures*

#### *Memory tasks*

Two measures were used to assess verbal and visuospatial short-term memory and working memory: the WISC-IV Digit Recall task (Wechsler, 2003) and the Corsi Blocks task (Kessels, van Zandvoort, Postma, Kappelle, & de Haan, 2000). The digit recall measure was selected based on its prior connection with patterning (Lee et al., 2012; Rittle-Johnson et al., 2013), and the Corsi Blocks task was selected because of its similarity with the digit recall task, enabling comparability between visuospatial and verbal memory tasks (Lezak, 1995).

#### *Digit recall*

The WISC-IV Digit Recall task measures verbal memory (Wechsler, 2003). The task asks the child to repeat a sequence of orally-presented numbers in forward and backward order to measure verbal short-term memory and working memory, respectively. Sequences increase in length from 2 digits up to 9 until the child is no longer able to correctly repeat a sequence of a particular length on two consecutive trials. Test-retest reliability for children ages 6–16 is high ($r$ = 0.83; Williams, Weiss, & Rolfhus, 2003), and both the forward and backward digit recall tasks have been used successfully with preschool children in prior research (Alloway et al., 2006; Gathercole, Brown, & Pickering, 2003). Children's scores were calculated as the total number of sequences correctly repeated, with separate scores for the forward and backward tasks.

#### *Corsi blocks*

The Corsi Blocks task measures visuospatial memory. The task asks the child to tap a visually-presented series of blocks, arranged in a scattered array, in forward and backward order to measure visuospatial short-term memory and working memory, respectively. The same procedure and materials used in prior standardization work were used (Kessels, van den Berg, Ruis, & Brands, 2008; Kessels et al., 2000; Pagulayan, Busch, Medina, Bartok, & Krikorian, 2006). Sequence lengths increase from 2 blocks to a maximum of 9 blocks until the child is unable to correctly tap a sequence of a particular length on two consecutive trials. Test-retest reliability for older children, ages 11–16, is moderately high ($r = 0.70$–$0.79$; Orsini, 1994), and the task has been used successfully with preschool children in prior research (Bull et al., 2008; Orsini et al., 1987). Children's scores were calculated as the total number of block sequences correctly tapped, with separate scores for the forward and backward tasks.

Children's performance on the Digit Recall – Forward task served as the verbal short-term memory measure, and performance on the Corsi Block – Forward task served as the visuospatial short-term memory measure. To create a unitary working memory composite score (based on the findings of Alloway et al., 2006), we standardized children's scores on the Digit Recall – Backward task and the Corsi Blocks – Backward task and calculated the mean.

*Inhibitory control*

Inhibitory control was measured using the *Head-Toes-Knees-Shoulders* (HTKS) task (Ponitz et al., 2008). In this task, children play a game akin to Simon Says, but are told to do the opposite of what the experimenter says (e.g., touch their toes when the experimenter says to touch their head, and touch their shoulders when the experimenter says to touch their knees). Each statement is a trial; children complete up to 26 trials, depending on their performance. Full credit (2 points) is given for any correct response, and partial credit (1 point) for any self-corrected response. Items are summed for a total score (maximum = 52), with higher scores indicating greater inhibitory control. Inter-rater reliability with preschool children is high ($r = 0.95$–$0.98$; Ponitz et al., 2008), and prior work has shown the measure to be correlated with other measures of attention and behavioral regulation. Specifically, performance on the HTKS during the fall of kindergarten was correlated with teacher-reported classroom self-regulation during both the fall ($r = 0.34$, $p < 0.01$; Matthews, Ponitz, & Morrison, 2009) and spring ($r = 0.29$, $p < 0.01$; Ponitz, McClelland, Matthews, & Morrison, 2009), and was also correlated with parental reports of attention and inhibitory control ($r = 0.25$, $p < 0.01$; $r = 0.20$, $p < 0.01$, respectively; Ponitz et al., 2009). Though modest, correlations of this magnitude are consistent with prior research comparing caregiver reports with direct assessments of behavior (Eisenberg, Smith, Sadovsky, & Spinrad, 2004; Howse, Calkins, Anastopoulos, Keane, & Shelton, 2003; Smith-Donald, Raver, Hayes, & Richardson, 2007).

