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Turbulent Imaging Simulation at DRDC

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Outline

- Theory
- Measurement trial results
- The passive imaging simulator
- The active imaging simulator
- Directions for digital imaging

Theory

Turbulence theory

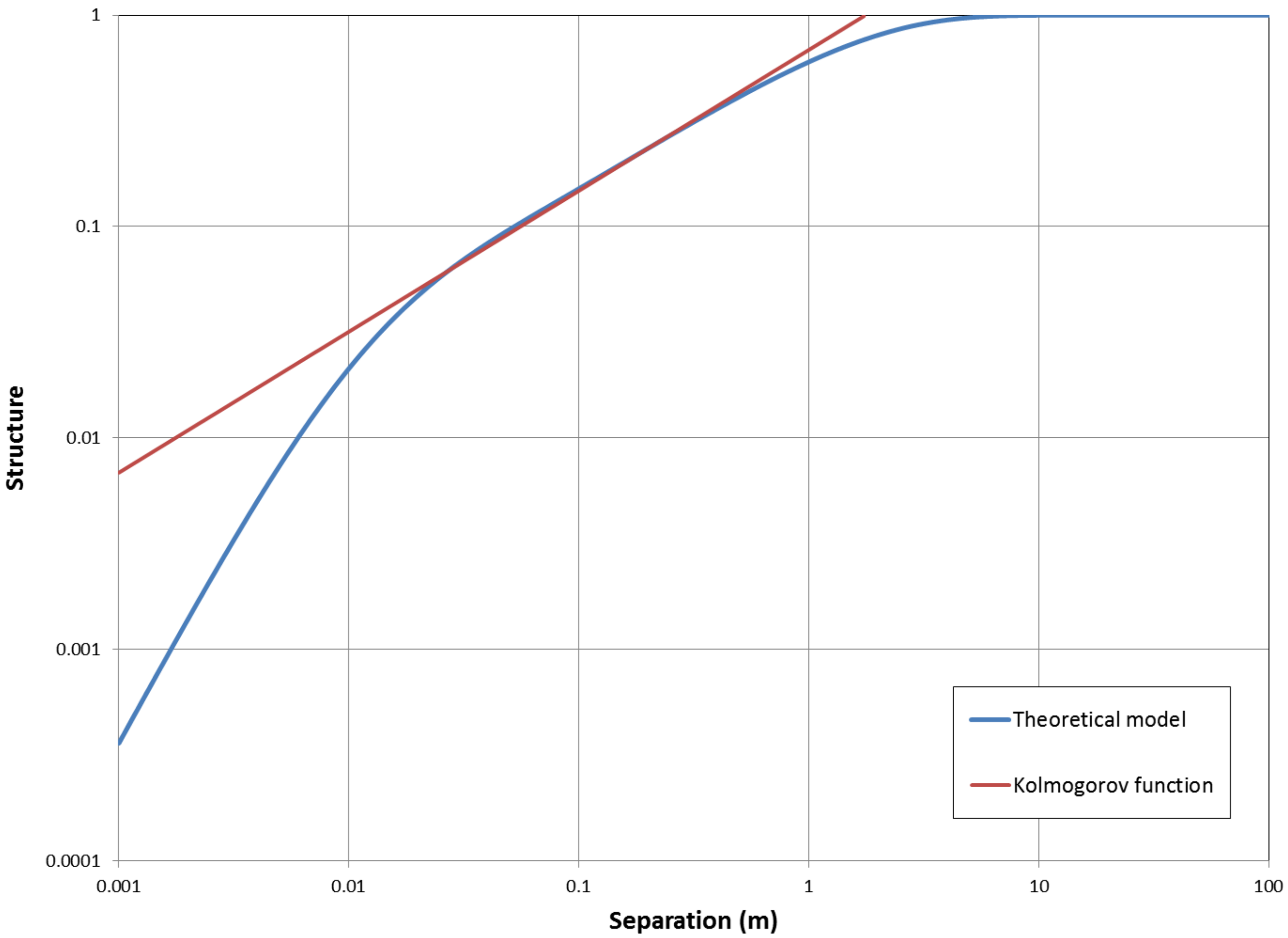
- The index of refraction is a random field, $n(\vec{x},t) = \bar{n}(\vec{x}) + n_1(\vec{x},t)$, where $\langle n_1 \rangle = 0$.

- The structure function is: $D_n^2(l) = \langle \left(n_1\left(\vec{x}+\vec{l},t\right) - n_1(\vec{x},t)\right)^2 \rangle$.