*Patterning*

A modified version of Rittle-Johnson et al.'s (2013) pattern assessment was used. Children completed 12 trials that included four different tasks (3 trials each) administered in the following order, as per previous research: Duplicate, Extend, Transfer, and Unit Isolation. Fig. 1 provides an example of each task. For each task, there was one ABB pattern, one AAB pattern, and one AABB pattern, presented in different orders for each task type. According to procedure in prior research, children were not given practice trials on Duplicate, Extend, and Transfer items. However, pilot data suggested that children needed extra scaffolding on the unit isolation task in order to understand what was being asked of them; thus, children were given up to three opportunities to practice and receive feedback on the unit isolation items before being scored on the task. The experimenter recorded the child's exact answer for each item.

*Error coding*

After all data were collected, one research assistant reviewed video recordings of the sessions as well as written records of children's answers (i.e., written explanations of the responses generated, such as "circle-circle-square-square-circle-square-square," which were recorded at the time of testing) in order to code incorrect responses as one of three kinds of errors: one-to-one, relational, or off-task. *One-to-one errors* included errors that demonstrated attention to individual items in the pattern but disregarded their role as part of a repeating unit. This category of errors included switching the order of two blocks in a unit (partial correct); beginning correctly but stopping too soon and failing to reproduce the entire correct answer (incomplete correct); or treating the entire sequence as the unit and reproducing the entire pattern when asked to produce the unit (duplicate; note: duplicate errors could only occur on Extend and Unit Isolation tasks). *Relational errors* included errors that demonstrated attention to repeating units, with individual elements ordered as a pattern but containing an incorrect repeating unit. This category included all instances when children created an incorrect, but viable, pattern (e.g., presented with an AABB-AABB pattern the child produced an AB-AB pattern or some other pattern, such as ABB-ABB). *Off-task errors* included errors in which responses were unrelated to the task and showed no awareness of patterns. The most common errors in this category were sorting the blocks/cubes by shape/color, placing the blocks/cubes in random orders, or using the blocks or cubes for other purposes (e.g., building shapes with the blocks). Each incorrect trial received exactly one error code reflecting the most relevant error type.

To assess reliability, the principal investigator double-coded trials for 10% of sessions. Inter-rater reliability between the primary coder and the principal investigator, calculated as the percentage of trials which received the same code from both coders, was extremely high (98% agreement, *Cohen's* $k = 0.93$).

## Results

Preliminary analyses indicated no experimenter effects: a MANOVA with experimenter as the fixed effect and our 12 measures (cognitive tasks, patterning task accuracies, and error types) as the dependent variables found no overall effect of experimenter (*omnibus* $F = 1.39$, $p = 0.13$). Further, analyses indicated no overall effects of gender, race/ethnicity, neighborhood SES, or school on the measures, omnibus *F*s ranging from 1.09 to 1.37, *p*s ranging from 0.08 to 0.39. Thus, these variables were not considered in subsequent analyses. Age was moderately correlated with patterning performance, $r(64) = 0.44$, $p < 0.001$; thus, age was used as a covariate in the main analyses.

Despite the nested nature of the data (children within classrooms), we did not use multilevel modeling in the analyses for practical and empirical reasons. Practically, the sixty-six children were recruited from nine classrooms across five preschools. Thus, each school had just 10–18 participating children, with one classroom having as few as three participating children. Thus, cell sizes were inadequate for multilevel modeling (Bickel, 2007; Heck & Thomas, 2000; Hox, 2002). Empirically, a MANOVA testing the effect of school on the outcome measures (cognitive tasks, patterning task accuracy, and error types) found no overall effect of school (*omnibus* $F = 1.37$, $p = 0.08$). This finding indicated that a misestimation of effects based on nesting was unlikely.

*Accuracy on patterning tasks*

Overall, children were correct on 67% of trials, with a mean total score of 8.1 correct trials ($SD = 2.91$) out of a possible score of 12. A repeated-measures ANCOVA with number of correct trials per task as the within-subject factor and controlling for age found that, as expected, accuracy varied as a function of task, $F(3, 62) = 12.89$, $p < 0.001$, $d = 0.90$. Children were most accurate on Duplicate tasks ($M = 2.67$ trials correct out of 3, $SD = 0.71$), and less accurate on Extend tasks ($M = 1.92$ trials correct, $SD = 1.15$), Unit Isolation tasks