- Using dimensional arguments, we find that $D_n^2(l) = C_n^2 l^{2/3}$ where C_n^2 is the *refractive index structure parameter* and is a measure of the strength of the optical turbulence. It ranges from 10^{-15} to 10^{-13}.

- This result is only valid in the inertial range; $l_o \ll l \ll L_o$, where l_o is the inner scale where dissipation becomes important, and where L_o is the outer scale where the turbulence is generated.

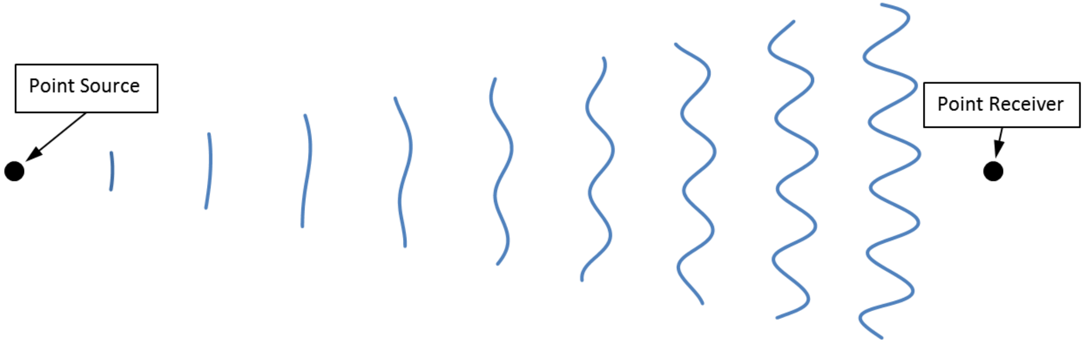
Structure Function with 10 m outer scale, 1 cm inner scale



Propagation theory

- A point source emits a wave through turbulence, which becomes progressively more distorted as it propagates.

- A point receiver measures a field: $E = E_0 \exp[\chi + iS]$, where E_o is the wave without turbulence, χ is the log-amplitude fluctuation and S is the phase fluctuation.



Paraxial propagation

- For visible and IR propagation, the wavelength is much smaller than the inner scale, $\lambda \ll l_o$, which means that depolarization effects are negligible and that the electric field can be modeled as a scalar: $\nabla^2 E - \frac{n^2}{c^2}\frac{\partial^2 E}{\partial t^2} \approx 0$
- Remote sensing imaging systems are well described using the paraxial approximation.
- If the line-of-sight is along the *z*-axis and the turbulent fluctuations are much slower than the period, we can write $E \approx \exp[ik_0 z - i\omega t]u$, where $k_0 = \omega/c$ and the function u obeys the paraxial propagation equation:

$$i\frac{\partial u}{\partial z} + \frac{1}{2k_0}\nabla_\rho^2 u + k_0 n_1 u \approx 0$$

The Rytov approximation

- For weak turbulence, the Rytov approximation is $u = u_0 \exp[\psi_1]$, where $\psi_1 = \chi_1 + iS_1$ is first-order in n_1:

$$\psi_1(\vec{\rho}_o, 0; \vec{\rho}, L) = \frac{k_0 L}{2\pi} \int_0^1 \mathrm{d}\,\eta \int d^2\, q \exp\left[\frac{i}{2} q^2\right] n_1\left(\vec{r}_s + \vec{q}\mu\sqrt{\eta(1-\eta)}, L\eta\right)$$

- where $\mu = \sqrt{L/k_0}$ is the Fresnel zone and $\vec{r}_s = (1-\eta)\vec{\rho}_o + \eta\vec{\rho}$ is the straight line connecting the start and end points.

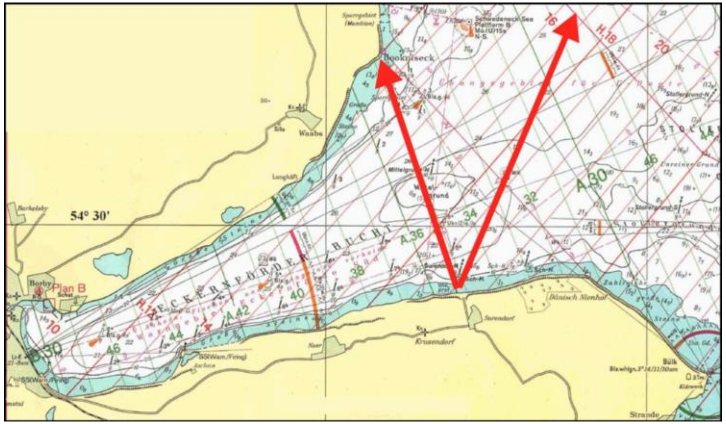
- For a spherical wave the scintillation is $\sigma_\chi^2 = 0.124 k_0^{7/6} L^{11/6} C_n^2$.