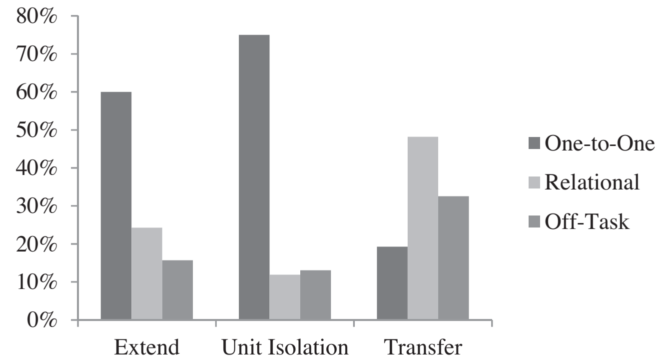


**Fig. 2.** Percentage of errors of each type by task type. *Note*: Because children performed near ceiling on Duplicate tasks (89% correct), these items were not included in error analyses by task type.

($M = 1.77$ trials correct, $SD = 1.27$) and Transfer tasks ($M = 1.74$ trials correct, $SD = 1.22$). Post hoc analyses showed that children were significantly more accurate on Duplicate tasks than on any of the other three tasks ($p$'s < 0.001). They were equally accurate across Extend, Transfer, and Unit Isolation tasks.

*Error analysis*

First, we examined the frequency of three types of errors: one-to-one, relational, and off-task. Five children did not make any errors and were thus excluded from these analyses. The remaining 61 children made errors on 35% of trials. One-to-one errors were most common, accounting for 49% of errors; relational errors were second most common, accounting for 31% of errors; and off-task errors were least common, accounting for 20% of errors. Within the one-to-one error category, the most common specific error type observed was duplicating errors (inappropriately duplicating the entire pattern when asked to show only one unit), which accounted for 44% of one-to-one errors. Overall patterning scores were negatively correlated to the percentage of errors that were off-task, $r(59) = -0.28$, $p = 0.027$, suggesting that very low levels of accuracy were due to failure to execute any strategy rather than inaccurate execution of a one-to-one or relational similarity strategy.

Next, we examined whether the frequency of the types of errors varied as a function of task. Duplicate tasks were not included in these analyses dividing errors into types because children made few errors overall (89% correct); this decision led one additional child, who did not make any errors on the remaining three task types, to be excluded. As illustrated in Fig. 2, the kinds of errors children made did, as expected, differ by task. A series of repeated-measures ANCOVAs, controlling for age and using pattern task as the within-subjects variable and the percentage of one-to-one errors, relational errors, and off-task errors as the dependent variables found an effect of task for all three error categories: relational errors, $F(2, 57) = 7.68$, $p = 0.002$, $d = 0.75$; one-to-one errors, $F(2, 57) = 10.31$, $p < 0.001$, $d = 0.85$; and off-task errors, $F(2, 57) = 3.85$, $p = 0.02$, $d = 0.49$. Post hoc analyses confirmed that children made significantly (all $p$'s < 0.05) more relational errors on Transfer tasks ($M = 0.67$, $SD = 0.13$) than on Extend ($M = 0.28$, $SD = 0.08$) or Unit Isolation ($M = 0.17$, $SD = 0.07$) tasks. Conversely, they made significantly fewer one-to-one errors on Transfer tasks ($M = 0.27$, $SD = 0.07$) than on Extend ($M = 0.70$, $SD = 0.12$) or Unit Isolation ($M = 1.05$, $SD = 0.15$) tasks. There were no significant post hoc comparisons for off-task errors, though children made more off-task errors on Transfer tasks ($M = 0.45$, $SD = 0.12$) than they did on Extend tasks ($M = 0.18$, $SD = 0.08$) at trend-level ($p = 0.06$), suggesting that with a larger sample size this difference may have been significant.

*Memory and inhibitory control*

*Absolute performance*

On the Digit Span – Forward task (verbal short-term memory), children's average raw score was 5.74 ($SD = 1.64$), with a range from 0 to 10. As expected, they averaged fewer correct digits in the Digit Span – Backward condition (verbal working memory), with a mean raw score of 2.33 ($SD = 2.02$) and a range of 0 to 6. On the Corsi Block – Forward task (visuospatial short-term memory), children's average raw score was 3.59 ($SD = 1.52$), with a range from 0 to 7. They performed similarly on the Corsi Blocks – Backward task (visuospatial working memory), but with greater variability, with a mean raw source of 3.10 ($SD = 2.34$, range from 0 to 8). As described in the methods section, the two working memory scores, correlated at $r(64) = 0.40$, $p < 0.001$, were standardized and averaged in order to create a working memory composite. Inhibitory control scores ranged from 0, the lowest possible score, to 52, the highest possible score, with a mean of 37.21 ($SD = 12.11$).