- Turbulence is considered weak if $\sigma_\chi^2 \leq 0.3$. In this limit ψ_1 is acceptable.

Measurements

NATO VAMPIRA trial

- The NATO Validation Measurements on Propagation in the Infrared and Radar (VAMPIRA) was conducted across the Eckernforde bay in Northern Germany, from March to April 2004.

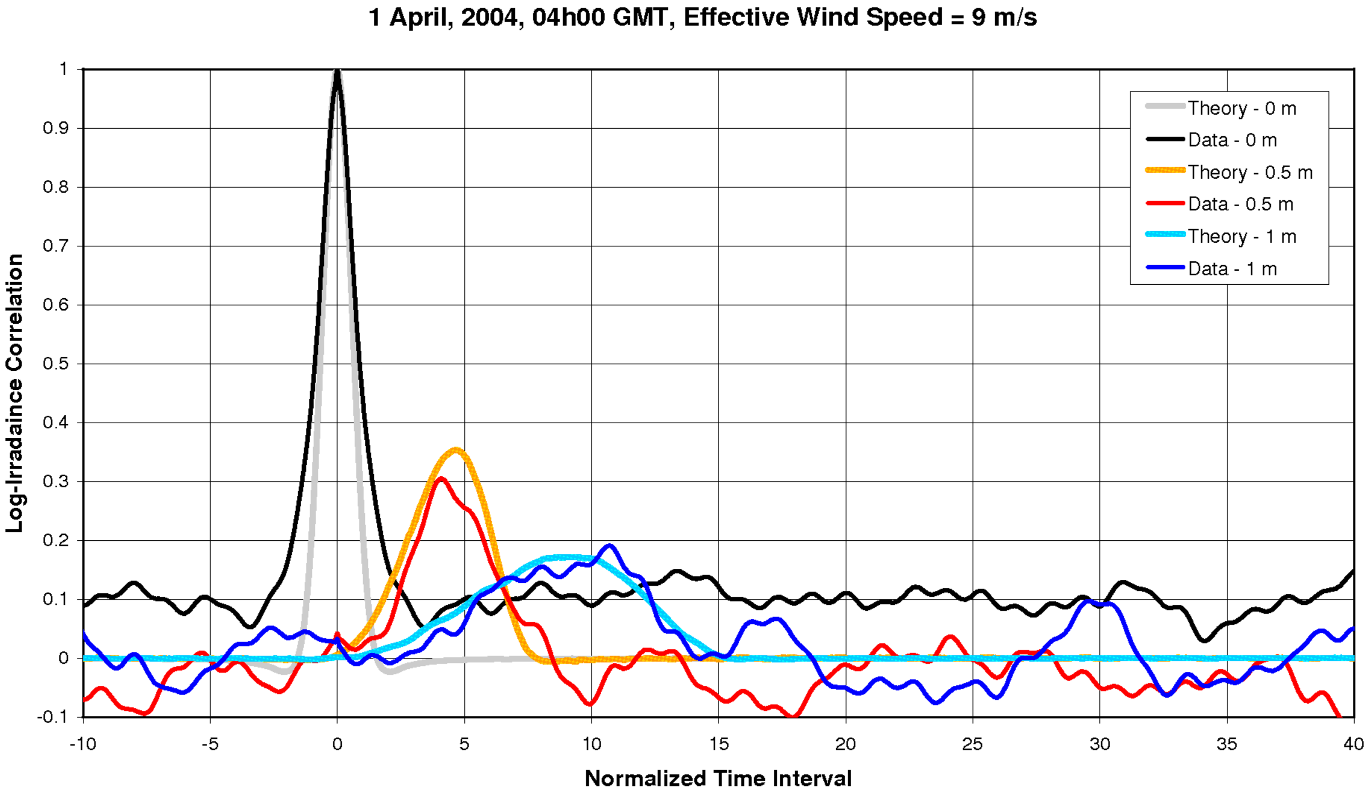


- High-speed digital image sequences were taken of an array of lights on the opposing side of the bay, about 8 km away.