*Relations with patterning accuracy*

Table 1 presents the correlation matrix for all patterning and cognitive measures. The only cognitive measure not correlated with patterning accuracy was verbal short-term memory, which was at trend-level, $r(64) = 0.21$, $p = 0.09$. The pattern of correlations indicated stronger correlations between the measures of visuospatial memory and patterning accuracy than between the measures of verbal memory and accuracy. Using the Meng, Rosenthal, and Rubin (1992) method of comparing dependent correlations, we found that visuospatial short-term memory was more strongly correlated with patterning accuracy than was verbal short-term memory ($z = 2.28$, one-tailed $p = 0.01$).

To determine if individual differences in memory and inhibitory control also predicted individual differences in accuracy, we conducted a series of ordinary least squares (OLS) regressions. As shown in Table 2, together with age, verbal short-term memory, visuospatial short-term memory, working memory, and inhibitory control accounted for nearly half (49%) of the variance in overall accuracy on the patterning assessment, $F = 11.39$, $p < 0.001$, $d = 1.96$. Visuospatial short-term memory was the strongest predictor of accuracy, $\beta = 0.26$, $p < 0.001$, and working memory was significantly related as well, $\beta = 0.22$, $p = 0.03$. Inhibitory control, $\beta = 0.19$, $p = 0.07$, predicted patterning accuracy at trend level. Verbal short-term memory, in contrast, did not account for any unique variance in accuracy.

We also conducted a series of hierarchical regressions to determine the amount of unique variance accounted for by each of the cognitive processes. When entered last into the hierarchical regression, visuospatial short-term memory accounted for the greatest variance above the other processes. The $R^2$ change when visuospatial short-term memory was entered last was 11%, while the $R^2$ changes when working memory, inhibitory control, and verbal short-term memory were entered last were 4%, 3%, and 0.3% respectively.

To better understand which processes might be involved in different kinds of patterning tasks, we conducted separate OLS regression models for each patterning task, with the four cognitive processes as the predictors and accuracies on the four task types as the outcomes. As shown in Table 2, together with age, verbal short-term memory, visuospatial short-term memory, working memory, and inhibitory control predicted accuracy on all four task types. Only visuospatial short-term memory accounted for variance in Extend and Unit Isolation tasks, although working memory was related to accuracy on Unit Isolation tasks at trend level ($\beta = 0.23$, $p = 0.06$); whereas, on the Transfer task, working memory accounted for unique variance, $\beta = 0.27$, $p = 0.03$, while

**Table 1**
Correlations between measures.

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| A. Patterning: overall | 1.00 | | | | | | | | |
| B. Duplicate | 0.55*** | 1.00 | | | | | | | |
| C. Extend | 0.69*** | 0.25* | 1.00 | | | | | | |
| D. Transfer | 0.66*** | 0.24t | 0.22t | 1.00 | | | | | |
| E. Unit isolation | 0.69*** | 0.15 | 0.31* | 0.25* | 1.00 | | | | |
| F. Verbal short-term | 0.21t | -0.09 | 0.19 | 0.11 | 0.26* | 1.00 | | | |
| G. VS short-term | 0.55*** | 0.32** | 0.35** | 0.39*** | 0.38** | 0.09 | 1.00 | | |
| H. Working memory | 0.43*** | 0.18 | 0.14 | 0.39*** | 0.38** | 0.22t | 0.33** | 1.00 | |
| I. Inhibitory control | 0.30*** | 0.09 | 0.21t | 0.22t | 0.26* | 0.41*** | 0.12 | 0.17 | 1.00 |

VS, visuospatial.
t $p<0.10$.
* $p<0.05$.
** $p<0.01$.
*** $p<0.001$.
($df=64$).

visuospatial short-term memory was related only at trend level ($\beta=0.23$, $p=0.07$).