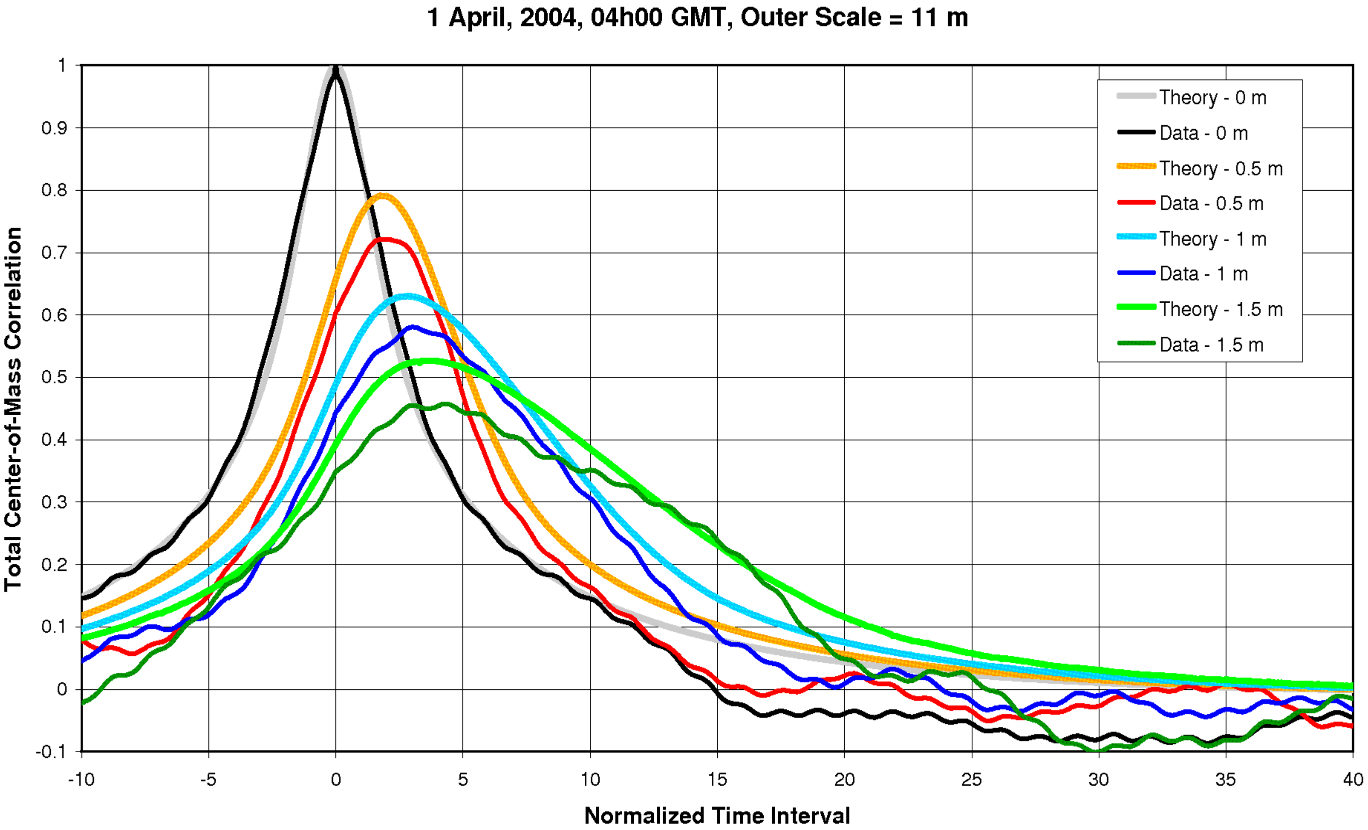
March 31, 2004
23h01 GMT

- A cross-correlation analysis of the lights on a weak turbulence case served as the basis of the passive turbulent imaging simulator (Potvin *et al.*, 2007a).