*Relations with patterning errors*

Last, we examined relations between memory and inhibitory control and children's errors using Pearson's bivariate correlations. The pattern of correlations indicated distinct processes related to each kind of error. The number of one-to-one errors was negatively related to visuospatial short-term memory, $r(59)=-0.29$, $p=0.03$, and verbal short-term memory at trend level, $r(59)=-0.25$, $p=0.052$. The number of relational errors children made was related to their visuospatial short-term memory, $r(59)=-0.28$, $p=0.030$ and inhibitory control, $r(59)=-0.36$, $p=0.004$, with working memory showing a negative correlation at trend level, $r(59)=-0.24$, $p=0.06$. The number of off-task errors was related to visuospatial short-term memory, $r(59)=-0.26$, $p=0.046$, and working memory, $r(59)=-0.34$, $p=0.007$. When considering all cognitive processes simultaneously in multivariate regressions and controlling for age, however, the only significant relation remaining was between inhibitory control and the number of relational errors children made, $\beta=-0.41$, $p=0.003$.

## Discussion

Motivated by the argument that early patterning activities support later mathematics performance through their promotion of relational reasoning (Fuchs et al., 2012; Lee et al., 2011; Mason, 1996; Mulligan & Mitchelmore, 2009; Orton & Orton, 1999; Warren & Cooper, 2006; Whitin & Whitin, 2011), this study aimed to better understand the strategies young children use to solve various patterning tasks. The results were consistent with the hypothesis that preschoolers' predominant approach for solving visual repeating patterning tasks is a one-to-one appearance matching strategy, but that they are more likely to use a relational similarity strategy on complex patterning tasks that require the mental representation and manipulation of the repeating unit. In this concluding section, we discuss implications of these findings for understanding what patterning instruction is most likely to support relational reasoning.

*Patterning strategies*

A task analysis of the kinds of visual repeating patterning tasks often used in early childhood programs suggested preschoolers may solve them using two different strategies: a one-to-one appearance matching strategy and a relational similarity strategy. The results suggested that preschoolers do indeed use both types of strategies. Preschoolers in the present study committed some errors on the patterning assessment that would result from using an appearance matching strategy and others that would result from incorrectly executing a relational similarity strategy. Similarly, the cognitive processes that were related to patterning performance suggested use of both strategies: both visuospatial short-term memory and working memory were related to overall accuracy, indicative of an appearance matching strategy and a relational similarity strategy, respectively. It is noteworthy that verbal short-term memory was not related to patterning accuracy, suggesting that children were visually processing information, rather than verbally recoding it (e.g., orally or mentally articulating the words blue, red – blue, red).

An analysis of performance on the different patterning tasks suggested that the strategy preschoolers used varied depending

**Table 2**
Standardized OLS regression coefficients for cognitive processes predicting patterning performance scores, overall and by task type.

| | Overall | Patterning task types | | | |
|---|---|---|---|---|---|
| | Total score | Duplicate | Extend | Unit isolation | Transfer |
| Age | 0.28** | 0.33** | 0.14 | 0.10 | 0.20t |
| Verbal STM | 0.06 | −0.15 | 0.13 | 0.14 | −0.02 |
| Visuospatial STM | 0.36*** | 0.20 | 0.28* | 0.24* | 0.23t |
| Working Memory | 0.22* | 0.08 | −0.02 | 0.23t | 0.27* |
| Inhibitory Control | 0.19t | 0.10 | 0.12 | 0.13 | 0.14 |
| $R^2$ | 0.49 | 0.23 | 0.18 | 0.27 | 0.29 |
| $F$ | 11.33*** | 3.64** | 2.56* | 4.44** | 4.78*** |

*Note*: STM, short-term memory.
t $p<0.10$.
* $p<0.05$.
** $p<0.01$.
*** $p<0.001$.

on the task. The pattern of results indicated that children tended to use a one-to-one appearance matching strategy when asked to duplicate and extend visual repeating patterns. Children were most accurate on these tasks; when they did make errors, however, their errors were those likely to be related to a one-to-one approach (e.g., leaving out or doubling a single block). Further, visuospatial short-term memory was related to accuracy on these tasks but verbal short-term memory, working memory, and inhibitory control were not related, suggesting that minimal mental processing was involved other than simply storing visual information. In contrast, the pattern of results indicated that children used a relational similarity strategy more often on the transfer task. On this task, children were less accurate, which is consistent with previous findings showing preschoolers' difficulty in accurately using relational similarity reasoning (Gentner & Rattermann, 1991; Kotovsky & Gentner, 1996), and were also most likely to make relational errors. In addition, accuracy on the transfer task, but not the other tasks, was predicted by working memory, suggesting that on this task children engaged mental processing beyond superficial appearance matching.