Data Analysis: NATO VAMPIRA Trial



Data Analysis: NATO VAMPIRA Trial

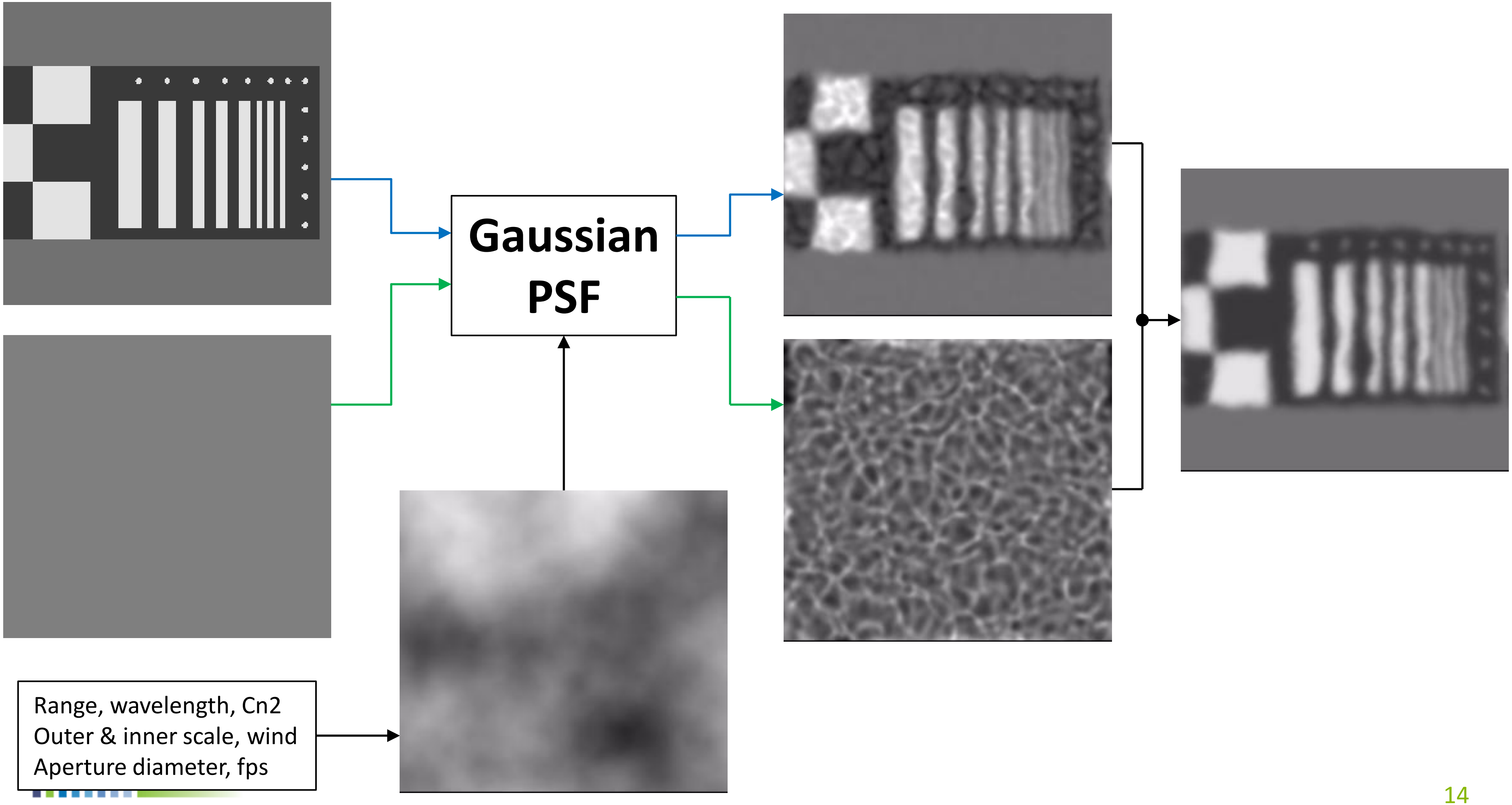


Passive imaging simulator

Optical Turbulence for Imaging Systems Simulator (OTISS)

- OTISS is based the following premises:
  - The optical turbulence is weak.
  - The target is Lambertian (incoherent)
  - The turbulence is ‘frozen’ and is transported by a constant wind.
  - The turbulent PSF has a Gaussian form.
- OTISS produces a turbulent image sequence by:
  - Creating a scalar field that encodes information about range, wavelength, aperture diameter, wind, Cn2, outer scale and inner scale.
  - Deriving the moments of the PSF from gradients of the scalar field.
  - Applying the PSF to the image and to a blank image. This creates a ‘reference’ image that will flatten the bumps in the original image.

OTISS image processing flowchart



Gaussian PSF

Range, wavelength, Cn2
Outer & inner scale, wind
Aperture diameter, fps

Important notes and warnings!

- The turbulent displacements come from the gradient of the scalar field, making them irrotational (Potvin *et al*., 2007b).
- The scintillation comes from the Laplacian of the field, and the variable blurs come from second-order derivatives of the field (Potvin *et al*., 2011).

- As mentioned before, the reference image method is to eliminate bumps in uniform regions of the image.
- This is because, due to ‘energy conservation’ uniform regions in a Lambertian target stay that way when subject to turbulence. Turbulent effects can only be seen around edges and gradients.
- However, this is not necessarily the case for specular or partly specular targets!

Active imaging simulator

Active imaging simulation

- OTISS is undergoing development to simulate active imaging scenarios.

- This means that the *Bidirectional Reflectance Distribution Function* (BRDF), which was conceived in terms of radiometry (i.e. non-interfering rays), must now be expressed in terms of coherent propagating waves.

- We accomplish this using Wigner functions, which transforms the *Mutual Coherence Function* (MCF) of an electric field into a quasi-probability function in phase-space (Potvin, 2018).

- The Wigner function is real and has the correct marginal distributions, but may be negative in places.

The paraxial Wigner function

- It is useful to define a Wigner function for paraxial propagation as:

$$W(\vec{R},\vec{p},z) = \left(\frac{k_0}{2\pi}\right)^2 \int \mathrm{d}^2\Delta \exp[-k_0\vec{p}\cdot\vec{\Delta}]\, u\left(\vec{R}+\frac{\vec{\Delta}}{2},z\right) u^*\left(\vec{R}-\frac{\vec{\Delta}}{2},z\right)$$

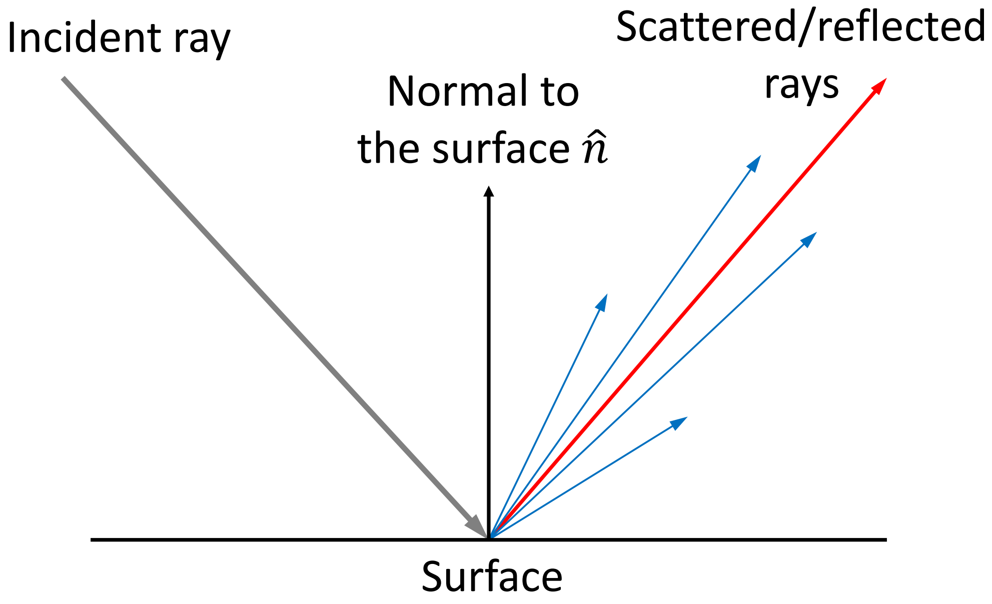
- where $\vec{p}$ is approximately the slope of the ray with respect to the *z*-axis and is much less than unity.
- The paraxial Wigner function approximately obeys the phase-space equation:

$$\frac{\partial W}{\partial z} + \vec{p}\cdot\vec{\nabla}_R W + \vec{\nabla}_R n_1 \cdot \vec{\nabla}_p W \approx 0$$

- It can also be shown that $\int d^2p\, W(\vec{R},\vec{p},z) = \left|u(\vec{R},z)\right|^2$, which represents the energy flux along the *z*-direction.

The paraxial BRDF, part I

- We construct a simple BRDF for paraxial active imaging systems by assuming that the outgoing rays (blue) are scattered in an isotropic way about the reflected ray (red).
- We also assume that the target surface has a small gradient vector, $\vec{g} = \vec{\nabla}_C f$, where $|\vec{g}| \ll 1$.
- Therefore, the slope of the reflected ray is approximately $\vec{p}_0 \approx -\vec{p}_i - 2\vec{g}$.



The paraxial BRDF, part II

- The incident and reflected paraxial Wigner functions relate as:

$$W_r(\vec{R},\vec{p}_r) \approx \int \mathrm{d}^2 p_i \, W_i(\vec{R},\vec{p}_i)\sigma(\vec{R},\vec{p}_i,\vec{g};\vec{p}_r)$$

$$\sigma(\vec{R},\vec{p}_i,\vec{g};\vec{p}_r) = \alpha(\vec{R})\frac{k_0^2 l^2}{2\pi}\exp\left[-\frac{k_0^2 l^2}{2}(\vec{p}_r+\vec{p}_i+2\vec{g})^2\right]$$

- The function α is the surface's reflectivity, and $l(\vec{R})$ is the surface's *coherence length* that describes the diffusiveness of the reflection/scattering process.