When children seemed to use a relational similarity strategy, preschoolers with better inhibitory control were more likely to execute it accurately than those with poorer inhibitory control: greater inhibitory control was related to fewer relational errors. This finding is consistent with prior research linking inhibitory control to analogical reasoning (Richland & Burchinal, 2013). As with other tasks that involve relational reasoning, it is likely that on patterning tasks children must inhibit the tendency to focus on salient characteristics in order to focus on underlying structure. More specifically in the case of visual repeating patterns, it is possible that inhibitory control helps children execute a relational similarity strategy in two ways. One possibility is that inhibitory control helps children overcome the tendency to produce familiar patterns (e.g., AB-AB) in favor of a more complex pattern (e.g., ABB-ABB) on transfer tasks where they are asked to generate the same pattern using different materials. A second possibility is that it helps children ignore a smaller salient repeating unit in order to identify a larger more complex one in which it is embedded (e.g., the recurrence of AB within a more complex ABB pattern). Future studies that explore the question of how inhibitory control contributes to better execution of a relational similarity strategy in patterning tasks could provide insight into ways to direct children's attention to support their use of this strategy.

Another question raised by the results is why children appear to use the one-to-one appearance matching strategy on some tasks and a relational similarity strategy on others. Similar strategy variability has been found on other mathematics tasks, such as arithmetic. Studies of children's addition strategies indicate, for example, that the same child will sometimes retrieve an answer from memory for some problems and use a counting strategy on others (Siegler, 1987). Problem difficulty and individuals' proficiency in relevant capabilities (e.g., counting skill, working memory) have been found to influence children's strategy choice in arithmetic (Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Kerkman & Siegler, 1993; Shrager & Siegler, 1998; Siegler, 1996).

The present pattern of results suggest that children's choice of which patterning strategy to use is also influenced by problem difficulty and individuals' proficiency in relevant capabilities. Overall, the preschoolers in this study seemed to use an appearance matching strategy more frequently than a relational similarity one. This result makes sense given that an appearance matching strategy was as likely as a relational reasoning strategy to lead to an accurate response on half of the tasks, and that preschoolers are generally more capable at appearance matching than relational reasoning (Gentner & Rattermann, 1991). This finding suggesting that appearance matching was children's predominant strategy is consistent with other research examining preschoolers' understanding of patterns. For example, Rittle-Johnson et al. (2013) found that preschoolers are more likely to refer to visual appearance similarity ("It goes yellow-yellow-blue," often while pointing to each individual pattern element) than to make relational comments ("Yellow is really like blue.") when they are asked to explain how two patterns are related. In addition, the current results suggesting more frequent use of a relational reasoning strategy on the transfer task than on the duplicate and extend tasks could be attributable to the complexity of the task. On this task, a relational similarity strategy, while more difficult for preschoolers, would be more likely than an appearance matching one to generate an accurate response. Thus, the pattern of children's strategy use on the different tasks may reflect adaptive strategy choice in which children try to maximize accuracy while considering efficiency (Shrager & Siegler, 1998).

It is especially striking that children seemed to have adaptively selected a relational similarity strategy on the transfer task given the order in which the tasks were administered. All children completed the patterning tasks in the same order: Duplicate tasks, Extend tasks, Transfer tasks, and finally Unit Isolation tasks. This order was used because it was consistent with prior use of the assessment (Rittle-Johnson et al., 2013) and with the order in which patterning is typically introduced in early childhood programs (Frye et al., 2013; NCTM, 2000). Because the less complex tasks, which could easily and accurately be solved with an appearance matching strategy, were administered first, an appearance matching strategy could have become the fixed response set. Yet, despite this potential cognitive obstacle, it seems that the added demands of the transfer task and the need for a relational similarity strategy to achieve accuracy on it were great enough to prompt a strategy change.