- A short coherence length corresponds to a very diffuse surface (i.e. $l \to 0$ indicates a Lambertian surface), while a long coherence length corresponds to a highly specular surface ($l \to \infty$ indicates a mirror).

Modeling the turbulence, part I

- We assume that the turbulence is weak enough to by described by the Rytov approximation, where χ_1 and S_1 are the log-amplitude and phase fluctuations.

- We also assume that the surface is ‘weakly specular’ in that its coherence length l is shorter than the correlation lengths of χ_1 and S_1.

- The incident wave can then be locally approximated as a plane wave over the coherence length.

- This simplifies the transfer between the incident and reflected Wigner functions.

Modeling the turbulence, part II

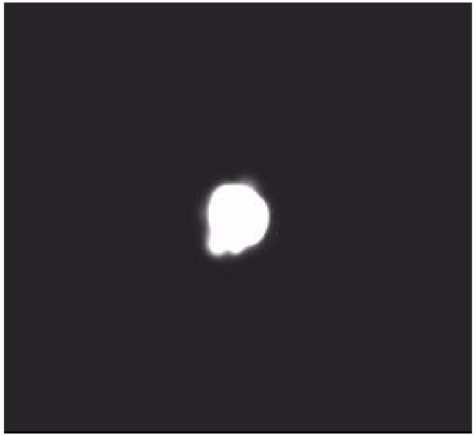
- We now test the active imaging simulation on a gray test panel with a variable coherence length.

Gray test panel, 2.25 X 2.08 m

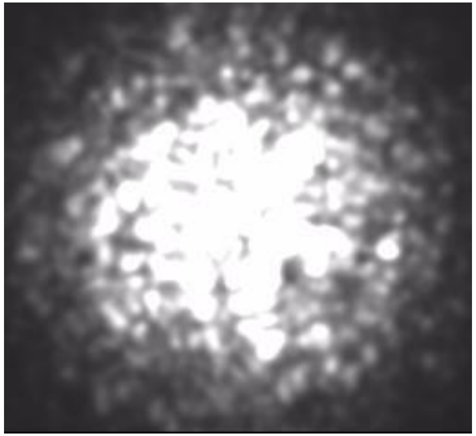
Range	2.3 km
Wavelength	4.2 µm
IFOV	3.83 µrad
Imager aperture diameter	18.36 cm
Cn2	5 e -14 m^(-2/3)
Turbulence Outer scale	10 m
Turbulence Inner scale	6 mm
Frame rate	30 Hz
Wind speed	1.5 m/s