### *Implications for instruction*

Although limited by potential order effects and a lack of direct strategy reports, the present study provides insight into how young children approach patterning activities, which are prevalent in early childhood mathematics curricula. Practice guidelines for early childhood mathematics programs emphasize the importance of instruction about patterns (Frye et al., 2013; NCTM, 2000). Learning about patterns is thought to help lay a foundation for future mathematics instruction (Charles, 2005; Kidd et al., 2013; Sarama & Clements, 2009). Indeed, some evidence indicates that children's ability to solve patterning tasks is positively related to their performance on numerical tasks (Fuchs et al., 2012; Lee et al., 2011). For example, Kidd et al. (2013) found that struggling first-graders who received pattern instruction performed better on an assessment of mathematics concepts at the end of the school year than students in a variety of control groups. An important question relevant to planning pattern instruction that the present results can help address is: which kind of pattern instruction is most likely to have the desired benefits?

The cognitive alignment framework for instructional design proposed and tested by Laski and Siegler (2014) suggests that to address this question it is necessary to consider which kinds of patterning experiences are most likely to elicit the kind of thinking that would support later mathematics learning. According to this theoretical framework, instruction is most effective when (1) the desired mental representation is identified, (2) instructional materials instantiate the key features of the representation, and (3) learning activities promote attention to the key features. If, as it has been posited, pattern instruction benefits later mathematics learning because it promotes understanding of relational reasoning, then instructional tasks that elicit a relational similarity strategy would lead to the greatest benefits.

The present results suggest that not all patterning tasks are comparable in terms of their alignment to relational reasoning. As we predicted, tasks asking children to duplicate and extend patterns seemed to elicit an appearance matching strategy. Thus, instruction focusing on these kinds of tasks might not have the desired benefits to later mathematics learning. In fact, they may even lead children to develop a misconception about patterns – that patterns are simply a collection of individual elements rather than a repeating unit or rule. If this is the case, then the order of the tasks used in the present study may have undermined children's use of a relational similarity strategy and our results may be a conservative measure of children's relational reasoning in patterning tasks. More importantly, if this is the case, then it is also possible that the typical order of instruction also undermines children's use of relational reasoning on patterning tasks because duplicate and extend patterning activities are precisely the kinds of patterning tasks used most frequently in early childhood programs (Fox, 2006).

Theoretically, tasks that require children to identify the core unit seem more closely aligned to the desired goal – understanding the relation between elements in a pattern and the underlying rule. Yet, our results suggest that preschoolers used an appearance matching strategy on the unit isolation task: visuospatial short-term memory was related to greater accuracy and the most common error on the task was exact duplication of the example pattern. The pattern of findings suggested that children did not do any mental analysis of the intrinsic structure of the pattern and the relations among elements to identify the unit, but rather simply relied on visual segmentation of the pattern. It is possible that they did not understand the task or that it was so cognitively demanding for them that they fell back on an appearance matching strategy. A potential implication is that these tasks may not be the best instructional lever for preschoolers. Another is that instruction using these kinds of tasks in preschool classrooms may not promote relational reasoning without explicit didactic statements that direct children's attention and support their reasoning.

On the other hand, as predicted, transfer tasks demonstrated the greatest alignment between the task demands and the desired relational reasoning; they seemed to elicit the greatest use of a relational similarity strategy. The implication is that greater use of these kinds of tasks in early childhood mathematics instruction would be most likely to have the desired benefits of including patterning in the curricula. These types of activities may push children past basic procedural knowledge of patterns as a visual matching task, toward a more conceptual understanding of patterns as relational systems (Rittle-Johnson, Siegler, & Alibali, 2001). Of course, this prediction remains to be tested empirically. Future studies should test what kind of pattern instruction leads to the greatest benefits to relational and numerical reasoning, using both pretest–posttest and longitudinal designs. Such research would lead to a better understanding of the relations among patterning, children's relational reasoning, and later mathematics learning as well as the ideal ways to order and structure patterning instruction to maximize its benefits.

## Conclusion

Early patterning activities are included in early education classrooms with the intention of supporting the development of relational reasoning. The present study found that the extent to which children actually employ relational strategies when completing patterning activities depends on the demands of the task; more specifically, results suggest that only activities requiring children to mentally represent and transform the repeating unit may promote relational reasoning. These results lay the foundation for future research exploring cognitive processes underlying patterning and the longitudinal links between different types of patterning experiences and future mathematics achievement.

## Acknowledgements

The authors would like to thank all participating preschools and children, as well as Kaitlin Mulhall and Jessica Shapiro for their assistance in data collection and coding.

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