Active imaging model outputs

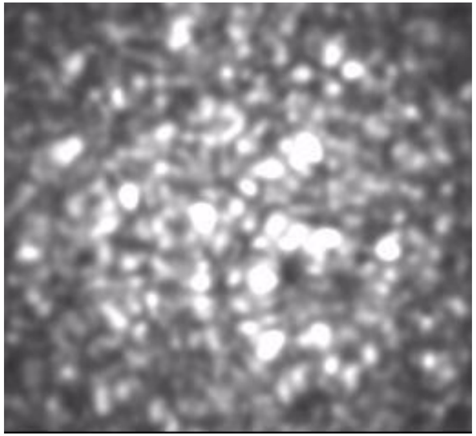
Coherence length:
8 mm



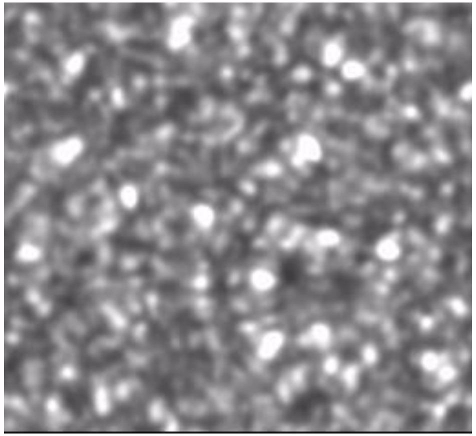
Coherence length:
800 µm



Coherence length:
400 µm



Lambertian



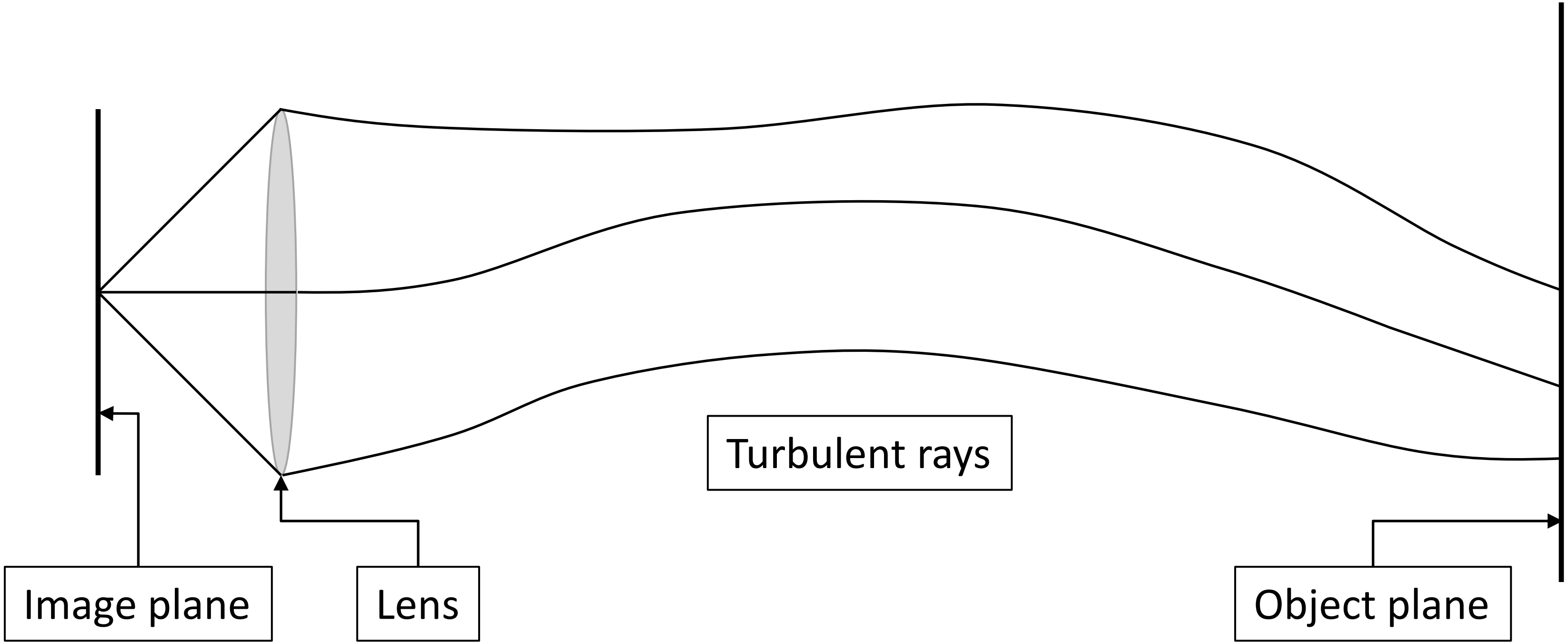
Important notes and warnings! part II

- We have shown how the BRDF can relate to the Wigner function in a physically consistent way.
- We have developed a simple model BRDF suitable for active imaging simulation using the paraxial approximation.
- The model introduces the coherence length of a surface.
- At present, the model is constructed for:
  1. Weak turbulence,
  2. Weak specular target surfaces,
  3. Weakly inclined target surfaces.
- More work is needed to extend the simulator for cases with target surfaces that are highly inclined, specular and seen through moderately strong turbulence.

Directions for turbulence in remote sensing digital imaging

- OTISS is based on fields whereas some digital imaging simulators use rays.
- The two approaches are physically complimentary but numerically can be quite different.
- A ray-based turbulent image simulator was developed by the ARL (Tofsted, 2001; Tofsted & O’Brien, 2004).
- While the ray method is not limited to the weak turbulence regime, it has some problems:
  - The turbulence needs to be spatially smoothed.
  - The finite aperture size must be taken into account.
  - The blur is a constant that must be added after the processing.
  - Bumps must also be removed.

Illustration of ray tracing imaging through turbulence



Possible solutions

- For each point in the image plane, a remote sensing simulator could trace a set of rays originating from various points in the aperture with the corresponding orientations and trace these rays through the turbulence.
  - Computationally demanding

- Rather than tracing rays, it could trace a single beam with a center, a width and a collimation that are affected by the turbulence as it propagates.
  - Demands some kind of parameterization scheme, a method to handle beam splitting

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